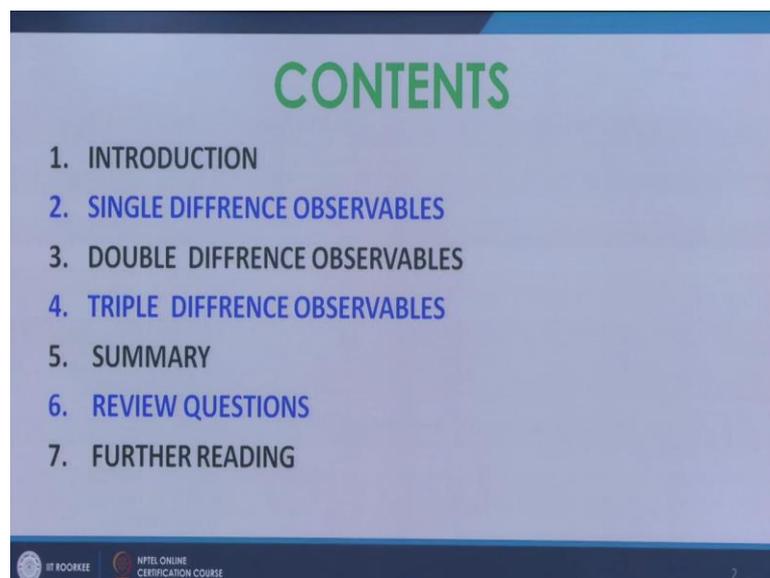


Digital Land Surveying and Mapping (DLS and M)
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Lecture – 12
GPS Data Pre-Processing Differencing

Welcome students to lesson number 12. In which I am going to discuss on GPS data preprocessing. There are different ways how GPS data has to be can be processed among then one of the method is the method of differencing which I will be discussing today. Now the method of differencing will be discussed under the following heads like introduction.

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Then there are 3 types of differencing which we will be discussing. Single difference observables, double difference, triple difference. Then followed by summary of this class now as we have discussed in last classes that GPS observables are plot with errors; that means, there are some in hand errors that comes into the GPS observables. And that reduces the quality of the GPS observable. And as a result if we make use of those GPS observable for GPS positioning the position information we will be associated with some error.

So, in order to reduce the amount of error in GPS positioning, we need to reduce or eliminate the errors associated with the GPS observables. And that can be done by

transforming the GPS data by define taking the different types of differences. So, now actually this differencing can be done in different ways, but we have seen that there are 3 ways how differencing can be taken and really end up with some effective observable which can be used for further processing. Now as we had discussed in the last class GPS observable there are 2 types one is that pseudo range pseudo range between c bar R and satellite i.

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The slide contains the following content:

$$PR_r^i(t) = \rho_r^i(t) + c \delta t_r - c \delta t^i + I_r^i(t) + T_r^i(t) + c dt_r + c dt^i + c dt_r^i + \epsilon_r^i$$

Single Differencing → $\textcircled{1} \frac{2R - IS - It}{S_r - S_{ref}}$

SS-Segment

Differencing

The diagram shows a ground station labeled 'a' and a receiver labeled 'b' connected by a wavy line representing a signal path. The ground station 'a' is marked with a triangle and the receiver 'b' is marked with a circle. The signal path is labeled with 'a' and 'b' at the respective points.

These pseudos range basically consist of the geometric range between the supplied i to receiver i and at any instant of time t. Suppose this is the observable we have in the GPS observation and at any instant of time t or any property this is the value we get which these value we consist of this geometric range plus the receiver clock error, receiver clock error then it will consist the satellite clock error. Then it we will contain the ionospheric error, then tropospheric error. Now here you can see that all these tropospheric error ionospheric error these are dependent on the epoch of observation that mean at each epoch the errors may be different. Then satellite here re receiver hardware error then satellite hardware error then multipath error and random error.

So, this is the expression this is the contents of the pseudo range observable. Already we have discussed in the last class similarly we can also go for the carrier phase observables. So, I will like to take only the example of pseudo range similar expression can be written for pseudo range carrier phase other will also. Now in single some differencing as I told

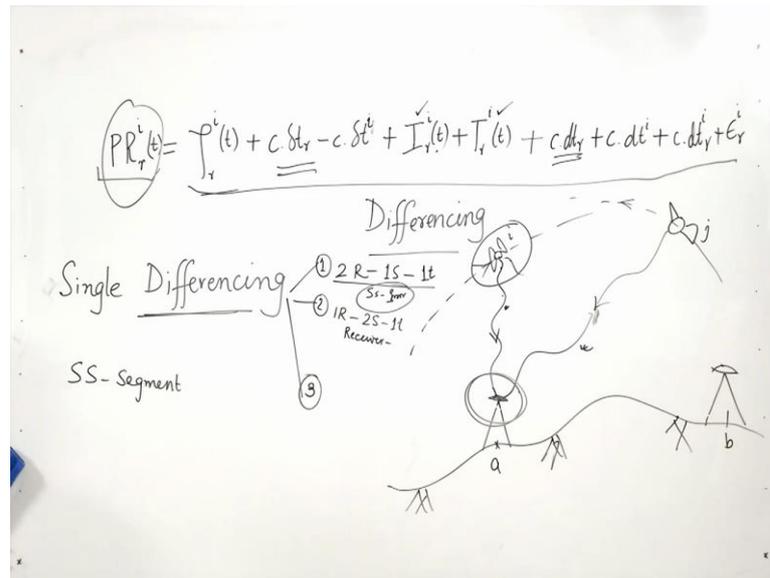
you differencing method we will provide ask the pseudo range free from the many of this errors and there are 3 ways how differencing can be done first one is call single differencing then double differencing then triple differencing, but we start with single differencing there are 3 ways how single differencing can be done.

Now, in the first case we may take the differencing means taking the difference of this observable that is called differencing. Now differencing single differencing can be done in 3 ways. First we may take suppose this is the terrene, where we will we may take observation have number some stations. Some of this is the station an and this is the station b now and the satellite is moving suppose this is the satellite i. So, in case of the first 1, let us take the first case where we will take the difference of observable between the receiver place that a and receiver place that b from the same satellite and at the same instant of time; that means, 2 receiver from one satellite and from the same epoch.

So, what will happen if we take the observable at the same instant of time, then the observable that is coming out from the satellite, we will be having the same error space segment error as I started as we have study in the last class, that the observables I associated with the space segment error atmospheric error receiver error and other error. So, the space segment error we will be identical for this signal and this signal. And if we take the difference of this observable then space segment error; that means, error due to satellite hardware error due to satellite clock a few many is error those error maybe will be eliminated. And also depending upon the distance of this you see where the amount of atmospheric error also will be reused.

Now, the second case instead of taking observable from 2 receiver, if we take the one receiver, but 2 satellite at the same instant of time.

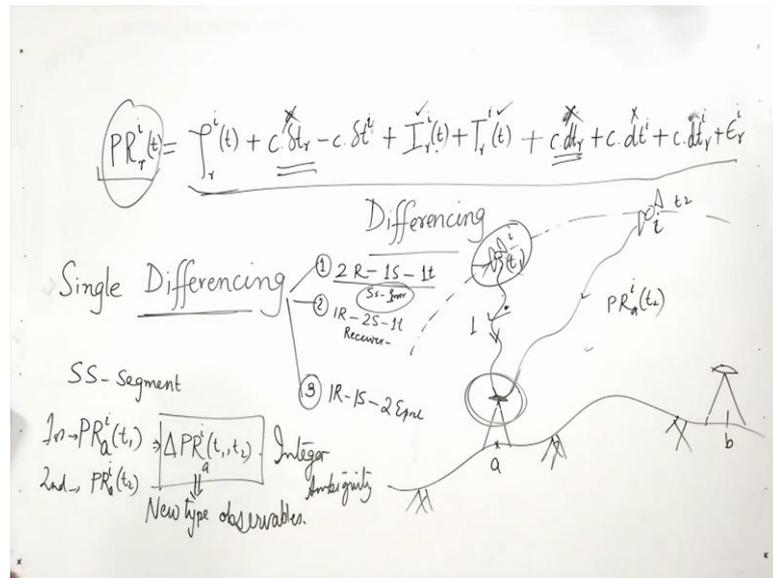
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So, what will happen. So, in this case we are receiving the signal at the same instant of time. So, the error associated with the user segment a specifically the receiver. So, receiver error will be eliminated that error due to satellite receiver clock and other hardware receiver error; that means, this one than this one we will be eliminated as well as if the distance of the satellites are more or less near about then they will the signal will travel from the same atmosphere more or less.

So, the amount of ionospheric error and tropospheric error from in this signal as well as in the signal will be more or less same. So, they will get reduced. So, that is your idea behind this thing. Third one in this case what we will happen instead of taking 2 receiver or 2 satellite.

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What we will take we will take one receiver one satellite, but 2 epoch; that means, they signal that is coming at epoch t 1, and it will travel some distance at time t 2 we will take the signal same say same satellite i. So, the pseudo range from the satellite i to receiver a at t 1 is this signal or signal this is the first signal and this second signal will be pseudo range from satellite i to receiver a at any time t 2.

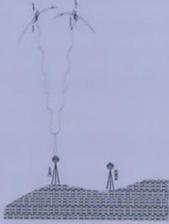
So, in this case as the signal we will be travelling from the from the same satellite. The second will be PRait 2. So, del p pseudo range t 1 and t 2. This is the difference this minus this will be this is the difference. So, that we this type of signal we will be free from integer ambiguity integer ambiguity.

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SINGLE DIFFERENCE OBSERVABLES...

Observables at two different epochs

- difference observables by taking difference of the observables from the same satellite observed at any station but at different epochs of observations.
- satellite as well as receiver clock errors and satellite as well as receiver hardware errors get cancelled
- Other errors also get reduced depending on the epoch of observations and interval
- in case of carrier phase observables, single difference between epochs also helps in eliminating the cycle ambiguities as it does not change with time.



$$PR^s_i(t_1) = \rho^s_i(t_1) + c\delta t_{r_1} - c\delta t_{s_1} + I^s_i(t_1) + T^s_i(t_1) + dt_{r_1} + dt^s + d^s_{r_1} + e^s_i(t_1)$$

$$PR^s_i(t_2) = \rho^s_i(t_2) + c\delta t_{r_2} - c\delta t_{s_2} + I^s_i(t_2) + T^s_i(t_2) + dt_{r_2} + \delta t^s + d^s_{r_2} + e^s_i(t_2)$$

$$\Delta PR^s_i(t_1, t_2) = PR^s_i(t_2) - PR^s_i(t_1) = [\rho^s_i(t_2) - \rho^s_i(t_1)] + [I^s_i(t_2) - I^s_i(t_1)] + [T^s_i(t_2) - T^s_i(t_1)] + [dt_{r_2} - dt_{r_1}] + [d^s_{r_2} - d^s_{r_1}] + [e^s_i(t_2) - e^s_i(t_1)]$$



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So, here it is written difference observables by taking difference of the observable from the same satellite absorbed at any station as I written here shown here, but at difference epoch of observation t_1 and t_2 . Satellite as well as receiver clock error because it is coming from the same satellite as well as to the same receiver the satellite receiver clock error and satellites as well as receiver hardware error get cancel.

So, this will get cancelled, then this will cancel, this will we getting cancel, this will be getting now not this one, but this will get cancel, this well get cancel. So, and in case of carrier phase observable integer ambiguity will be removed. So, this is what in short about the method of single difference. By taking these differencing we will get a new type of observables, and these observables are gone as single difference observables. So, by single differencing we get 3 types of observables.

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$$PR_r^i(t) = P_r^i(t) + c \cdot \overset{\times}{\delta t_r} - c \cdot \delta t^i + I_r^i(t) + T_r^i(t) + c \cdot \overset{\times}{\delta t_r} + c \cdot \overset{\times}{\delta t^i} + c \cdot \overset{\times}{\delta t_r} + \epsilon_r^i$$

Differencing

Single Differencing

- ① 2R-1S-1t $\rightarrow \Delta PR_{ab}^i(t) = PR_a^i(t) - PR_b^i(t)$
St. Sm
- ② 1R-2S-1t Receiver $\rightarrow \Delta PR_a^{ij}(t) = PR_a^j(t) - PR_a^i(t)$
- ③ 1R-1S-2t $\rightarrow \Delta PR_a^i(t_1, t_2) = PR_a^i(t_1) - PR_a^i(t_2)$

We will get single difference between 2 receivers; that means, a and b from the same satellite at the same instant of satellite. So, this is the new observable we will be able to get that is what it is called transformation and in these case single receiver, but from 2 satellites.

So, here I can say the difference t minus difference pseudo range i p t. And in these case pseudo range at than t minus pseudo range from the to the receiver a from satellite i to from j. So, now, in the third case the so, it is the difference it is the difference pseudo range difference t 1 pseudo range to from satellite i to receiver a at time t 2. So, these by taking difference you can say there are different ways how we have taken the difference. So in these case we have taken the difference at the same instant of time from the same satellite by 2 receiver.

In these case we have taken the observables to the same receiver, but at from 2 different satellites at the same instant of time and in these case observable we have taken the difference of observable to these same receiver from the same satellite, but at 2 instant of time. And by using these way we can reduce we can remark we can remark some of the others and we can reduce some other errors; however, in because the multi path error depends on the geometry of the satellites as well as the configuration of receiver with respect to adjoining objects. So, in no cases we can avoid the multi path error. So, after these another method of differencing we can arrive at that is call double differencing.

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DOUBLE DIFFERENCE OBSERVABLES

DOUBLE DIFFERENCE OBSERVABLES

- Observables from differences between two single differences
- Two broad categories of double differencing.
- First category using single differenced observables of two different stations from two different satellites. There may be two types.
 - First type consists of taking differences of the single differenced observables obtained from a satellite observed simultaneously by a pair of receivers at any epoch of observation;
 - Second type, differences may be considered between the single differenced observables obtained from observing same pair of satellites simultaneously by a receiver at same epochs of observation
- Double difference observables are free from satellite and receiver clock errors
- Other errors in observables get reduced.
- Double difference observables are mostly used for further processing to obtain desired parameters and to carry out ambiguity resolution.



$$\Delta PR_{ab}^i(t) = \Delta PR_{ab}^i(t) - \Delta PR_{ob}^j(t)$$

$$\Delta PR_{ab}^i(t) = \Delta PR_{ab}^i(t) - \Delta PR_{ab}^j(t)$$



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Now, I will explain about the second type of differencing observable that is the double difference observable.

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Double Differencing Observables

①	$\Delta PR_{ab}^i(t)$	$\left\{ \begin{array}{l} 2R-2S-1t \leftarrow 2R-1S-1t \\ 2R-2S-1t \leftarrow 1R-2S-1t \end{array} \right.$
②	$\Delta PR_{ab}^i(t_1, t_2)$	$\left\{ \begin{array}{l} 2R-1S-2t \leftarrow 1R-1S-2t \end{array} \right.$

Single Differencing Observables

$\Delta PR_{ab}^i(t) = PR_a^i(t) - PR_b^i(t)$
$\Delta PR_a^i(t) = PR_a^i(t) - PR_a^j(t)$
$\Delta PR_a^i(t_1, t_2) = PR_a^i(t_1) - PR_a^i(t_2)$

Now, double difference observable double differencing observables.

Actually double differencing observables are the difference of the single difference observable. Now in in this case we have taken 2 receiver one satellite in a single epoch of time. In these case we have taken single receiver of observation from 2 satellites at the same instant of time. And in these case we have taken one set this one a station one

satellite 2 instant of time. Now in the in case of double differencing actually we will be taking the difference of this or this or this. So, now, if we take the difference of this. So, it will lead to 2 receiver 2 satellite one single instant of time or here also we will get 2 receiver 2 satellite one instant of time.

So, we can see here that if we take the difference of this for 2 satellites, and if we take the difference of this for 2 receiver ultimately we will end of with the same type like del difference abijt. So, whether we take the difference of these type of single observable or these type of single observable, we will end up with the same double difference observable. And if we take this one like one, $R_2 t$ and $2 R$ ones. So, we will take these for 2 receiver. So, we will end up with the double difference observable $a b i t 1 t 2$. So, what I want it to say is that for by single differencing actually we will be end up with 3 types of observables was are whereas, with the double differencing we will end up with the 2 types of double difference observables.

So, now if we see in these figure we see here that 2 broad categories of double differencing observables from differences between 2 single differences, we will end up with the double difference observable and 2 braid categories of double differencing. So, this is the way because if we take the difference of this for 2 satellites or the difference of these for 2 receiver you will end up with the same value or same durable of error. So, it is this same type of observable. This is one category and another category is that we are taking these single difference observable which is taken for a single receiver and we are taken if we take for the another receiver for the same satellite for the 2, 2 epoch of time we will end of with this double difference observable and this is the second category.

Now, in the first category consist of taking differences of single difference observable obtain from a satellite obtain from a satellite, observe simultaneously by a pair of receiver. So, from the same satellite we are taking a pair of satellite pair of receiver and then from the another satellite for the same pair of satellites for from the same pair of receiver and taking the difference of that we will give the double difference observable the first type consist of taking the difference of single difference observable obtained from a satellite observe simultaneously from a pair of receiver at any epoch of observation. So, from a/. So, one satellite. So, difference of this and the second type differences may be cons. So, they though this is the same out output, but we can error of

these from 2 different parts one by making use of these another by taking use of these that is what it is written under first type and second type.

Under first category first category of double difference can be obtained from this first type of single difference of this first steps single difference of difference of this second type of single difference now double difference are free from as a result of this the double difference will be free from satellite as well as receiver clock errors. So, this is the another error get reduced. So, this is the first category of the double difference observable in the second category as I have already told we will be taking the difference of the first single difference observable which has been obtained from a receiver from a sun satellite, but at 2 instant of time. So, we will take first a receiver a then another receiver b and then difference of that. So, we have taken like this.

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DOUBLE DIFFERENCE OBSERVABLES....

DOUBLE DIFFERENCE OBSERVABLES

- Second category single differenced observables of two different stations from the same satellite but at two different epochs.
- Free from integer cycle ambiguities
- Residual effects of receiver clock error due to time tag bias on the computation of the range term need to be considered if the separation between stations is large.
- Multipath error, a function of satellite-receiver geometry, does not get cancelled in the double-difference observables and noise gets increased with each DD operation.
- Systematic effects due to unmodeled atmospheric errors are generally increased as compared to the SD.
- DD is the most popular differencing technique for relative positioning during GPS surveying.

$$\Delta PR_{ab}^i(t_1, t_2) = \Delta PR_a^i(t_1, t_2) - \Delta PR_b^i(t_1, t_2)$$

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So, we are taking here or from the same satellite to the 2 different receiver, but to pack up time. So, that is the second category single difference observable at 2 different stations from the same satellite from the same satellite from the same satellite at 2 different stations, but at 2 instant of time. So, t 1 during t 1 the satellite I was here during t 2 the satellite I was here during t 2 the satellite I was here and the observables from that we will be taking from this is your and this is your and then we will take the difference. So, as a result of this because we are using the same satellites the integer ambiguity will be removed. So, the differencing will provide us; that means, for the carrier phase

observable, if we look take into consideration the career phase observable then the integer ambiguity will be nullified.

And the residual effects for receiver clock error due to time lack time tack bias on the competition of the range term need to be considered if the separation between the station is large. Now if the separation is large then we have to we (Refer Time: 22:58) causes about doing this thing the multi path error which is a function of satellite and receiver geometry now you can see this satellite and receiver geometry will be different if I go from here to here does not get cancelled in the double differencing observable noise get increased with each double difference operation. So, this is another thing which just to be marked because the geometry will be changed. So, the multi path error will not be cancelled systematic error due to un modeled un modeled atmospheric error are generally increased as compare to single difference. Another thing is there; however, double difference is the most popular differencing technique for LED positioning during GPS surveying and most of the GPS positioning we do based upon the double differencing method. So, double differencing observables are very much important for our GPS surveying, and we make use of double differencing observable for our GPS positioning and for precious positioning specifically we should consider the double differencing observable. So, that is double differencing observable and the third and the another kind which is also available that is call triple differencing observable.

(Refer Slide Time: 26:45)

The image shows a handwritten derivation on a whiteboard. At the top, the receiver position vector is given as $PR_r^i(t) = P_r^i(t) + c \cdot dt_r - c \cdot dt^i + I_r^i(t) + T_r^i(t) + c \cdot dt_r + c \cdot dt^i + c \cdot dt_r + \epsilon_r$. A horizontal line is drawn under the entire expression. Below the line, the word "Differencing" is written. Underneath that, "Triple Differencing" is written above the number "3", and "(Observables)" is written below the "3". An arrow points to the right, leading to a boxed expression $2R - 2S - 2t$. Below the boxed expression, "(One)" is written. A downward arrow points from the boxed expression to the text "Difference of double difference".

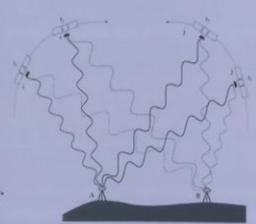
Now, triple differencing observable from the name it is (Refer Time: 24:37) you can on triple difference the difference of difference of double differences double differences.

(Refer Slide Time: 27:32).

TRIPLE DIFFERENCE OBSERVABLES

TRIPLE DIFFERENCE OBSERVABLES

- *involves differencing of two double differences at two different epochs
- *only one type of triple difference observables.
- *Obtained from TWO receivers from same TWO satellites at the same TWO epochs.
- *errors eliminated in single- and double differences thus get eliminated also during triple differencing.
- *carrier phase pseudo-ranges, integer ambiguity gets cancelled
- *cycle slips get mapped as individual outliers thus helps in their elimination during further processing.
- *introduces correlations between observations and reduces the data weights.
- *triple difference observables lose some geometrical strength and thus, lead to degraded precision.
- *primarily useful as preprocessing technique to get good approximate positions for further processing of DD observables.



$$\Delta PR_{ij}^k(t_1, t_2) = \Delta PR_{ij}^k(t_2) - \Delta PR_{ij}^k(t_1)$$



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So, in this triple differences observable we have to take 2 receiver, 2 satellite and 2 epoch of time. So, only one variety only one variety we will get 1 variety observable we get. So, in the triple difference is like this we may (Refer Time: 25:22) triple differences show define channel because first we can take one R 2 s 2 t 1 t for single difference.

(Refer Slide Time: 27:41)

$$PR_r^i(t) = \underbrace{P_r^i(t) + c \cdot \Delta t_r - c \cdot \Delta t^i + I_r^i(t) + I_r^i(t) + c \cdot \Delta t_r^i + c \cdot \Delta t^i + c \cdot \Delta t_r^i + \epsilon_r}_{\text{Differencing}}$$

Triple Differencing (Observables) \Rightarrow (One) $2R - 2S - 2t$

$\begin{matrix} 1R - 2S - 1t & \xrightarrow{2} & 2R - 2S - 1t & \rightarrow & 2R - 2S - 2t \\ 2R - 1S - 1t & & & & \\ 1R - 1S - 2t & \rightarrow & 2R - 1S - 2t & & \\ & & & & 1R - 2S - 2t \end{matrix}$

Difference of double differences

Then we can go for 2 R 2 s 1 t and then we can go for 2 R 2 s 2 t. Or we can go for 2 s one actually 2 R one s one t difference of this then we can go for 2 R 2 s one t.

So, one I may go from here to here or I may go from here. So, I will be reaching to this same these are the same. So, and then we know we have 1 R 1 s 2 t this is the single defines. So, from here I can go for 2 R 1 s 2 t or we may go for 1 R 2 s 2 t. So, this is the way how we can get double difference. So, you will see and then from there we may go for to either way we can react with this. So, triple difference only one type of observable we can get now these triple observables are actually as we have seen here a lot of reduction in data will be there. So, the geometric a stability of the satellites which we define by dob value we will be much decorated.

So, generally we do not go for we make use of 3 d triple difference observable for identifying the cycle slips. And as a preprocessing for the GPS available because thought triple differences also we will be able to find out what are the GPS observables contains some errors or mistakes. So, if we can identify those observables will be taken out and later we will make use of double differencing of the observables for our further GPS processing. So, that is while about in this way we can reduce the amount of if we can eliminate many of the errors we can reduce many of the errors.

And we can clear our we can improve the quality of our GPS available then that is all about todays class now let me summarize that GPS observables are for to it errors. And the quality of GPS observations or the GPS point positioning will depend upon the quality of the GPS observables; that means, we want need to eliminate or reduce the errors as much as possible one of the simplest way how this can be done is the method of differencing; that means, by taking the differences of GPS observable we may generate new types of observables and there are 3 ways we can generated single difference observables double difference observables and triple difference observables.

Of this the double difference observables are most useful for GPS data positioning; however, the triple difference observables helps us to identify the presence or absence of cycle slip error as well as the mistakes or gross error, and as soon as those are being identified through triple difference observables we can take out those observables and the existing observables we may make use of double difference observable for further processing to find out the GPS position. With these I like to conclude todays class. And

the next class I will like to take on GPS point positioning in which I will like to mathematically deduce, how the position of GPS receiver is being determined making use of GPS observables.

Thank you.