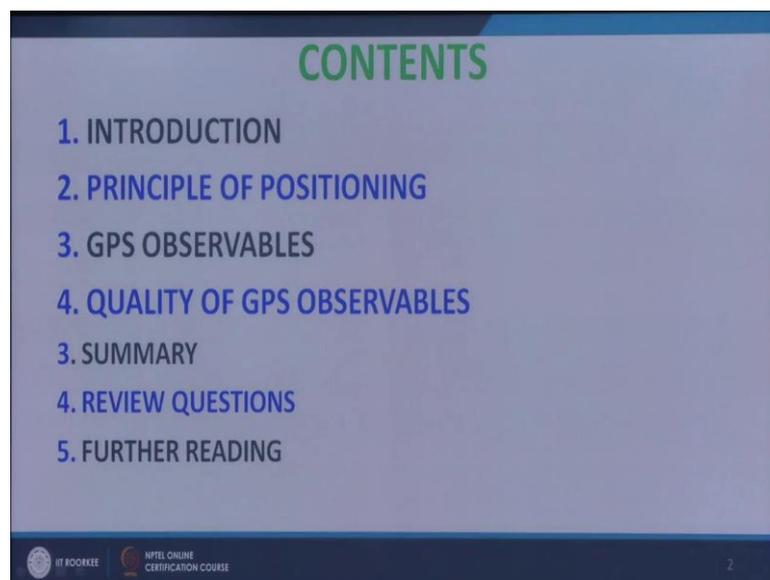


Digital Land Surveying and Mapping (DLS&M)
Dr. Jayanta Kumar Ghosh
Department of Civil Engineering
Indian Institute of Technology, Roorkee

Lecture – 10
Principle of GPS Positioning and GPS Observables

Welcome students, today I am going to talk on lesson number 10; in this lesson I will discuss the principle of GPS positioning; that means, how really the position get determined using the GPS satellites and GPS receiver and next I will like to discuss on really what are the observables or the what are the parameters we really look for from GPS observation.

(Refer Slide Time: 01:07)

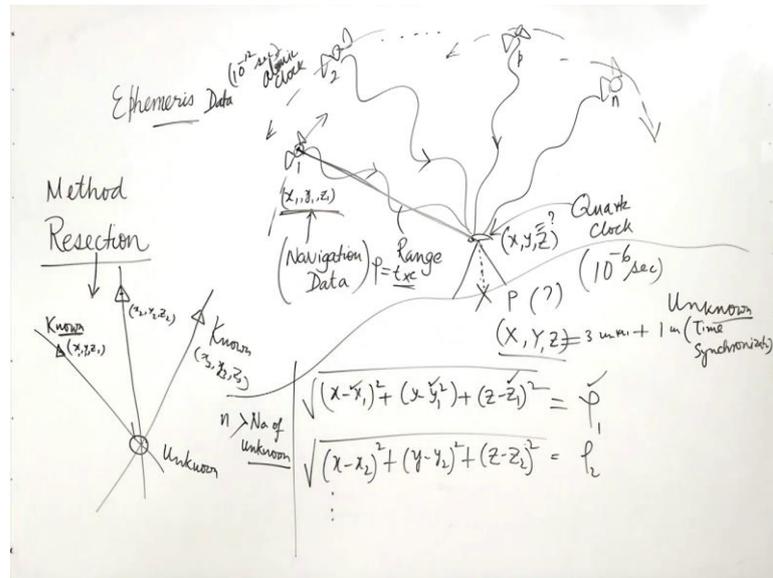


Now this class that principle of GPS positioning and GPS observables we will be discussed under following heads; first I will like to tell you about GPS positioning and observables then principle of positioning; that means, how we do determine the position of any point or location using GPS satellite and what are the observables really we look for in GPS observations and how the quality of GPS observable ascertained followed by summary review question and further reading

Now, as we all know from the beginning that we do carryout GPS surveying or we do make use of GPS to find out the position of any point and that position we find out by taking observation from GPS satellites to GPS receiver and that data we do process

further to find out the location of the point. Now in GPS this determination of the location of points is done by the method known as resection on more or we say it more precisely the analytical resection; now what is that method?

(Refer Slide Time: 02:39)



Now, suppose this is the surface of the earth and this is the point P whose location we want to determine. So, what we will do in case of GPS surveying we will setup a GPS receiver with centering on this point and leveling and after making it on this receiver we will receive signals from GPS satellites. So, as I told you that we need at least 4 satellites data from four satellites, so and there may be other satellites.

So, say GPS signals from at least 4 satellites suppose 1, 2, P n; that means, these are the n is the number of satellites. So, the signals are coming from there and the in surveying you have learnt you must have learnt or those who have started surveying they know this term resection and what is this method of resection; in method of resection what we do or from in the method of resection, we do determine the unknown position from known position. So, if we know the position of two objects from there if we draw a line the intersection of these two line will give the position of this point.

So, from known point to unknown point location can be determined, so in case of planimetric position we need two objects in case of three dimensional object we know we need to have three objects. So, and then intersection of these three objects will give you the 3D location of the point. Now, in case of GPS if we correlate these thing then

also we should be saying if there are 3 satellites; we should be able to find out the location of this point because the position is determined by 3 coordinates, but we need 4 satellites at least in case of GPS, we will be explain I will be explaining it next why it is.

Anyway, so in case of method of resection which; that means, we need to know the location of this point means we need to know the coordinates of this point $x_2, y_2, z_2; x_3, y_3, z_3$ and we are drawing this line that is the graphical method. Now in case of GPS we know the location of the satellites x_1, y_1, z_1 this location of the satellites are available these are available in the navigational data. So, we have learnt in the last our class some other class that from satellites we do take the, we do get the GPS observation and in that GPS observation we do have the navigation data and nav data or c nav, l nav. So, in that data actually we have the ephemeris information; ephemeris data inside the GPS nav data and this ephemeris data provides us the location of GPS at an instant of time.

Now, if we this information is known and these distance which is called range; this distance we get from the satellite observation by measurement of time, the time of travel from this point to this point if we can compute that is available in the GPS data and time multiplied by the velocity of propagation of this signal which is the electromagnetic signal, so it is the velocity of light very approximately if we say; so from this; that means, now we need to know the location of this point. So, suppose x, y, z are the location of this point which we do not know, so if we apply the distance equation. So, we can say x minus x_1 whole square, y minus y_1 whole square, z minus z_1 whole square square root is equal to this range.

Now, here in this equation this x_1, y_1, z_1 and ρ these are known there are three unknowns. Now if we take signals from 3 satellites x_2 whole square, y minus y_2 whole square, z minus z_2 whole square, ρ_2 . So, in that way we can get n numbers of equation from if we have signals from n satellites and it should be number of n should be greater than number of unknown. Now in case of GPS it is the X, Y, Z ; that means, three unknown plus one unknown is due to time synchronization; what is this (Refer Time: 09:53); that means, one more unknown.

Actually in GPS we use atomic clock in satellite and we use quartz clock, quartz clock in receiver and quartz clock is of the order of minus precision of the order of 10 to the

power minus 6 and atomic clock is maybe of the order of 10 to the power minus 12, 7. So, we can see there is a high mismatch in synchronization; as a result there will be a large error in time measurement or in due to synchronization and the error is so big that we considered this as a unknown.

Whenever in unknown and that unknown is associated with the particular receiver. So, actually we need to have 4 satellites at least to have that error due to time, so we need to have at least 4 satellites in GPS position determination and it is the solution of this equation provides us the location of the receiver and also the error synchronization error. Now this GPS observation, this is the geometric range; geometric range means the actual distance between the; so this is what is analytical resection method and that method is being adopted for GPS to find out the position of unknown points.

(Refer Slide Time: 11:58)

PRINCIPLE OF GPS POSITIONING

- Unknown position determined from known distances of three pre well-defined positions.
- Three known positions and their three known distances from the unknown position (represented by three unknown coordinates) provide three distance equations. Solution of these three equations provides location of unknown position.
- In GPS, satellites positions and their ranges (distances of the satellites from the receiver) are available in their respective signals.
- From these data, unknown position of the receiver get computed applying the method of resection.
- Clock synchronization error is considered as fourth unknown thus need signal from at least four satellites.

IIT ROORKEE
 NIEL ONLINE CERTIFICATION COURSE

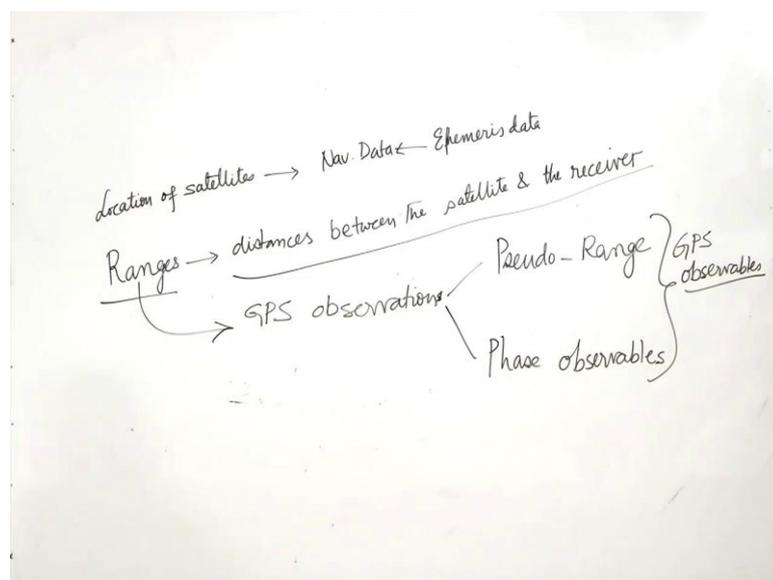
4

So, that is what has been shown here in this figure you can see that this is a point $x_r; y_r; z_r$ this is a receiver position which we want to determine and this at this location a receiver is being placed which is receiving signal from satellite s_1, s_2, s_i, s_n ; that means, n satellites and corresponding to each superscript we have assumed the location of this satellite as like this and now as I told you then r_1, r_2, r_3 are the geometric range, so from this $x_r; y_r; z_r$ is unknown and this is known the location of this satellites are known and these are available in the GPS data, navigational data or ephemeris data and the range can be computed by finding out the time of travel of the signal from satellite to

receiver; this is what is being computed from the GPS code information as well as phase information and if the time is can be calculated then the velocity of propagation we know electromagnetic signal is the velocity of light. So, from that we can get these ranges and if we know the ranges then that equation distance equation can be solved and by solving this equation we can get the location of the receiver.

So, as I told you clock synchronization error is considered as a fourth (Refer Time: 13:48) because as I told you that the clock that is used in the satellite is atomic clock and the having the precision of the order of 10 to the minus 12, 10 to the minus 11, 10 to the minus 13 depending upon different type of atomic clock and the clock that we use in GPS receiver is generally quartz clock. Now one question may arise why we do use such different type of clock, the reason behind is that atomic clocks are costs huge; if we use the same type of clock in the receiver we will not able to buy their the receiver. So, to reduce the cost of the receiver, we use a very chip in comparison to atomic clock; so that is the quartz clock we use. So, there is one more unknown (Refer Time: 14:51) and that unknown is the clock synchronization error which we take as an another known and we resolve that by taking data from one more satellite. So, an incidentally whenever we go for positioning, it is the time synchronization error see that there is a mistake here actually this bracket will be coming here, so like this anyway. So, in this way we do determine the position of unknown point by method of analytical resection

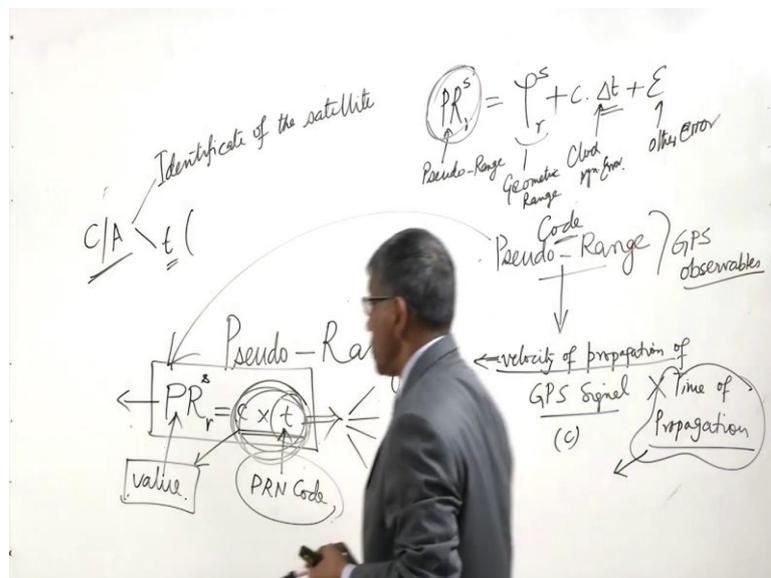
(Refer Slide Time: 15:39)



So, now you have seen that to find out the unknown position using GPS we need to have the location of satellites and these are available in the nav data navigation data or within the nav data it is the ephemeris data which provides us the location of this satellite and we need to have ranges; that means, the distances between the satellite and the receiver. Now these ranges which is the distances between the satellites this is also available in GPS observations. Now these ranges are available in two ways one is called pseudo range and the other is called phase observables and this two constitutes the GPS observables. So, GPS observables are nothing, but these code pseudo range or pseudo range this is also called code pseudo range or pseudo range and the phase observable.

Now, I will like to discuss on code pseudo range as I told you that the pseudo range or range we is determined from the relation velocity of propagation of GPS signal multiplied by time of propagation; that means, time of propagation means the time taken by the signal to come from the satellite to a receiver.

(Refer Slide Time: 17:47)



Now, as we know that already I have told you that there is a large synchronization error in GPS clock and the quartz clock in the receiver. So, instead of there is a mistakes; there is a systematic time error in the time of propagation. So instead of range; range mean geometric range the actual distance we do get a pseudo range means the a range associated with error. So, it is called pseudo range the range the actual distance between the satellite and the receiver plus some distance due to error in synchronization of time.

In fact, we ideally we say that the velocity of propagation of GPS signal is the velocity of light. Now velocity of light the assumption is that when the signal will travel in (Refer Time: 19:53) but as you know that when the signal travels from satellite to receiver, it will pass through the atmosphere. So, ideally it is c , but practically it will not be c ; that means, there will be change in velocity of propagation when there will be a velocity change in velocity propagation; that means, time of propagation is also be differing from actual.

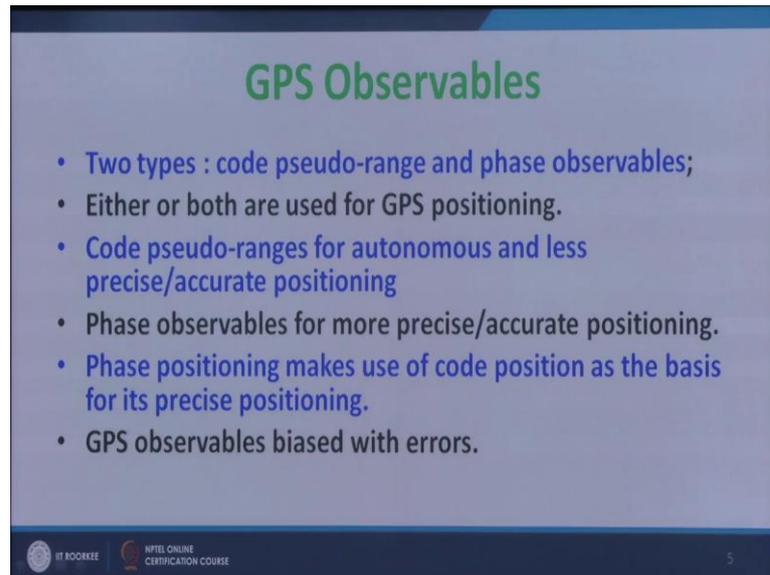
So, these time of propagation or the velocity c into time this time measurement will be associated with different types of error like synchronization error and many other error atmospheric error and hardware error so many other errors which we will learn next some other classes. So, the range we want to calculate actually it is the pseudo range, so $P R$; $P R$ between the satellite s , to receiver r . So, and this is the thing which we get in the GPS observation and that is called code pseudo range and because the time that is measured from the PRN code; pseudo random code, time is measured from the pseudo random code; so it is called code pseudo range and this is one of the observables as I told you that all GPS receiver should have the capability to decipher the C A code.

So, the idea is that that C A code is a code which gives the identification of the satellite from which the signal is coming to the receiver plus these C A code also provide the time t and this measurement of time is not that very accurate; however, it also solve many purpose and to start with we have to measure some time and that is the thing, later this time gets refined by making use of phase measurement anyway.

So, this is the idea behind code pseudo range and in GPS observation we get this value c into t some value we get and that is what is the pseudo range, that means the error distance a geometric distance plus some error distance from the satellite to the receiver. So, we do we can write the pseudo range between satellite s to receiver r ; it is the geometric range plus c into Δt ; that means, error in measurement in time plus some other error, this is the clock synchronization error and this is the geometric range and this is your pseudo range.

So, this is the we get a single value which represent this one and that value is consisting of this part, this part, this part; so this is what we should remember.

(Refer Slide Time: 24:01)



The slide is titled "GPS Observables" in green text. It contains a bulleted list of six points. The first point states there are two types: code pseudo-range and phase observables. The second point says either or both are used for GPS positioning. The third point notes that code pseudo-ranges are for autonomous and less precise/accurate positioning. The fourth point says phase observables are for more precise/accurate positioning. The fifth point states that phase positioning uses code position as the basis for its precise positioning. The sixth point says GPS observables are biased with errors. At the bottom left, there are logos for IIT ROORKEE and NPTEL ONLINE CERTIFICATION COURSE. At the bottom right, the number 5 is visible.

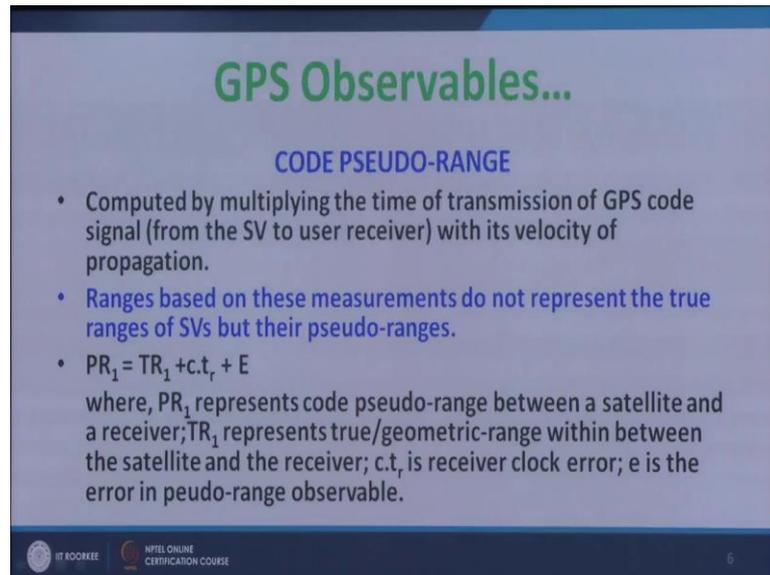
GPS Observables

- Two types : code pseudo-range and phase observables;
- Either or both are used for GPS positioning.
- Code pseudo-ranges for autonomous and less precise/accurate positioning
- Phase observables for more precise/accurate positioning.
- Phase positioning makes use of code position as the basis for its precise positioning.
- GPS observables biased with errors.

So, this is what is written here that code GPS observable there are two types; code pseudo range and phase observable. So, I have till now I have discussed about the code pseudo range either or both used for GPS positioning, either I should say rather code or both only carrier frequency cannot be used for to make use of the carrier frequency; we should first decipher the code. So, ah correctly it should be code or both are to be used for GPS positioning; code pseudo range are for as I told you that these measurement of time will be course in nature as well as it will be associated with error.

So, it will be having it is used for autonomous or less precise accurate positioning and phase observable for precise positioning and as I told you GPS observables are biased with errors.

(Refer Slide Time: 25:15)



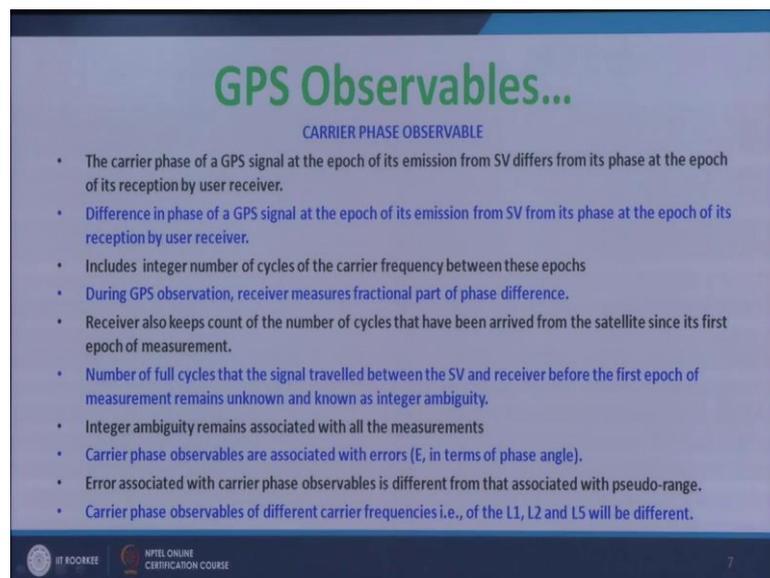
GPS Observables...

CODE PSEUDO-RANGE

- Computed by multiplying the time of transmission of GPS code signal (from the SV to user receiver) with its velocity of propagation.
- Ranges based on these measurements do not represent the true ranges of SVs but their pseudo-ranges.
- $PR_1 = TR_1 + c.t_r + E$
where, PR_1 represents code pseudo-range between a satellite and a receiver; TR_1 represents true/geometric-range within between the satellite and the receiver; $c.t_r$ is receiver clock error; e is the error in pseudo-range observable.

IT ROORKEE | NPTEL ONLINE CERTIFICATION COURSE | 6

(Refer Slide Time: 25:27)



GPS Observables...

CARRIER PHASE OBSERVABLE

- The carrier phase of a GPS signal at the epoch of its emission from SV differs from its phase at the epoch of its reception by user receiver.
- Difference in phase of a GPS signal at the epoch of its emission from SV from its phase at the epoch of its reception by user receiver.
- Includes integer number of cycles of the carrier frequency between these epochs
- During GPS observation, receiver measures fractional part of phase difference.
- Receiver also keeps count of the number of cycles that have been arrived from the satellite since its first epoch of measurement.
- Number of full cycles that the signal travelled between the SV and receiver before the first epoch of measurement remains unknown and known as integer ambiguity.
- Integer ambiguity remains associated with all the measurements
- Carrier phase observables are associated with errors (E , in terms of phase angle).
- Error associated with carrier phase observables is different from that associated with pseudo-range.
- Carrier phase observables of different carrier frequencies i.e., of the L1, L2 and L5 will be different.

IT ROORKEE | NPTEL ONLINE CERTIFICATION COURSE | 7

Now, code pseudo range as I have shown that it is the true range geometric range plus error due to receiver synchronization error and other error.

(Refer Slide Time: 25:33)

$$(PR^s) = \phi^s + c \cdot \Delta t + \epsilon$$

Carrier Phase Observables

$$\phi_y^s = X \cdot 360 + (\phi^s - \phi^r) + N$$

Integer Ambiguity

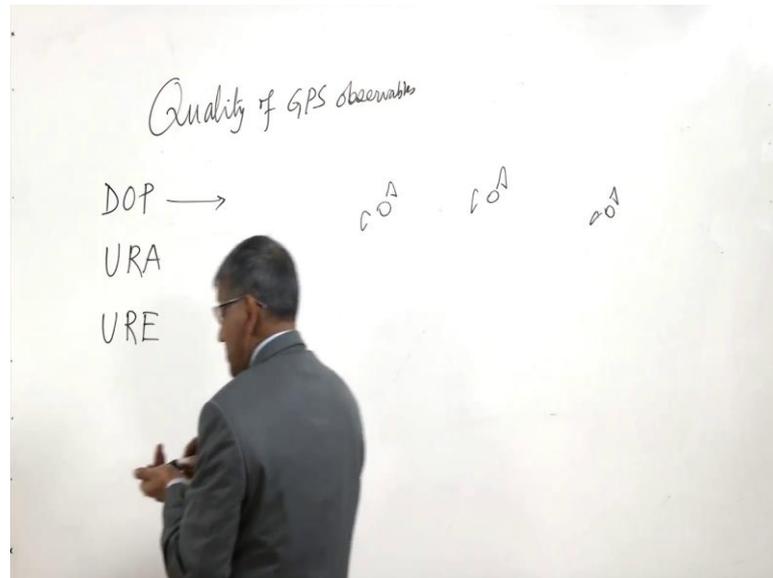
Now carrier phase observable other observable is carrier phase it is the carrier phase observable from the name itself carrier phase observables. It is the difference in phase; that means, signals are coming from the satellite to receiver. So, it will at some phase a 5, 1 it will be leaving and at some phase 5, 2 it will be reaching, so the receiver we can measure the difference in phase between the signal it is left and the signal it is being received.

But point is that in between these there will be some X numbers of suppose full sets, cycles. So, the X numbers of full cycles means X into 360 degree, so that will be the actual phase between the satellite and the receiver, but problem is that the full numbers of cycles are difficult to count, what we do in this receiver there is a counter which can counts the numbers of full cycles that between two consecutive resection; how many full cycles are being arrived; counting that we can get the value of this, but there will be a problem for the first signal to be received because in the beginning how many full numbers of cycles it comes (Refer Time: 27:34) it was trustable it is not known so there will be an error associated with each GPS observable which is called integer ambiguity.

So, whenever we will go for carrier phase observables then we will have the x numbers of full cycles this is the fractional cycle plus a error which is called integer ambiguity. So, next the quality of GPS observables as I told you that the GPS observables are associated with errors, now the amount of error that GPS observable contain we will

define its quality; lesser the error, better is the quality. So, how do you quantify about the quality? There are three parameters which actually and provides us some indigestion about the quality.

(Refer Slide Time: 28:52)

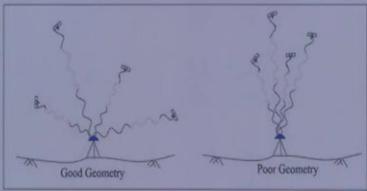


That is called DOP; Dilution Of Precision, URA these are range accuracy and URE, these are range error. These are the three parameters which provides us some indigestion about the quality of the error, now this DOP; Dilution Of Precision. Actually during the observation of GPS data, GPS receiver provides us this value; now these value depends upon the geometry of the satellites with reference to the receiver.

(Refer Slide Time: 29:45)

QUALITY OF GPS OBSERVABLES: SATELLITE GEOMETRY

- Quality of GPS observables also depends on the satellites-receiver geometry at the epoch of its observation.
- Good geometry if these are spatially well distributed in the sky considering zenith point of the receiver as the centre for distribution.
- Poor geometry if SVs are in clustered form
- Good geometry provides better quality observables and poor of degenerated quality.
- Geometry quantified through dilution of precision (DOP).
- DOP values defines the quality of the GPS observables.
- Lower the DOP value, better is the quality of observables.
- Guiding DOP values depend on the type of surveying.



IT KOOBEE | NPTEL ONLINE CERTIFICATION COURSE | 9

Now, here you can see that in this case the satellites are well spread and this time and in this case satellites are narrowly spaced. Now, when the satellites are widely spaced that provides us better geometry and the value of the dilution of precision will be less, but here the geometry is poor and we will get the high DOP value. So, whenever we take the GPS observation, we should look into the DOP value, what it is showing; if the value are less means 2 or 1 or 3; something like that then we can assume that the quality of the GPS observables will be good and if it is more like 7, 8, 9, 10, so then we can say that the quality of the GPS observables are not good.

So, during observation itself we can restrict our observation if the DOP value is more or we can take our observation when the DOP values are less. Now once you start taking the GPS observable even if you are found that the DOP values are less then also you have some other characteristics which you can see to find out whether the collected data is good or not and one of the criteria is that user range accuracy.

(Refer Slide Time: 31:22)

QUALITY OF GPS OBSERVABLES: URA

USER RANGE ACCURACY (URA)

- URA of any particular GPS signal is being specified in its navigation message.
- In word three, bits 13 through 16 provide the URA index of the SV.
- URA value is a conservative estimate of RMS error for estimation of accuracy in pseudorange measurements.

IIT ROORKEE NPTEL ONLINE CERTIFICATION COURSE 10

Now, this is a parameter which is available in the GPS navigation message; now in the navigation message there are some words, word number three, bits number 13 to 16 provide an index which shows the URA index and this is a value which provides the user range accuracy and about the quality of the GPS data.

(Refer Slide Time: 32:10)

QUALITY OF GPS OBSERVABLES: URE

User Range Error (URE)

- Error budget present in measurement of range between a user's receiver and the observation satellite is known as user range error (URE).
- URE is being computed by taking square root of the sum of the squares of the individual errors.

IIT ROORKEE NPTEL ONLINE CERTIFICATION COURSE 11

Another parameter which is called user range error, this is also a parameter actually user range error as I told you that during GPS observation the errors and the sources of errors are different. So, what we do for each source we do try to find out how that source is

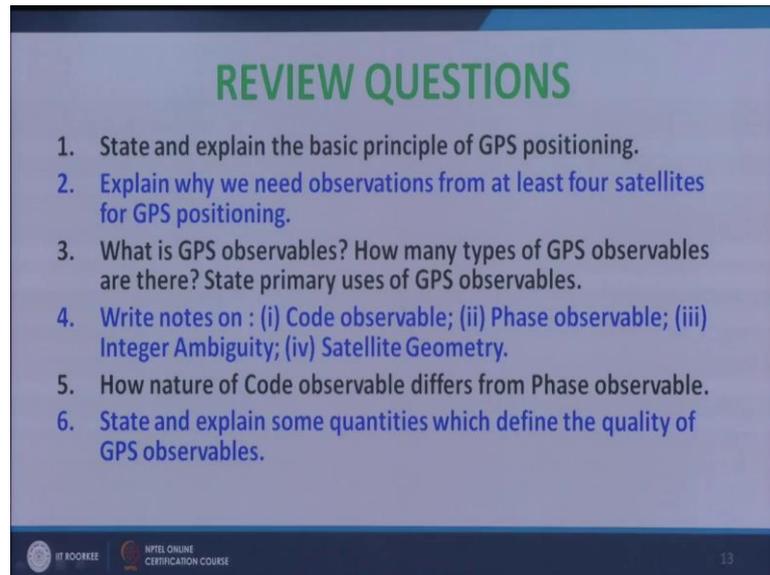
contributing to the error in measurement of range and taking square of those errors and square root of the summation, we do compute the user range error and if it is less then we say that the quality of GPS observable is good, otherwise it is not that good. In that way we can control or we can judge about the quality of the GPS observable which is very important for carrying out good GPS surveying; that means, if we want to end up with very precise solution, very accurate solution; the GPS observation should be good quality, good quality means it should be associated with least amount of errors and these are the parameters which gives us an indication, so with this I like to complete this class.

Now, we will summarize that GPS we do go for GPS surveying to find out the position of any point and the position of a point using GPS is being done by using the method of analytical resection. The method of analytical resection needs the unknown location or known location of the satellites which are available from GPS ephemeris data, which is available in the GPS observation as well as we need to know the range; that means, the distance between the satellite and the receiver, but since there are different types of error associated with the signal propagation as well as the system, the range we get not the actual range, but a pseudo range.

And that pseudo range can be computed from in two ways one from code and that is called code pseudo range and other from carrier actually in GPS observable we get the phase of the GPS signal or difference in signal or range and if you multiply that phase observable by the wavelength of the signal that will provide us the that carrier pseudo range carrier pseudo range and the position from GPS observable depends upon the quality of the GPS observable and the quality of the GPS observable depends upon what is the amount of error associated with lesser the error, better is a quality and the extent of error that is available or that is what in the GPS observation can be indicated through can be visualized we have a some idea through some parameters like DOP value or URA User Range Accuracy, User Range Error.

So, during observation we should always try to find out what is the DOP value and we should take the GPS observation and the DOP value is low. With these I want to conclude today's class.

(Refer Slide Time: 36:00)



REVIEW QUESTIONS

1. State and explain the basic principle of GPS positioning.
2. Explain why we need observations from at least four satellites for GPS positioning.
3. What is GPS observables? How many types of GPS observables are there? State primary uses of GPS observables.
4. Write notes on : (i) Code observable; (ii) Phase observable; (iii) Integer Ambiguity; (iv) Satellite Geometry.
5. How nature of Code observable differs from Phase observable.
6. State and explain some quantities which define the quality of GPS observables.

IIT KOOBEK NPTEL ONLINE CERTIFICATION COURSE 13

There are some review questions which you should be able to answer after this class and if you want to know more, you can go through this book and in the next class I will be talking on the errors, different types of error as I already discussed today that GPS observables are associated errors, what are the different sources of errors that I will discuss in the next class.

Thank you.