

# Analysis and Design of Bituminous Pavements

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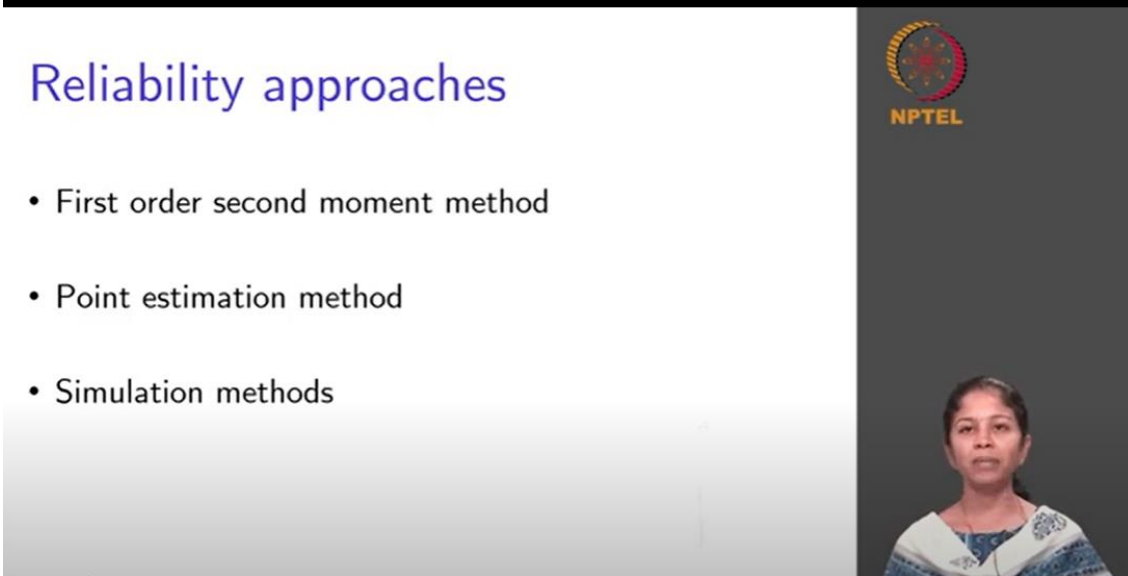
Department of Civil Engineering

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Lecture – 36

Reliability in Pavement Design - Part 06

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Reliability approaches

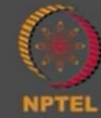
- First order second moment method
- Point estimation method
- Simulation methods

Hello everyone, welcome back. In this lecture we are going to talk about the other reliability approaches that are used in design. So, far we have seen the consideration of reliability in IRC37, how reliability is taken into account in AASHTO 1993 design procedure and how it is taken into account in AASHTO 2004 design procedure. And now we are going to see other design procedures which are not used in any standard specification, but done at a research level. So there are three common reliability approaches. One is a first order second moment method, second one is a point estimation method and the third one is simulation method. I will briefly touch upon the first two methods, but we are going to discuss in detail about the third method.

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## First Order Second Moment

- The inputs and outputs are not expressed in the form of any distribution
- Mean and standard deviation values are used
- Derivatives associated with this method complicated
- Assumption – input parameters are mutually independent




So, what is the first order second moment or it is called as FOSM method. So, the input and outputs in this particular design method is not expressed in the form of any distribution. We are going to use only a single point value for the input and we are going to get the corresponding single point value for the output. In this case, mean and standard deviation values are used, but we are not going to arrive at any distribution like how we will be using in the Monte Carlo. The derivatives associated with this method are complicated. So, we use a Taylor expansion series, we have to eliminate the higher order terms, then a lot of post processing is associated with arriving at the equation. The assumption which is used in the FOSM method, is that the input parameters are independent. But we all know that there is a lot of relation between the input parameters, especially with regard to pavement design.

When we are seeing the Monte Carlo simulation method, we will be considering the interaction terms; interaction between modulus and modulus of one layer to modulus of another layer, modulus of one layer to thickness of another layer. So, there are going to be a lot of dependencies. Again, even in AASHTO 1993, we had seen that when we are designing for one particular layer. For example, the layer coefficient for layer 2, depended on the resilient modulus of subgrade and thickness of the above lying layer. So, there are a

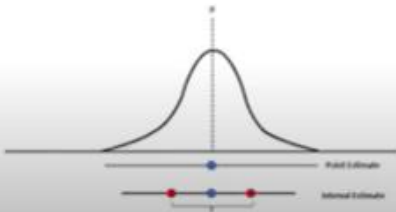
lot of dependencies especially with respect to the stress state in a pavement system. So, these dependencies are not considered in this particular method. So, this is one method which is theoretically defined.

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## Point Estimation


- Measures the values of independent variables at specific points to estimate the respective parameters for the dependent variables



The diagram shows a normal distribution curve. A vertical dashed line extends from the peak of the curve down to a horizontal axis. Below this axis, there are two more horizontal lines. The first line has a single blue dot in the center, labeled 'Point Estimate'. The second line has a blue dot in the center, with red dots on either side, and a bracket underneath, labeled 'Interval Estimate'.


The next method is a point estimation. Here also the values of independent variables are measured, but only at specific points and again the distribution is not considered. Previously, if we use the mean and standard deviation, here we are going to use specific values. If it is done at some point, it is called as a point estimate and if it is done over the range of an interval, it is called as an interval estimate. So, again in this method, it uses only single point measurements and the corresponding outputs are again obtained here, but the distribution is not taken into account. So, this is a second method which talks about point estimation. Then we are going to talk about the simulation methods.

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## Simulation

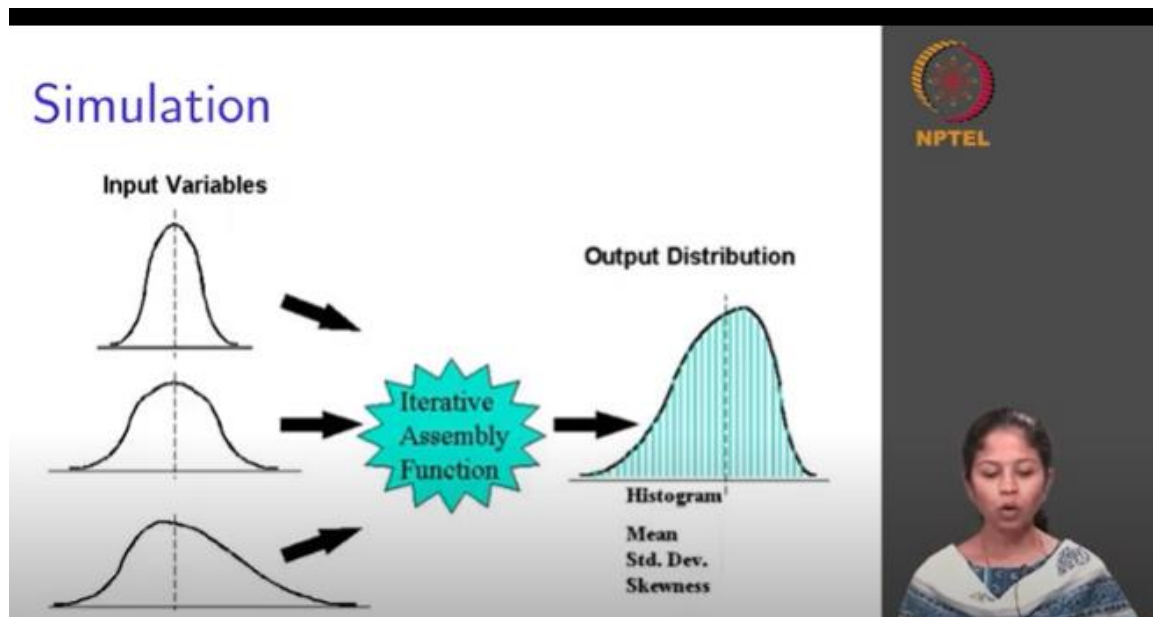
- Includes numerical integration and simulation process. The entire distribution is required for each input parameter instead of representing it by some statistical values.
- The simulation process involves artificial generation of a distribution of each input parameter with the help of random number generators (Monte Carlo Simulation)
- These distributions are then used to generate the probability distribution of the system



This includes numerical integration and simulation process. So, the first two methods may be done manually, but this method cannot be done manually. It is like high tedious and we need a computer or a simulation process to carry out this method. So, the entire distribution is required for each input parameter instead of representing it by some statistical values. If you recollect, the first two methods were using one-point measurement or measurement at specific points.

So, only in this case, we are going to use the entire distribution. The simulation process involves artificial generation of a distribution of each input parameter with the help of random number generators. This Monte Carlo simulation is nothing but a random number generator. It gives a particular mean and standard deviation value; it is going to give you the distribution. So, that is the function of a Monte Carlo simulation and this approach is also commonly referred to in that particular name. So, these distributions are then used to generate the probability distribution of the system.

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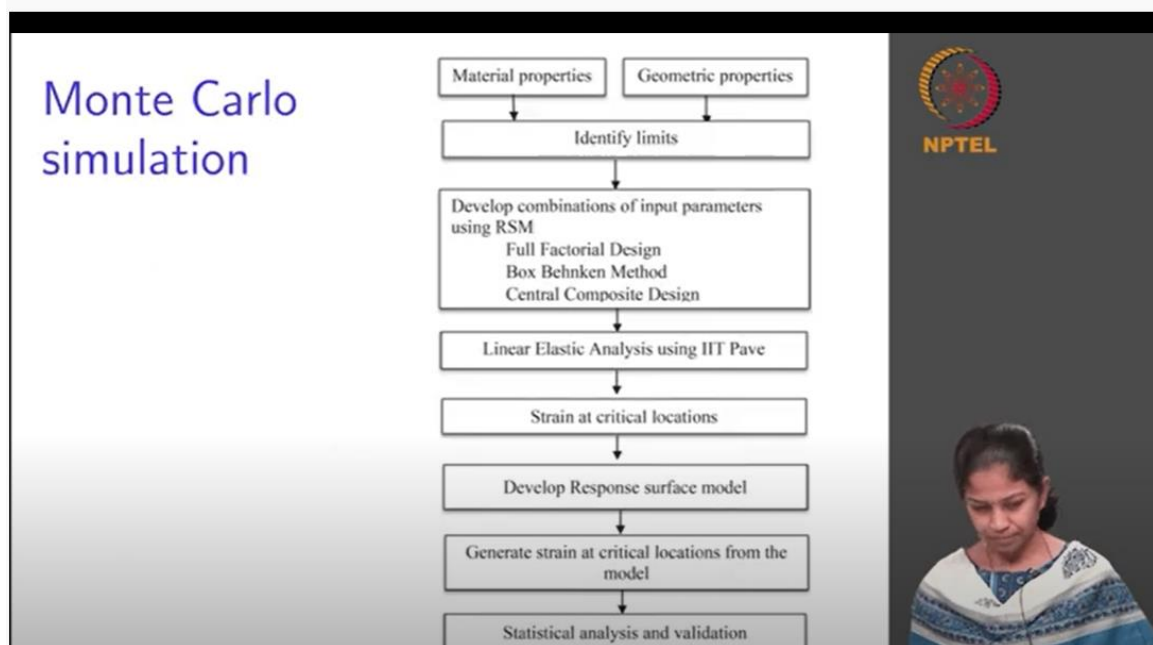


We will go through this procedure in a step by step manner. Now we can see for each of my input variable, I am arriving at this distribution. Again, how do I get this distribution? To arrive at the distribution, we should have done some homework before. We typically tend to collect data from field.

Let us say my input parameter is thickness of asphalt concrete layer. So, how do I say which is the kind of distribution which represents the variability and thickness of my asphalt concrete layer? I go to the field, I measure the thickness at different points, I collect the data, I plot a frequency distribution of the data. So, to this frequency distribution, I will be able to fit in any distribution. So, whichever is a distribution which closely represents this variation, we choose that particular distribution. Let us say I am going to assume a normal distribution. If I fit it for this frequency distribution of input parameter a normal distribution, I am going to get a mean and a standard deviation value. So, these values can be used subsequently used to represent the variation and thickness of a given layer. Again, this is specific to that particular site. I have to make plots like this at multiple locations and from those locations, I have to get the value and cumulative effect of all those on the mean and standard deviation I can compute and use it as a representation which is more generic in nature and would suit any of the locations that we will be using for design.

So, like that you can see here, a normal and skewed normal distribution curve. We can see here that the distribution of thickness of asphalt concrete layer is a skewed normal distribution. So, for all of the input variables, I will be using individual distributions. Then there is a design process. We can associate it with any design process that we will be using and we will use it to generate the output. So, we will again get a histogram here. We can fit an appropriate distribution, compute the mean, standard deviation and skewness. So, this is what the simulation techniques do.

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Now, let us go through the step by step procedure that is used in a Monte Carlo simulation. We have material properties and we have geometric properties. We decide what is the modulus of each layer that we will be using and we also arrive at the thickness, the type of layer that is to be used and the thickness of individual layers. So, based on these two inputs, we identify the limits. So, what is the range over which the modulus can vary? What is the range over which the thickness can vary? So, using all of it, we use a design of experiments approach to obtain the combinations. So, I hope many of you will know what a design of experiments approach is. It is used, let us say we have a number of factors, we want to identify the influence of each of those factors. We typically tend to do a factorial design.

Let us say I have 3 factors and each of them have 3 values; so we will have 9 experimental combinations. This number is very small, sometimes we will have large experimental combinations, which might not be realistic to carry out especially when it involves testing. So, in that case, we use a factorial design, which will limit the number of combinations that we have to use.

There are different methods. One is a full factorial design, which will consider all the combinations. There are other factorial designs like a Box-Behnken design method (BBM) or a central composite design (CCD) method, which will give us lesser number of combinations compared to the full factorial case. So, maybe in this case, we might be getting 5 or 4 designs. So, that is a factorial design. Again, design of experiments is a separate subject; a separate technique on its own. It is not possible to explain design of experiments within the scope of this particular course. You can read it from other sources. And when we are looking at the step by step procedure, I will also show you as to how this is done, done for the pavement system. Then we use a linear elastic analysis using IITPAVE.

We calculate the strains or critical strains corresponding to each of the combination. And then we identify the strain at critical locations. It is not necessary that we have to compute the strain only at critical locations. We can do it for multiple locations and identify which is the critical strain. Using these things, we develop a response surface model, which will show us the variation of critical strain with respect to the input variables.

So, we arrive at a response surface model. Using that model, we will be able to predict the strain for any other combination which is not considered in this experimental combination. So, within the range which is specified in this particular step, we will be able to interpolate and arrive at the critical strain values. Then we generate the strain at critical locations from the model. And finally, we carry out statistical analysis and validation. So, this is the overall procedure that we are going to follow.



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## Monte Carlo simulation

- Step 1: Assume a cross-section for design and the modulus values for the layers

| Input Parameters | Lower limit | Upper limit |
|------------------|-------------|-------------|
| $E_1$ (MPa)      | 1000        | 3000        |
| $E_2$ (MPa)      | 146         | 312         |
| $E_3$ (MPa)      | 49          | 100         |
| $h_1$ (mm)       | 80          | 180         |
| $h_2$ (mm)       | 400         | 450         |

- Each variable is considered as a factor and the range over which these factors vary is also specified as levels



And let us now look at the individual steps. So, step 1 is to assume a cross section for design and modulus value for the layers. Let us say that we are assuming only 3 layers, a subgrade, a base course and a bituminous course. Now, we can see here, the first step is to provide limits for each of these layers. So, what will be the  $E_1$ ,  $E_2$ ,  $E_3$ ,  $h_1$  and  $h_2$ ? So, what are the modulus values corresponding to the layers? Let us take the first layer which is the bituminous layer. I consider a lower limit that the modulus value will not be less than 1000 MPa, the upper limit is 3000 MPa. So, this is the range in which my modulus value will vary. Similarly, if I look at layer 2, I have given a value of 146 to 312 MPa. These values are again taken from IRC 37. If you look at the codal provision, depending upon the grade of bitumen, we can, we will have values between 1000 and 3000 MPa for the resilient modulus. So, that is why the same range is given here; similarly, for  $E_2$ . So all these are computed from IRC 37 for some assumed values of CBR and for some assumed grade of bitumen. Similarly, for  $E_3$ , it ranges from 49 to 100 MPa. So, these are the values, which have to be specified by the designer. We will be knowing from prior experience or from specifications, what will be the minimum value that is required and what is the maximum value that it should not exceed. So, this is given as the upper and lower limit. So, now if we translate it into the terminology used in design of experiments, these are called as factors. The input parameters which we will be varying in our experiment, we call



them as factors and for each factor, we have given 2 values. So, these values are called as levels. So, we have 2 levels for each factor.

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The slide is titled "Monte Carlo simulation" in blue text. Below the title is a bullet point: "Step 2: Use a suitable DoE approach to arrive at combinations of input parameters". There are three 3D wireframe cube diagrams illustrating different experimental designs. The first is labeled "a) FFD" (Full Factorial Design) and shows orange dots at each of the eight vertices of the cube. The second is labeled "b) BBM" (Box-Behnenken Design) and shows orange dots at the eight vertices and one dot at the center of the cube. The third diagram, positioned below the others, shows a central composite design with orange dots at the eight vertices and one dot at the center, with dashed lines connecting the center dot to each vertex dot. In the top right corner of the slide, there is a circular logo with a red and yellow border and the text "NPTEL" below it. In the bottom right corner, there is a small video inset showing a woman with dark hair, wearing a blue and white patterned top, looking down.

So, if we consider 2 levels and 5 factors, we will be having 32 combinations and we will also assume some center points. So, this is for a full factorial design, wherein all the combinations will be considered. Now, there are different approaches which use different methods to reduce the number of combinations. So, one is called as a full factorial design, wherein all the combinations will be considered. So, if we consider, this is a case which indicates for 3 factors and 3 levels. So, each node is one combination. So, this is going to be the one value, the lower limit of factor 1, lower limit of factor 2, lower limit of factor 3. This is my first combination. This combination is going to be upper limit of factor 1. So, this is F3, upper limit of factor 3 and lower limit of factor 2. So, like that we will be arriving at different nodes and different experimental combinations.

So, now this Box-Behnken design or a central composite design is going to give us a lesser number of combinations, but still not compromising on the influence of all these factors on the response. So, we will use a suitable design of experiments approach to arrive at the combination of input parameters. Let us say that we are using a Box-Behnken design. If you look at the paper by Ashwathy and Dr. A.K. Swamy, they have explained the



difference between the BBM and CCD for use in pavement design applications for certain reasons. You can see here, there are certain combinations which fall out of the experimental limits in case of CCD and it is suggested that Box-Behnken design is a better method when we use it for pavement design. So, as a designer, we can select the appropriate design of experiments methodology, but BBM has certain advantage in this case. So, let us arrive at a particular design methodology.

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## Monte Carlo simulation

- Step 2: Use a suitable DoE approach to arrive at combinations of input parameters

| Combination | $E_1$ (MPa) | $E_2$ (MPa) | $E_3$ (MPa) | $h_1$ (mm) | $h_2$ (mm) |
|-------------|-------------|-------------|-------------|------------|------------|
| 1           | 1000        | 146         | 75          | 130        | 425        |
| 2           | 3000        | 146         | 75          | 130        | 425        |
| 3           | 1000        | 312         | 75          | 130        | 425        |
| 4           | 3000        | 312         | 75          | 130        | 425        |
| 5           | 1000        | 229         | 49          | 130        | 425        |
| 6           | 3000        | 229         | 49          | 130        | 425        |
| 7           | 1000        | 229         | 100         | 130        | 425        |
| 8           | 3000        | 229         | 100         | 130        | 425        |
| 9           | 1000        | 229         | 75          | 80         | 425        |
| 10          | 3000        | 229         | 75          | 80         | 425        |
| 11          | 1000        | 229         | 75          | 180        | 425        |
| 12          | 3000        | 229         | 75          | 180        | 425        |
| 13          | 1000        | 229         | 75          | 130        | 400        |
| 14          | 3000        | 229         | 75          | 130        | 400        |
| 15          | 1000        | 229         | 75          | 130        | 450        |
| 16          | 3000        | 229         | 75          | 130        | 450        |
| 17          | 2000        | 146         | 49          | 130        | 425        |
| 18          | 2000        | 312         | 49          | 130        | 425        |

Now we arrive at the experimental combinations. You can see here combination 1, the  $E_1$  value is given  $E_2$ ,  $E_3$ ,  $h_1$ ,  $h_2$ . So, for all the 5 factors which we had given here, one value of all the 5 are used here and this is my first combination. In my second combination, you can see this is moved to the upper level. So, this is lower level of factor 1, this is upper level of factor 1, all the others are kept constant. Then I move on to the combination 3, wherein again this is kept at the lower level, but this value is kept at the upper level and remaining also you can see here all of them are at the lower level. This is at the upper level, remaining everything are at the lower level. Now this is a lower level, this is taken to upper level. So, like that for each experiment again this is the upper level, upper level and all of them are at the lower level. So, it is varied in a specific manner and all the combinations will be considered in this case. So, this is in a specific order, but then it will randomize

again these orders so that we do not run experiments in any specific order also. So, again I have shown only limited combinations, but we had got about 41 combinations in this case. So then we arrive at the experimental combination. So, now what am I going to do with this experimental combination? So, we know that we know how to do a pavement design using IRC 37, the IITPAVE software.

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The slide features a diagram of a three-layer pavement system. At the top, two downward-pointing arrows represent 'Wheel loads'. Below them, the 'Asphalt layer' is shown with thickness  $h_1$ , modulus  $E_1$ , and Poisson's ratio  $\mu_1$ . Horizontal arrows labeled  $\epsilon_1$  indicate the strain at the interface between the asphalt and granular layers. Below that is the 'Granular layer' with thickness  $h_2$ , modulus  $E_2$ , and Poisson's ratio  $\mu_2$ . At the bottom is the 'Subgrade' with thickness  $h_3 = \infty$ , modulus  $E_3$ , and Poisson's ratio  $\mu_3$ . Vertical arrows labeled  $\epsilon_2$  indicate the strain at the interface between the granular layer and the subgrade. The NPTEL logo is visible in the top right corner of the slide.

So, now let us take the first combination. I am going to take my IITPAVE software. I will be using only 3 layers, instead of E1 value I will be using 1000 MPa, in place of E2 value I will be using 146 MPa, in place of E3 I will be using 75 MPa and in place of the thickness of the first layer or the bituminous layer I will be using 130 mm, in place of  $h_2$  I will be substituting 425 mm and Poisson's ratio can be assumed in this case. So, I think we used a Poisson's ratio of 0.35 uniformly for all layers and we can assume a specific loading condition. We can assume an 80 kN single axle dual wheel case, maybe we can just go ahead with the 560 kPa tire pressure.

So, we can consider all these cases and carry out a design. So, for this we will be computing the critical strains. So, where we will be computing the critical strain? Horizontal tensile strain at the bottom of bituminous layer and vertical compressive strain at the top of

subgrade. Here it is suggested to compute the critical strains at the position which is directly beneath one tire here, below the wheel load and at a point which is exactly in between the 2 wheel loads. So, this we have to compute it at the bottom of bituminous layer and at the top of subgrade. So, this is done using a linear layered elastic analysis. So, we get horizontal maximum tensile strain ( $\epsilon_t$ ) directly under the load in between the wheel loads, vertical maximum compressive strain ( $\epsilon_v$ ) directly under the load in between the wheel loads. So, all these strains are now computed. Then we use a response surface methodology. Now let me show you this. So, here we will be getting the critical strain values;  $\epsilon_v$  and  $\epsilon_t$ . So, among the we get 2 values, one for single tire another for dual tire. So, I will have 4 values of critical strains. I can choose the one which is critical and then use it for design. So, against each combination, I will be having 4 values as listed here.

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**Monte Carlo simulation**

- Step 4: Use an RSM model to explain the variation of strain with the input parameters

$$y = \beta_0 + \sum_{j=1}^k \beta_j X_j + \sum_{j=1}^k \beta_{jj} X_j^2 + \sum_{i < j} \beta_{ij} X_i X_j$$

*Responses ( $\epsilon_t$  or  $\epsilon_v$ ) = Intercept +  $F_1 \times E_1 + F_2 \times E_2 + F_3 \times E_3 + F_4 \times h_1 + F_5 \times h_2 +$   
 $F_6 \times E_1 \times E_2 + F_7 \times E_1 \times E_3 + F_8 \times E_1 \times h_1 + F_9 \times E_1 \times h_2 +$   
 $F_{10} \times E_2 \times E_3 + F_{11} \times E_2 \times h_1 + F_{12} \times E_2 \times h_2 + F_{13} \times E_3 \times h_1$   
 $+ F_{14} \times E_3 \times h_2 + F_{15} \times h_1 \times h_2 + F_{16} \times E_1 \times E_1 + F_{17} \times E_2 +$   
 $F_{18} \times E_3 \times E_3 + F_{19} \times h_1 \times h_1 + F_{20} \times h_2 \times h_2.$*

The slide also features the NPTEL logo in the top right corner and a small video inset of a woman in the bottom right corner.

So, now we will use a response surface methodology to explain the variation of strain with variation in all the input parameters. So, we can see here, this is my y value, which is the predicted strain. Again, we can have, let us say that we have identified the critical strain and we have one equation for vertical compressive strain, one equation for horizontal tensile strain. Then this is expressed in terms of the individual factors, the square term among the individual factors and the interaction between the terms. So, when I was

discussing earlier using the FOSM method, we were saying that the interaction terms are not taken into account in this design procedure. But here we are taking into account of the interaction terms as well. So, using this for the example that was shown below, this will be the model that will be obtained using a response surface methodology.

$$y = \beta_0 + \sum_{j=1}^k \beta_j X_j + \sum_{j=1}^k \beta_j X_j^2 + \sum_{i < j} \beta_{ij} X_i X_j$$


We have individual equations for  $\epsilon_v$  and  $\epsilon_t$ . So, let us say this equation is my  $\epsilon_v$ , it has intercept  $F1 \times E1$ ,  $F2 \times E2$ ,  $F3 \times E3$  and so on. So, here you can see the square terms. If you look at this, it is  $E1 \times E1$  and if you look at this, we have  $h1^2$ . If you look at this  $h2$  and this is  $E3^2$ . So, all these square terms are there, then you can see here, this has the interaction terms, right? You can see here. So, all these are interaction terms. So, this model takes into account of the interaction among the parameters and the square terms which are used to predict the critical strain value.


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## Monte Carlo simulation

- Step 4: Use an RSM model to explain the variation of strain with the input parameters

| Model Parameters | Static load  |              |
|------------------|--------------|--------------|
|                  | $\epsilon_t$ | $\epsilon_v$ |
| Intercept        | 2.31E-04     | 2.32E-04     |
| $E_1$            | -6.61E-05    | -2.42E-05    |
| $E_2$            | -4.66E-05    | -2.61E-05    |
| $E_3$            | -5.94E-07    | -4.44E-05    |
| $h_1$            | -9.60E-05    | -6.68E-05    |
| $h_2$            | -5.94E-07    | -1.76E-05    |
| $E_1 \times E_2$ | 2.35E-05     | 5.20E-06     |
| $E_1 \times E_3$ | -1.00E-06    | 4.60E-06     |
| $E_1 \times h_1$ | 1.05E-05     | -2.13E-06    |
| $E_1 \times h_2$ | -5.00E-08    | 2.82E-06     |
| $E_2 \times E_3$ | 9.00E-07     | 1.06E-05     |
| $E_2 \times h_1$ | 2.86E-05     | 1.33E-05     |
| $E_2 \times h_2$ | -5.00E-08    | 3.25E-07     |
| $E_3 \times h_1$ | -1.08E-06    | 1.20E-05     |
| $E_3 \times h_2$ | 2.00E-07     | 3.20E-06     |
| $h_1 \times h_2$ | -7.50E-08    | 7.88E-06     |
| $E_1 \times E_1$ | 1.93E-05     | 5.44E-06     |
| $E_2 \times E_2$ | 9.71E-06     | 2.22E-06     |
| $E_3 \times E_3$ | -4.58E-08    | 8.77E-06     |
| $h_1 \times h_1$ | 1.87E-05     | 1.22E-05     |
| $h_2 \times h_2$ | -2.92E-08    | 2.48E-07     |





Now, let us see how this looks like. You can see here for the example which was shown before, these are the model parameters. We have an intercept. This is for a particular loading case. This is the coefficient that is obtained for the  $\epsilon_t$  that is to predict the



horizontal tensile strain and these are the set of coefficients that are obtained for vertical compressive strain. So, we have intercept  $E_1, E_2, E_3, h_1, h_2$ . So, we have the influence of individual terms, then we have the influence of interaction terms and the influence of square terms, So, again we have different set of values for  $\varepsilon_t$  and  $\varepsilon_v$ , right? So, this is how we get the output. Now, we have an equation. If you count, I think we will be having about 24 model parameters of the form which is shown. So, now if I know the values of  $E_1, E_2, E_3, h_1$  and  $h_2$ , I will be able to use this equation and predict the value of  $\varepsilon_t$  or  $\varepsilon_v$ . Provided the value of  $E_1, E_2, E_3, h_1, h_2$  are within the limits which are given in step 1. Then I will be able to use this model.

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## Monte Carlo simulation

- Step 5: Assume mean and CoV values for all input parameters

| S. No. | Parameter                                      | Mean value adopted        | COV (%)           |
|--------|--|---------------------------|-------------------|
| 1.     | Bituminous layer thickness ( $h_1$ )           | As per the section chosen | 5                 |
| 2.     | Granular layer thickness ( $h_2$ )             | As per the section chosen | 10                |
| 3.     | Elastic modulus for bituminous layer ( $E_1$ ) | 2500 MPa                  | 15                |
| 4.     | Elastic modulus for granular layer ( $E_2$ )   | 250 MPa                   | 20                |
| 5.     | Elastic modulus for subgrade ( $E_3$ )         | 60 MPa                    | 20                |
| 6.     | Commercial vehicles per day (CVPD)             | 1411                      | 20                |
| 7.     | Vehicle damage factor (VDF)                    | 4.5                       | 15                |
| 8.     | Lane distribution factor (LDF)                 | 1.0                       | 10                |
| 9.     | Growth rate                                    | 5%                        | 10                |
| 10.    | Design period                                  | 15 years                  | 0 (Deterministic) |

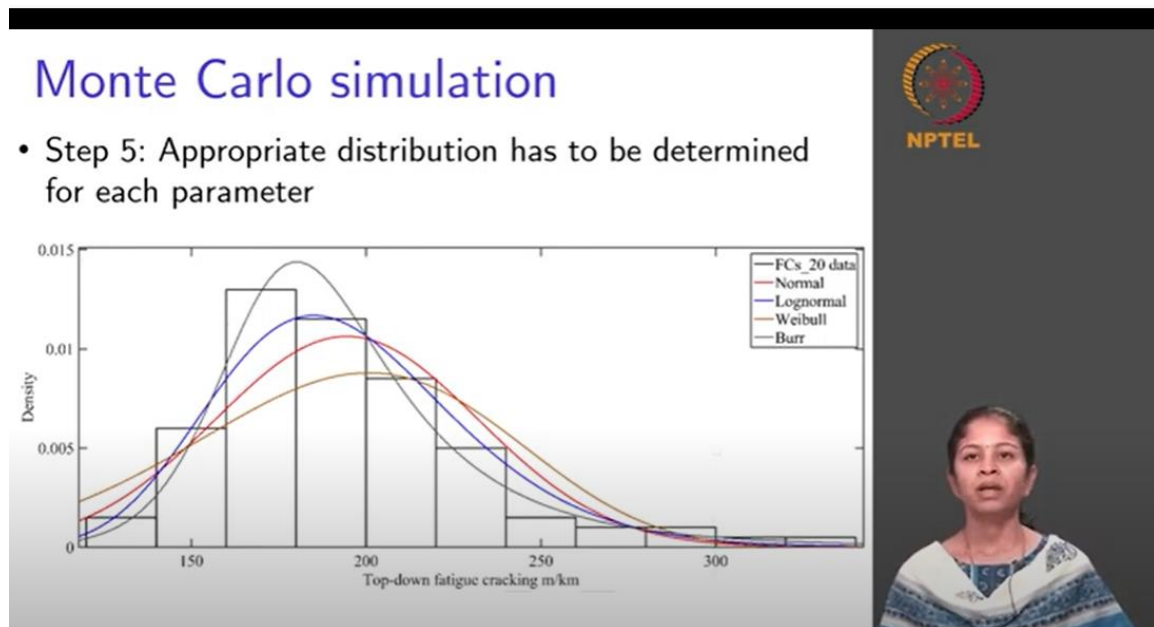

  


After arriving at the model constants and the RSM model, the next step is to use Monte Carlo simulation and obtain a distribution for each of the input variable. So, if you look at this, we have bituminous layer thickness. So, whatever is the section which is given in IRC catalogue, the thickness of the bituminous layer which is given in that catalogue can be chosen. A coefficient of variation of 5% is assumed for this particular case. So, we have a mean value and we have a CoE value. Similarly, for the granular layer thickness, elastic modulus of bituminous layer, elastic modulus for granular layer, for subgrade, for these 3 modulus values, the mean value is directly specified. It is assumed to be 2500 MPa for  $E_1$ ,

250 MPa for E2, 60 MPa for E3. The values are assumed here. But for the coefficient of variation also, in the previous lectures, when one of the previous lectures, I showed you a table wherein they had summarized the coefficient of variation from different literature and the typical values to be used was also given. So, those values are can be used or measurements can be made like I showed before a distribution can be fit and we can estimate the mean and standard deviation. And the commercial vehicles per day is also given. Again, a CoV is assumed for that vehicle damage factor, lane distribution factor, growth rate and design period. So, this design period is deterministic. It is exactly 15 years. So, the mean and CoV values are assumed. Using this mean and CoV value, we now generate a distribution using the, using the Monte Carlo simulation.

Previously, we had seen that when we had a histogram like this, we fit a distribution to this histogram, right? Now what we are going to do is we fit a normal distribution from the mean and CoV value that we know for this normal distribution, we are going to back calculate and arrive at a histogram. That is what we are going to do in this Monte Carlo simulation.

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Again, before fitting a distribution, it is not necessary that we have to always assume a normal distribution. There are many other distributions which can be used to explain the



data. Say for example, if this is the data which is shown in the form of histogram, we can use a normal distribution, log normal, Weibull, a Burr distribution, there are many other distributions which we can use. So, we have to first arrive at a procedure which will help us to select which distribution best represents the given data. Then we can use that particular distribution rather than approximating any data to a normal distribution. So, this has to be kept in mind and appropriate values from the distribution can be used in terms of in place of the main value.

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## Monte Carlo simulation

| Property          | Description              | Previous Investigation |                 |                               | Reference                                   |
|-------------------|--------------------------|------------------------|-----------------|-------------------------------|---|
|                   |                          | Range of COV (%)       | Typical COV (%) | Type of distribution          |   |
| Layer Thickness   | Bituminous surface       | 3 - 12                 | 7               | Normal                        | Timm et al. (2000), Noureldin et al. (1994) |
|                   |                          | 3.2 - 18.4             | 7.2             | Normal                        | Aguiar-Moya et al. (2009)                   |
|                   | Bituminous binder course | 11.7 - 16.0            | 13.8            | Normal                        | Aguiar-Moya et al. (2009)                   |
|                   |                          | 5 - 15                 | 10              | Normal                        | Noureldin et al. (1994)                     |
|                   | Gravel base              | 10 - 15                | 12              | Normal                        | Noureldin et al. (1994)                     |
|                   |                          | 6.0 - 17.2             | 10.3            | Normal                        | Noureldin et al. (1994)                     |
|                   | Gravel subbase           | 10 - 20                | 15              | Normal                        | Noureldin et al. (1994)                     |
| Overlay thickness |                          |                        | Lognormal       | Tighe (2001)                  |   |
| Elastic Modulus   | Bituminous Layers        | 10 - 20                | 15              | Normal                        | Noureldin et al. (1994)                     |
|                   | Gravel base              | 10 - 40                |                 | Lognormal                     | Timm et al. (2000)                          |
|                   |                          | 10 - 30                | 20              | Normal                        | Noureldin et al. (1994)                     |
|                   | Gravel subbase           | 5 - 60                 |                 | Lognormal                     | Timm et al. (2000)                          |
|                   |                          | 10 - 30                | 20              | Normal                        | Noureldin et al. (1994)                     |
|                   | Subgrade                 | 5 - 60                 |                 | Lognormal                     | Timm et al. (2000)                          |
|                   |                          | 10 - 30                | 20              | Normal                        | Noureldin et al. (1994)                     |
| CBR               | Base                     | 10 - 30                | 20              | Normal                        | Noureldin et al. (1994)                     |
|                   | Subbase                  | 10 - 30                | 20              | Normal                        | Noureldin et al. (1994)                     |
|                   | Subgrade                 | 10 - 30                | 20              | Normal                        | Noureldin et al. (1994)                     |
| Traffic           |                          |                        |                 | Extreme Value Type I          | Timm et al. (2000)                          |
|                   |                          |                        |                 | Normal, Lognormal and Poisson | Zollinger and McCullough (1994)             |

- Step 5: Appropriate distribution has to be determined for each parameter

So, once we get this distribution, you can see here again, for different parameters, studies have shown that different distributions best explain them. So, if you look at the thickness, the normal distribution has been commonly observed to fit the variation, but for overlay alone in one study, a log normal distribution has been seen to have a better representation. Similarly, for some parameters, normal distribution is used, for some parameters, log normal distribution is suggested. In the case of traffic data, studies have shown that extreme value distribution, normal, log normal, Poisson, all of them have been observed to better represent the data. So, based on the type of data that we have, we can fit an appropriate distribution and use the parameters. So, this is step 5 wherein we are assuming

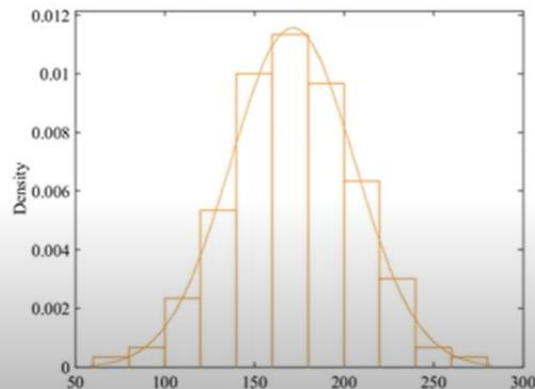


a distribution and using the corresponding mean and the coefficient of variation value for that distribution.

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## Monte Carlo simulation

- Step 5: Obtain a suitable distribution with any selected number of points (say, 1000) for the mean and CoV

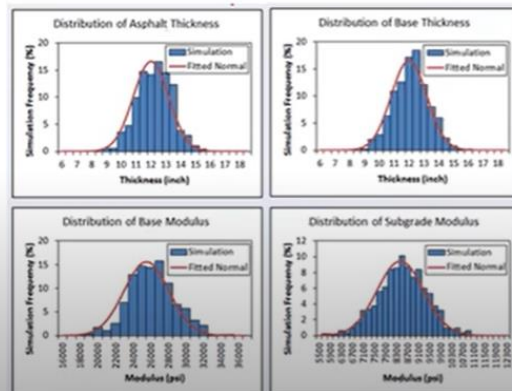


So, this is how we will get the output. So, if I know, let us say again, for the sake of discussion, let us come back and stick to normal distribution. So, this is the mean and standard deviation that I know for this distribution, I am going to use this. We can generate any number of points. Depending upon the accuracy, I can split this normal distribution into any number of data points. And since we said it cannot be done manually, we need the help of a computer; we are going to assume more number of points because that will give us better accuracy. So, in this case, let us say we are assuming 1000 data points. So, what this Monte Carlo simulation will do is give us 1000 points. That can be approximated using a normal distribution with the mean and standard deviation of this particular value. So, it will again split back this distribution into the form of histogram. So, if I count the frequency of this histogram, it is going to be 1000. So, this is what we will get from the Monte Carlo simulation. Now, this can be done for individual input parameters. As we had seen before, we have 3 modulus values, 2 thickness values. So, for all the 5 input parameters, I will be able to get a distribution like this. So, I will get 1000 data points for each input parameter.

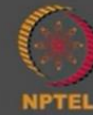
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## Monte Carlo simulation

- Step 5: For each input parameter 1000 points will be obtained for the selected mean and CoV corresponding to that parameter



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So, it will be something like this. So, I will have a distribution which I get for thickness of bituminous, again it does not correspond to the example which I had shown before, but it is just to illustrate what we will be getting from this Monte Carlo simulation. So, we will get a distribution of thickness, distribution of base thickness, distribution of subgrade modulus, base modulus, so like that. For all the 5 input parameters, we will be able to get distributions. Since we have assumed 1000 points, there will be 1000 points for each of these parameters. So, now what are we going to do with these 1000 points? So, I said we have 1000 points, so I will be now having 1000 combinations like this.

So, for my input parameters are  $E_1$ ,  $E_2$ ,  $E_3$ ,  $h_1$ ,  $h_2$ , these are my input parameters. So, in the 1000 points which are generated, again these 1000 points will not be in an order that is maybe this leftmost point will not be the first point, they will be in a random manner, but when I plot it, I will get a frequency distribution like this. The 1000 points will not be increasing from this end to this end, it will not be in a specific order. It will be in a random order and that is why we call Monte Carlo simulation as a random number generator. So, now when I have 1000 points, I will get the first point of point 1, first point of  $E_1$ . I will use it here, first point of  $E_2$ , first point of  $E_3$ , first point of  $h_1$ , first point of  $h_2$ . So, all these values I will be using it. So, this is going to be my first combination. Similarly, second

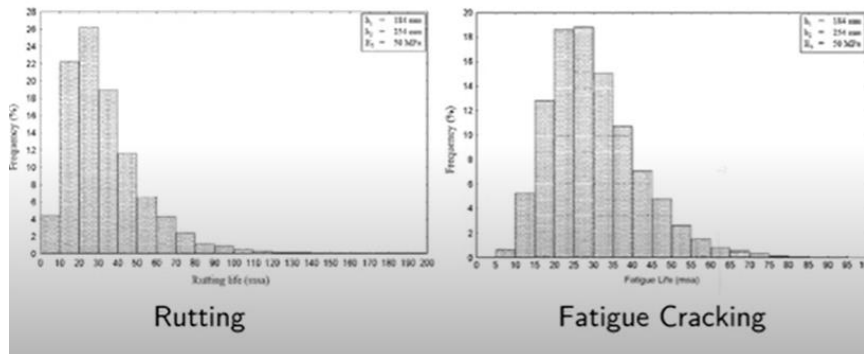
value of E1, similarly second value of E1 I will input here, second value of E2 like that I will get the second combination, like that I will get all the 1000 combinations. We do not have to worry about the randomizing the order because already these 1000 points are in a random order. So, the combinations that we get here will also be in a random fashion. So, we will get 1000 points and 1000 combinations. Now these 1000 points will not be the same as that we had used for calculating the model in this particular step. Before we had only around 41 combinations or so. But here we are going to have 1000 combinations. So, now how do we predict the  $\varepsilon_t$  and  $\varepsilon_v$  value for each of this combination? That is where we are going to use a RSM model.

We had earlier arrived at a model which we said we can use to predict the  $\varepsilon_t$  and  $\varepsilon_v$  value. So, we are going to use that model for each of the combination, input the values in the model, we will get the corresponding  $\varepsilon_t$  and  $\varepsilon_v$ . We are not going to use IITPave for this, we are only going to use the RSM model to arrive at  $\varepsilon_t$  and  $\varepsilon_v$  value. So, we get  $\varepsilon_t$  and  $\varepsilon_v$  for all the 1000 combinations. Now we can use this  $\varepsilon_t$  and  $\varepsilon_v$  to arrive at the corresponding.  $\varepsilon_v$  is vertical compressive, so we will get  $N_R$  and  $\varepsilon_t$  is horizontal tensile, so we will get  $N_f$ . So, for each combination we have 1  $\varepsilon_v$  and 1  $\varepsilon_t$ , so we have 1000 points for  $\varepsilon_v$  and 1000 points for  $\varepsilon_t$ . So, we are going to get 1000 values for  $N_R$  and we are going to get 1000 values for  $N_f$ . So, this is how the output is also generated in the form of a distribution. I will again come back and show you this.

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## Monte Carlo simulation

- Step 6: The distress transfer function is used and for each of the 1000 points, the corresponding  $N_f$  and  $N_r$  values will be calculated to obtain a distribution

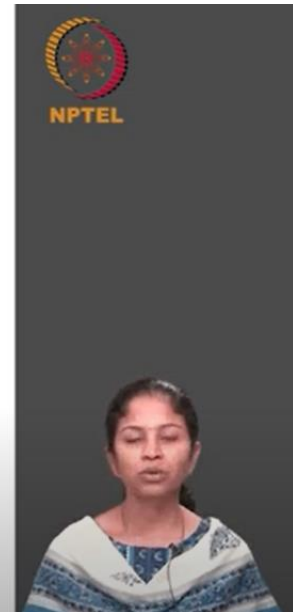
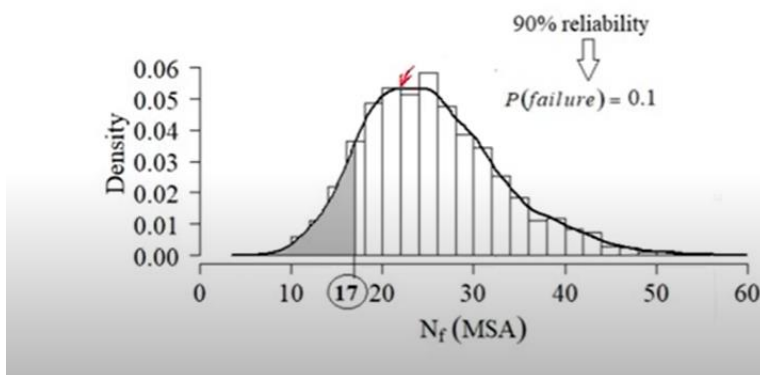


So, the next step is to arrive at 1000 combinations of input parameters for each combination. We can arrive at the 1000 combinations here using the RSM model that was developed. Then this will be the output or this will be the kind of output again not exactly the same output that corresponds to the data I was showing, but this will be the kind of output that we will get. So, this is for rutting, so this is my  $N_R$  and this is  $N_f$ . So, let us say if we had 1000 combinations, I will get a distribution like this for 1000 points. It is not exactly 1000, but let us assume that it is if I sum up all these frequencies I am going to get 1000. So, this will be the output for  $N_R$  and this will be the output for  $N_f$ . So, we get a distribution of the rutting or fatigue life and it is not one single point measurement as we usually used to get because we said this is a probabilistic approach. We are going to use a reliability based design. Let us now see how to use the distribution that we have arrived before for  $N_f$  and  $N_R$  to compute the reliability level for a given traffic condition or compute the traffic condition for a selected reliability level.

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## Monte Carlo simulation

- Step 7: Calculate the expected traffic and estimate the level of reliability associated

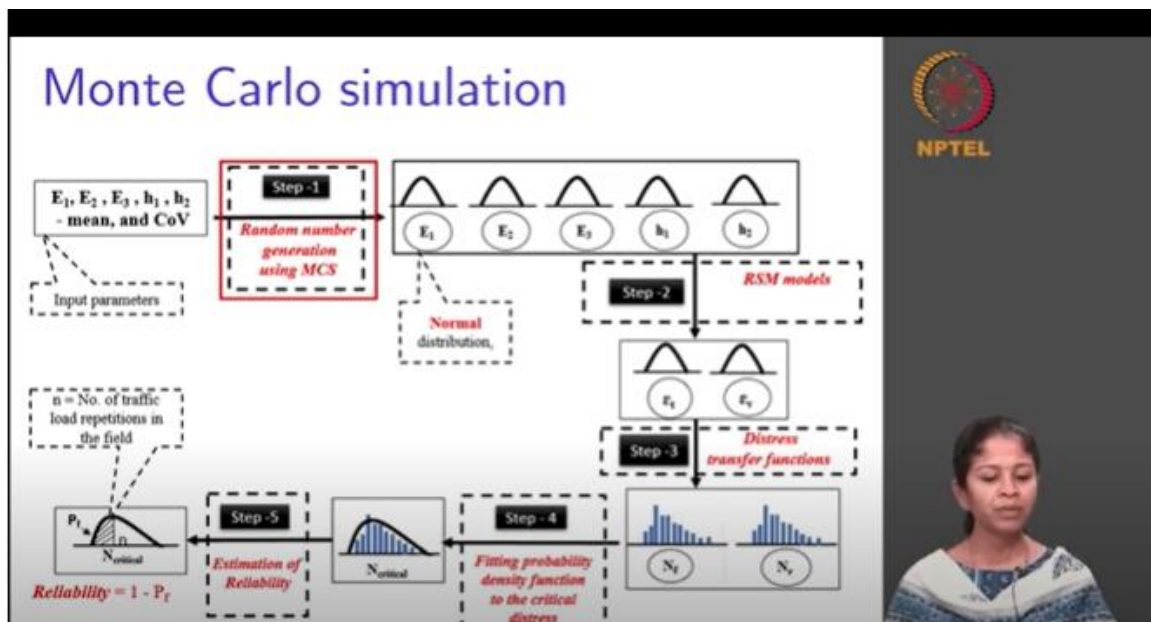


This is the distribution that we have got. So, this is the distribution that we have fit to the  $N_f$  and  $N_R$  values that we have got. Let us say that the traffic value that is observed in field is amounting to 17 msa. So, this is the design traffic that I am expecting in my pavement. Now, for this design traffic let us see what is the reliability level. So, this is 17 msa here. So, the area on the curve to the left of 17 msa which is shaded area is the probability of failure. So, reliability is nothing but  $1 - \text{probability of failure}$ , which is nothing but the unshaded portion. So, my reliability is high in this case, when my traffic is 17 msa. So, I will have a higher reliability level. On the other hand, so we have now computed reliability for a given traffic condition.

The second one what we are going to do now is to compute traffic for a selected reliability value. So, how are we going to do this now? We, let us say that we are interested in 90% reliability or the probability of failure is 0.1. So, we will go back to the standard normal table, find out what is the standard normal deviate corresponding to 90% reliability, we identify the value. So, let us say this unshaded area. This is my reliability. So, this should be 90%. Let us say this is where I will obtain my standard normal deviate. So, what is that traffic corresponding to this? Maybe around 12 msa, right? So, this is the traffic corresponding to 90% reliability. So, if I want 90% reliability, my expected traffic in field

should be 12 msa or lesser. So, this is how we use this information to compute the traffic level for a selected reliability. So, now this is comprehensively what a simulation technique can do and how we use the variability in input parameters, the actual variability observed for input parameters into the simulation technique and we also obtain a corresponding distribution for the output and how we calculate the reliability.

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Let me now put everything in a nutshell. So, this is a schematic diagram again by Donia, which shows how the distributions, how the Monte Carlo simulation is carried out and how the distribution varies. Let us say we have input parameters, we have E1, E2, E3, h1 and h2. So, for all these input parameters, we will be using a mean value and a coefficient of variation. Now, we will have a random number generation using Monte Carlo simulation. So, then for that, we will be arriving at a distribution for E1, distribution for E2, E3, h1, h2. Then we will be using RSM model, we will be predicting the  $\epsilon_t$  and  $\epsilon_v$  value. But before doing this, we would have done some steps earlier, wherein we will define limits for the parameters, use a design of experiments (DOE) approach. Arrive at combinations, use IITPave, obtain the  $\epsilon_t$ , so define limits, use DOE combinations, arrive at the combinations, obtain  $\epsilon_t$  and  $\epsilon_v$  value. So, for these  $\epsilon_t$  and  $\epsilon_v$  value, we will fit an RSM model. So, these things we would have done earlier before getting into this simulation

technique. So, then, we have the distribution for each of these input parameters, whatever is the RSM model that we have obtained here, we will use and we will obtain distributions for  $\varepsilon_t$  and  $\varepsilon_v$ . We will be using the distress transfer function because it is an equation. So, once you plug in  $\varepsilon_t$  and  $\varepsilon_v$  values, we will be able to get  $N_f$  and  $N_R$  values. So, for each combination, we will also have the modulus values. So, when we have to use the modulus value for computation of  $N_f$ , for that particular combination, whatever was the modulus in that combination, we will use that modulus to compute the  $N_f$  value.

Then once we get this, we will fit a probability density function. Again, it may not be a normal distribution like how we have fit for the input parameters. Here also we can fit any appropriate distribution, obtain the distribution for it. So, we have obtained a distribution. Now, we will calculate the n value which is number of traffic load repetitions that will be observed in field. So, now we plot this n on this plot. So, this is going to be the probability of failure and this will be the reliability which is nothing but 1 - probability of failure. So, this is in summary about the simulation technique and how to use it for reliability estimation.

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## Difference between different approaches

- IRC 37 : Mean value of inputs – 50% Reliability  
Higher reliability levels for distress prediction
- AASHTO: Mean value of inputs – 50% Reliability  
1993 Reliability for Traffic estimate
- MEPDG : Mean value of inputs with different degrees  
2004 of uncertainty  
Desired reliability for distress prediction  
Reduction in variability through calibration constants



Now, let us summarize the difference between different approaches that we have been discussing before. We have IRC 37, we have AASHTO 93 and we have MEPDG 2004. Again, I am not including the simulation technique, I am only restricting myself to the specifications or the guidelines given by different agencies. So, in IRC 37, we are using mean value for all input parameters. So, as far as the input parameters are considered, we are associating a 50% reliability to it. But we have different reliability levels for distress prediction, we have 80% and we have 90%. We have individual equations for 80% reliability and 90% reliability. But we will be able to predict only for these 2 levels of reliability. If I want a different reliability of 70%, then it may not be possible. So, this is with respect to the IRC approach.

Now let us see the AASHTO 1993 approach. There also we are only using the mean value of inputs. So, that also corresponds to 50% reliability only. But when it comes to the reliability, we can choose any reliability level. We can choose any reliability level and correspondingly we will be computing the standard normal deviate and using it for computing W18. Number of repetitions we have already defined this. So, we will be using it to compute W18. So, this is with AASHTO 93. Again, the similar kind of setup as we have for IRC 37, except that we have flexibility in choosing the reliability level.


Next comes MEPDG 2004. So, in this design procedure also we saw that we are not again using any distribution. We are using only mean value for all the input parameters. But the accuracy with which the input parameters are estimated is accounted through different levels of input. So that is to account for the degree of uncertainty associated with the input parameters. The second one is the reliability associated with distress prediction. So, we saw that we will be predicting the distress for the mean value, but we can correct it for any different reliability level using the standard error estimate and the standard normal deviate corresponding to a reliability level. So, the distress can be computed for any level of reliability. Then we will also be reducing the uncertainty associated with prediction through estimation of calibration constants.

We saw that we have global and local calibration constants. So, once we determine the global and local calibration constants, we will be able to account for as we had seen earlier, the variability in material parameters, the variability in traffic and environmental



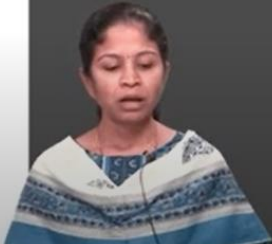
conditions and the variability associated with the distress prediction model. But for that, we have to collect field data. But once we collect field data, we will be able to estimate standard error, which will take into account of all these variations. So, this is the difference between different design approaches and consideration of reliability in each of these approaches.

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## Summary

- Pavement design – involves uncertainties with respect to input parameters, construction process, traffic and environmental conditions
- Impacts the predictive capability of the performance of pavement
- How are these uncertainties factored in the design procedure?
- Input parameters – mean values used in most cases which translates to 50% reliability




So, now let us summarize what we have seen about reliability in general. So, the first aspect that we said is with regard to the uncertainty, we said pavement design process involves lot of uncertainties. So, it could be with respect to input parameters, it could be with respect to traffic and environmental conditions or it could be with respect to the variation in construction process that is the difference between the as designed and as built case. So, there are again random variations in climate and traffic. So, there are many factors which contribute to the variation in the pavement design process. So, the variations in all of these parameters impacts the predictive capability of performance. So, if I am able to precisely specify values for all these input parameters, I will be able to precisely say what is the expected performance of the pavement. Since we have lot of uncertainties associated with all these input parameters, we also have the same degree of uncertainty or the combined influence of these uncertainties on the performance prediction also. So, the major challenge

that lies in here is how to factor in all these variations into the design process. So, we have seen different design procedures, different methodologies which attempt to consider all these variabilities into the design process.


So, the input parameters are the ones which are more commonly discussed here. The input parameter variability is quantified and it is represented using a mean and a standard deviation or a coefficient of variation value. Since this data could be collected or the typical variability observed in field from other locations can be used, more often this is considered in the design procedure.

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## Summary

- Distress models – corrected for desired reliability
- Distress models – calibrated from specific studies or specific data set; limited options to re-evaluate the constants
- Simulation techniques – Consider actual variability in input parameters as part of the design process
- Involves unreasonable computation time when inbuilt into the design procedure



Then we have distress models which are corrected for reliability. Again, as I was mentioning throughout in this lecture, the design models are empirical equations which are obtained based on specific studies conducted by individual agencies. So, they cannot be generalized for other conditions. To account for this variability, we have to calibrate the constants in the equation for the specific location. So, these values are given as specific numbers in certain design guidelines, whereas like MEPDG specifies it could be given as generic values also. And we have options to arrive at these generic values in terms of global and local calibration constants. So, once the model is adjusted, we will be able to

predict the performance with a higher degree of certainty. So, that is another set or another aspect wherein the certainty associated with the distress prediction can be improved.

Then we can use advanced techniques like simulation techniques. These simulation techniques again so far, none of the design methodologies consider the actual variation in input parameters. As I was mentioning, it is possible, but it is highly time consuming and any software will not be able to give a reasonable output in a practical amount of time. So, that is why this is not considered in any of the design guideline. But we have seen other techniques which actually take into account of the variation in input parameters and use it in the design process, identify what is the corresponding variability in the output and use it to compute for a specific, use it to compute the reliability for a traffic condition or compute the expected traffic for a given reliability. So, this is in summary about the reliability aspect. Thank you for your time.