Mechanical Characterization of Bituminous Material
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Lecture No 6
Linear Viscoelastic Response part 04

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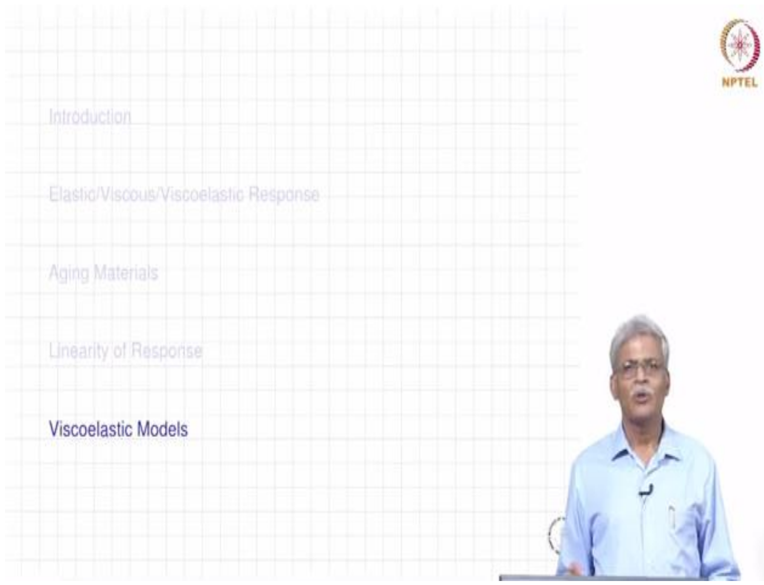

So what we are going to is to introduce some very elementary viscoelastic models here.

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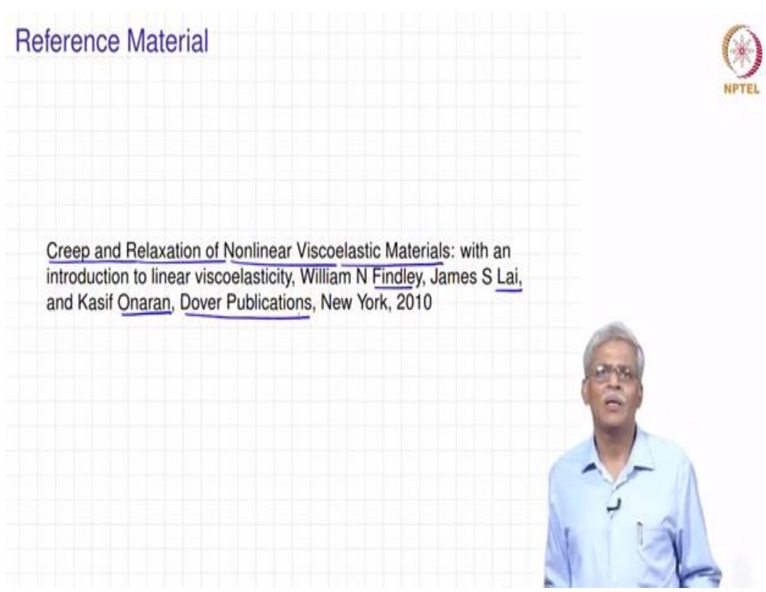

And I suggest that most of the material that is discussed. Henceforth, in this is based on the classic book on Creep and Relaxation of nonlinear viscoelastic materials: written by Findley, Lai and Onaran. Originally it was published by North hand land, and later Dover published it as a very cheaper addition, some portion of this material will also be shared as a course material here.

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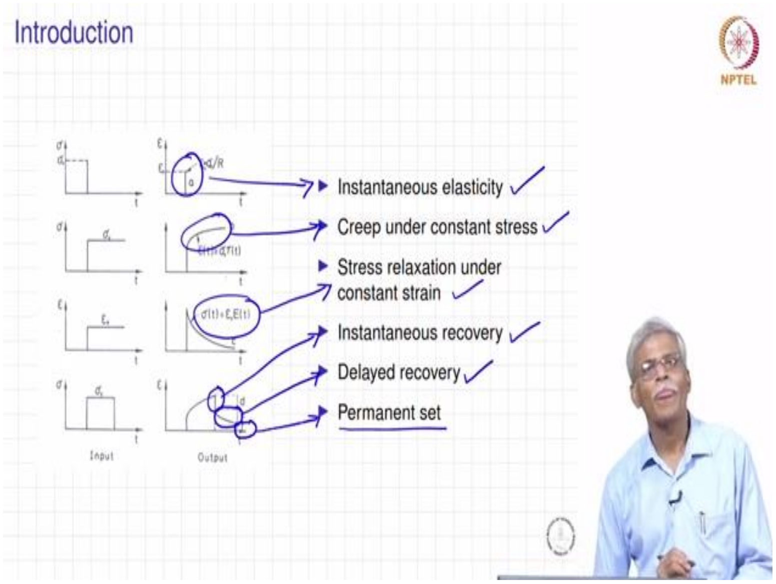

So, what we are going to do is, so let us write some basic things about what do we really expect from a material. Or what is our understanding of the fundamental future of any viscoelastic model. So the first and foremost thing is, this particular model should exhibit some kind of instantaneous elasticity. Right? Then this model should also be able to exhibit some kind of a time varying strain under constant stress.

And if you are applying a constant strain, there should be some kind of a stress relaxation. That can be instantaneous recovery. And there is also necessary for delayed recovery and some kind of response which is what you can call it as permanent set or residual strain or irrecoverable strength. So, what it means is, if you take a look at it, so this your instantaneous elasticity that you are looking at. This is your creep under constant stress that you are looking at.

And this is the stress relaxation that you are looking at. And we are also looking at some kind of instantaneous recovery and then a delayed recovery and a permanent set. So these are some of the basic features of any viscoelastic model that we are really looking at, so let us try and see whether we could build the model.

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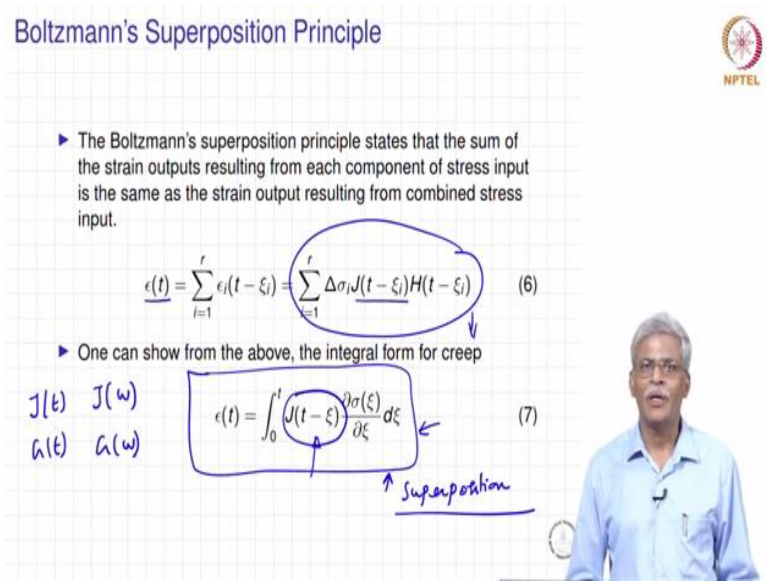

Now again, to reemphasize why we should know this because we yet do not know what kind of form for J that we need to use. So for that reason, we are going to construct some simple model,

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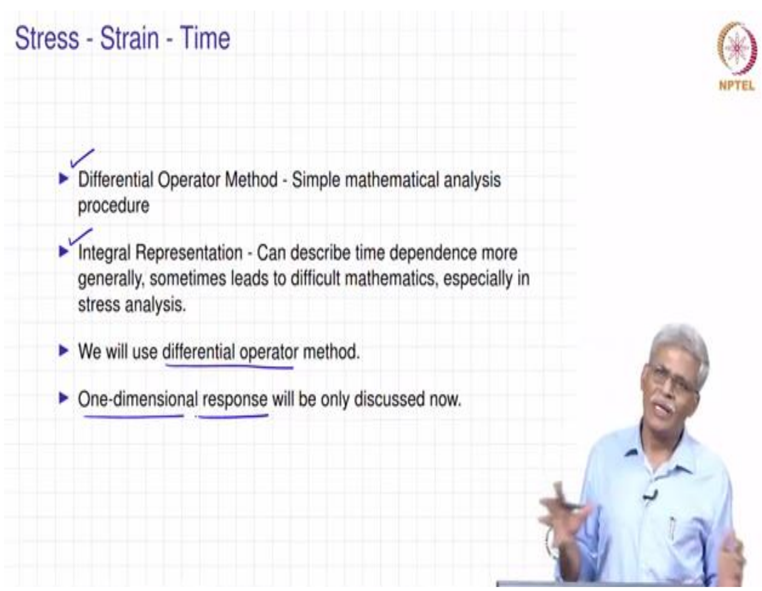

and we are not going to get into too much of the computation, here we will work with very simple differential operator method. So we could have differential operators method, or we could have integral representation. But sometimes, when we are doing stress analysis the integral representation can become little bit tricky. But as far as this particular course is concerned, we will stick to the differential operator method.

And there are many advanced books for instance theory of viscoelasticity by Christensen, and there is also a book on viscoelasticity by Lakes. So this book's cover more from the perspective of the integral representation. And we will also restrict our discussion, only to the one dimensional response. We need to emphasize this point again and again here because the main motive of this particular course is to characterize the response of bituminous binders and bituminous mixtures.

And whatever representations of the mechanics of this material that is needed to explain our characterization technique will only be addressed here, the theory of viscoelasticity is very detail, there are easily three to four semester course worth material that needs to be discussed. We will not get into all those details here.

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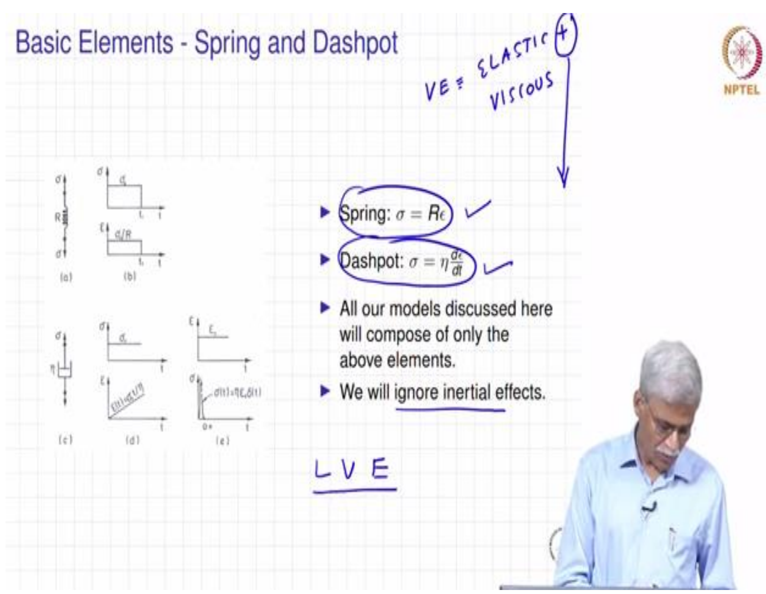

Okay, so recollect the discussion that we had in the earlier lecture on, the use of the Spring as well as the use of a Dashpot. We said that the viscoelastic response is sum total of elastic + viscous. Please understand that we assume that addictive decomposition here it is not necessary that it has to be addictive in nature. And we will ignore what is really called the inertial effects, we will ignore the body forces.

And we will be discussing only with the respect of springs and Dashpots. There are different types of tamper elements that are available. We will not be discussing all those things there are nonlinear springs as used nonlinear dashpots as used. There are also some viscoplastic elements that are available in the literature. We will not be discussing any of those things, as I repeatedly the emphasis. We will be looking into linear viscoelastic response.

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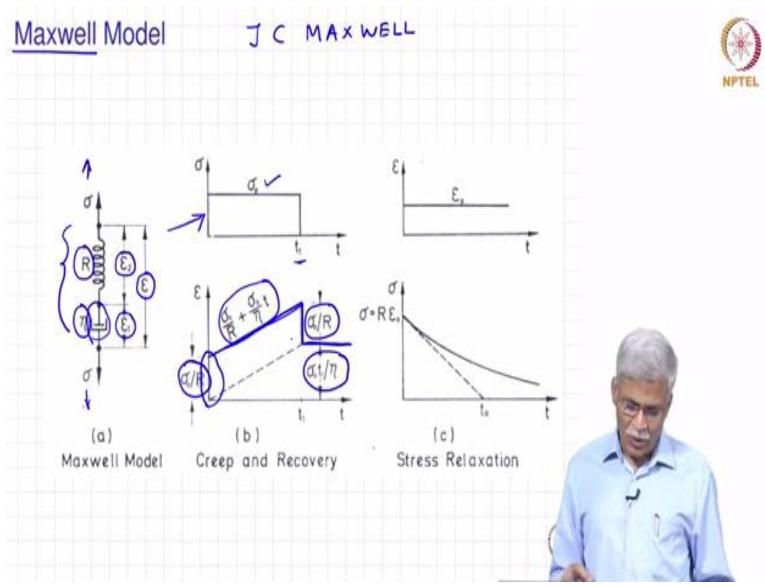

Right, the very first model that is available, is the model due to Maxwell, J.C Maxwell, one of the well-known scientist of our time. So, This is consist of a linear springs and dashpots in series okay. So we are going to call this as R, and we are going to call this as η, and we are going to subject this combination to force that will result in a stress of σ. So the strain here is ε_1, the strain here is ε_2 and the total strain is ε.

So, this is the kind of load that these applied here. So this is as far as creep and recovery is concerned, what could also do the experiment in which we apply only the stress relaxation. So now, as of now without even deriving anything we can actually take a look at this picture, and try and see whether one can guess what will be the response of this material. So, what we need to do is not worry too much about the value is that are written here.

But only, take a look at the shape of this material, shape of this particular graph. okay right. Do not worry about that stress relaxation graph that is given here, so it will get little more time to derive this and get involved in this, as of now you take a look at it so if you apply a σ of σ_0, in this particular form. Since, you have connected these two elements in series, the spring is going to instantaneously extend and that is given by this portion right?

Then since the load applied is constant, whatever is the extension that you are going to see the spring will be attained instantaneously. Then after that, what will really happen? The dashpots starts extending, and if you recollect, how we used to draw the picture for the dashpot in the viscous damper element that we discussed earlier, it is going to increase in this particular way.

Now what we are going to do this at this particular point, we are going to unload it at the t equals t1 and you are going to see that as you unload it, the spring is going to get back to his old position. So, it will exactly recover, whatever we got here instantaneously. And you are going to see that this strain is going to be like this, because we know for a fact that this strain will not be recovered as far as that damper is concerned.

Okay, so now we will find out how to get these values, so if we have to get these values, we need to do some elementary calculation.

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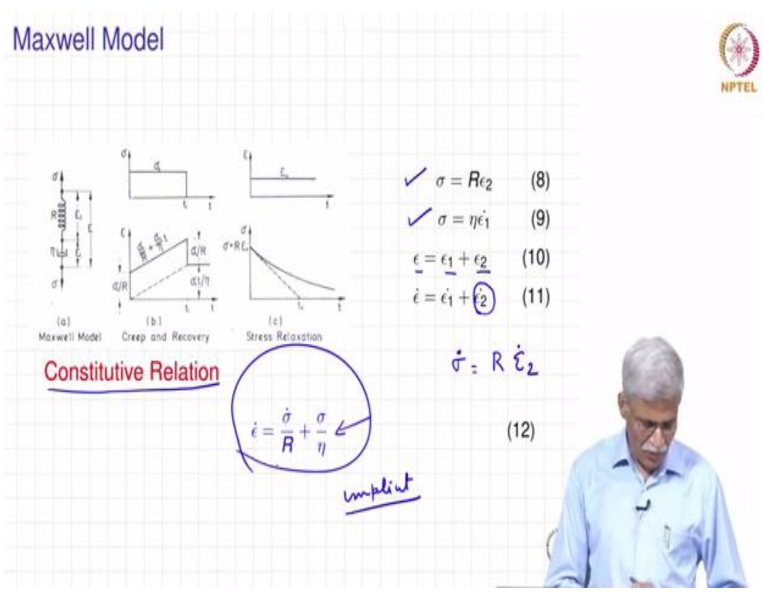

Okay. So we will do that now. So what we are going to do is, we are going to write $\sigma = R\varepsilon_2$, that is the model or the constitute expression for the spring and the $\sigma = \eta\varepsilon_1$, that is the model, or the constitute expression for that dashpot. Now since we have connected this series, $\varepsilon = \varepsilon_1 + \varepsilon_2$. So now what you really have to do is take the time derivative of equation 10 so it is nothing but it is ε_0 is $\acute{\varepsilon}_1 + \acute{\varepsilon}_2$. Now what we now need to do is to substitute ε_2 you see here, so we take a time derivative of this, so that is going to be σ dot = r times $\acute{\varepsilon}_2$. So you substitute it here, rearrange the terms, and you are going to get this particular expression, now this particular expression is written by me here, as the constitutive relation. Okay, now this is basically is the expression that you got.

And you see that this is that ordinary differential equation, the first order equation in time. And now we start feeling the slowly the complexity of this response. So we are used to writing constitutive models, which are explicit in the sense that you are going to have a dependent variable, you are going to have a independent variable and all you do is to substitute the appropriate parameters and get the response of this material.

But right now what you are seeing is the response is not explicit, but kind of implicit not implicit in the rigorous sense of the mechanics that is considered, but that it is not straightforward so that means for any given loading conditions, we need to solve this expression. And only then we will get the actual response that we are really looking at okay. So this is the model form that we got here.

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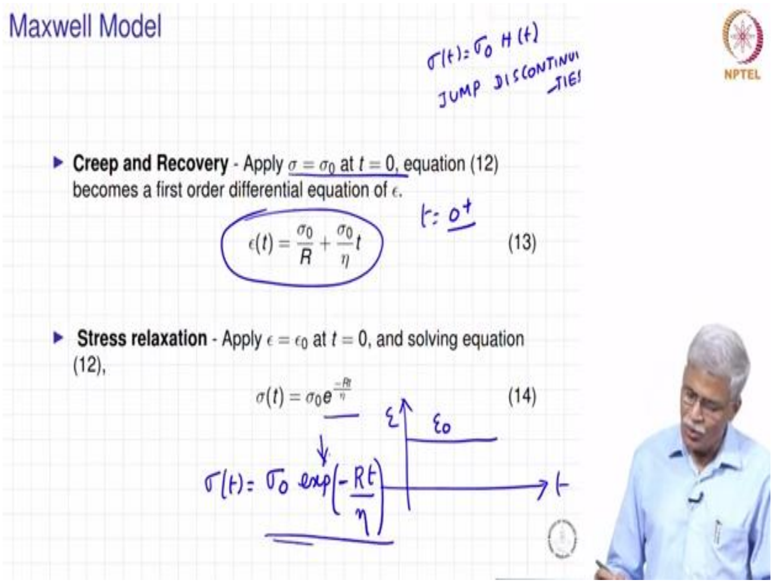

Now how do we really solve this creep and recovery and stress relaxation. Now, there are different ways in which this expression can be solved. You can solve it in a really, really rigorous way, because there is always going to be a jump. So how are you going to get it. So this σ_0 H (t). Now if you try to take the derivative of this because there is going to be derivative here.

You have to also consider what are really called as jump discontinuities. We are not going to do all those things here. So if you look at the book of Wineman and Rajagopal, which I refer to in this earlier part of the lecture, a clear cut definition of derivation on how to consider the jump discontinuities are given. We are not going to do all those things, but rather we are going to do it in a fairly straightforward way.

We are going to assume that $\sigma = \sigma_0$, at t = 0, ideally it should be t = 0+, but we are going to assume that it is. So when we substitute and solve for it, this is the expression that you are going to get here. And what do we do for the stress relaxation, for stress relaxation, what do we really apply, for you apply, ε_t and this is going to be ε_0. So when we apply this particular thing you are typically going to get an exponential term here, it is fairly well understood.

So, if I write it a little carefully, you are going to get exponential – r times t divided by η, so there is going to be some kind of an exponential decay. So now let us go back, take a look at this picture because we seem to have found out what are the values associated with a picture now. Right?

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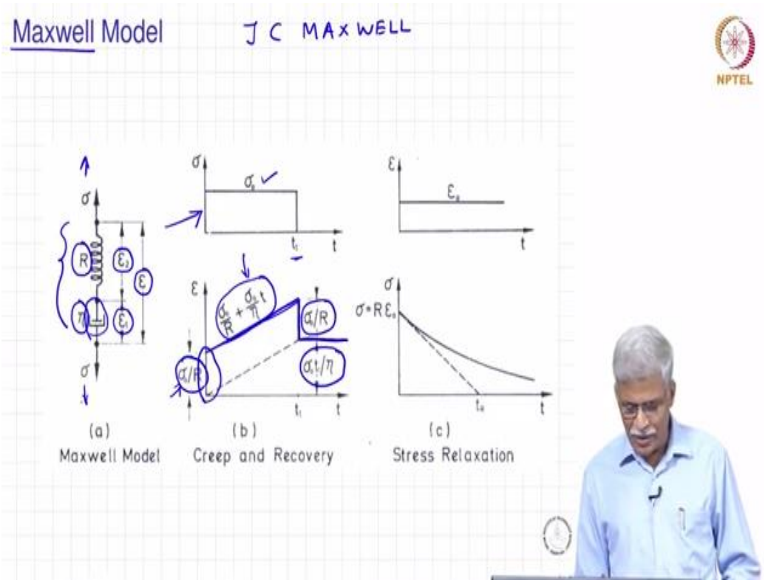

So you can actually see how we got to σ_0/R. How did we get this? This is this strain corresponding to this. Now you can also check that when you solve this, you got σ_0/R + (σ_0/η) t. You can actually see this particular thing here. And similarly, when we are unloaded we got another σ_0/R, and then you will also notice that the residual deformation that you see here is (σ_0t1) /η. Okay.

So now we will see how to go about getting there. The first thing is we need to see a little more about this particular expression. Okay, because this expression seems to be fairly straightforward because what you see here is the response, something like this. That is what you are seeing here. This is given by σ_0/R. And this expression takes care of this. But what about this expression? So, let us try to do some interesting calculation here, and we will see the following,

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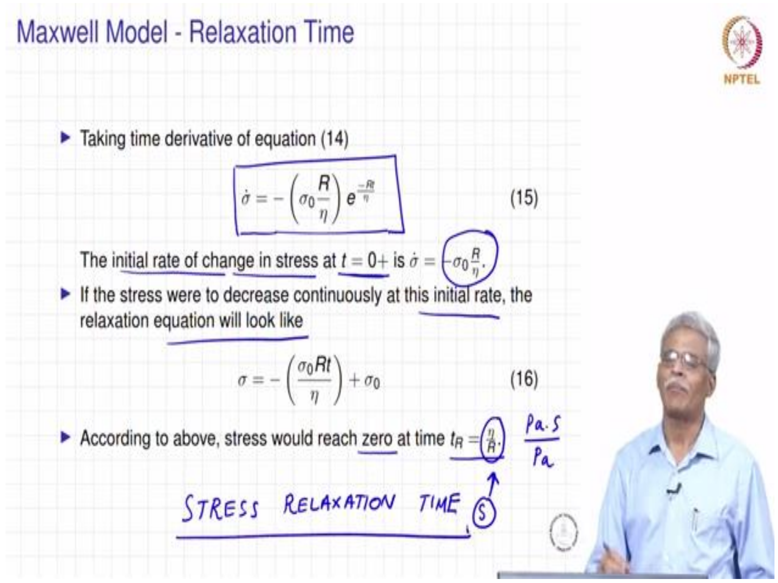

So we will take the time derivative of this expression. There is a reason why we want to do this particular procedure now. So when I take the time derivative of equation 14, you should check it out by yourself. It is necessary that you do all the calculations on your own and check it here. So, you are going to see that you get something like this. The initial rate of change in stress at t $=0^+$ is given as (σ_0/R)/ η.

So what it means is, now take a look at this particular figure that you see here. And this exponential decay was derived by S in the earlier expression okay. So we got what is really called this σ_0 exponential (–Rt/η). Now, What we really want to see as the time, t keeps moving in this direction. That is an exponential decay. Right? Now, what really happens here.

So there is a jump that is given by $R\varepsilon_0$. Now what we want to see is if the stress decays at the fastest rate possible. And what is the rate? You take the derivative of this expression and set t=0, and find out the slope, you are going to see that this line, it will go to zero at a specific point, and this particular time T_r has a specific meaning in viscoelasticity. Okay. So what it means is, according to the above, if the stress were to decrease continuously at its initial rate the relaxation equation will look something like this.

So according to the above, the stress would reach zero at a time, $\tau_r = \eta/R$. So what is the unit of this, this is Pascal second. And this is Pascal. So the unit for this is S second, and this is called stress relaxation time, very, very important parameter as for us viscoelastic response is concern.

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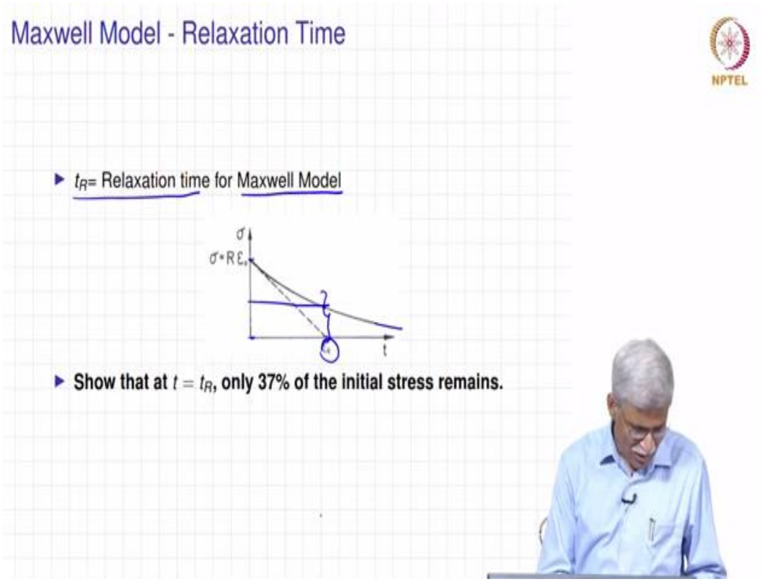

So, this is the stress relaxation time for the Maxwell model that we have derived here. Now as an exercise you can do this, it will be very easy for you to find out and in fact the solution for this is available in Findley, Lai and Onaran. So what it means is when you hit this particular thing you are going to be able to see that the amount of stress, that is get to be relaxed is going to be only 37% of the peak stress that you saw here.

In a sense, this is a very interesting parameter that tells you how fast the material will relax in time. Okay? And we need to understand that, since the stress relaxation can keep going for ever. We do not know when it will actually go to zero because this is going to be an asymptotic solution. So for us to understand and compare how fast stress is go to zero, it is always beneficial to have a parameter that could indicate the variation of the stress relaxation and that parameter is the stress relaxation time.

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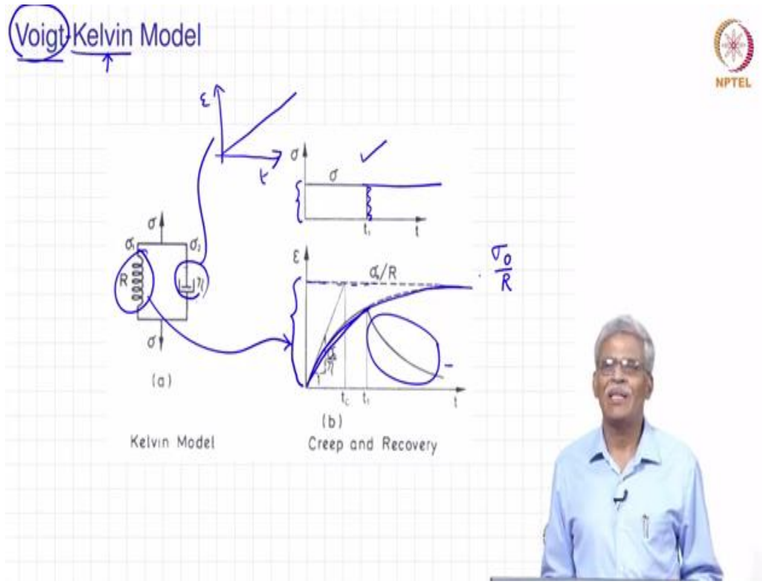

So now let's take a look at another model very interesting model, given by Voigt-Kelvin. So this has to be pronounced as Voigt and this is Kelvin, Lord Kelvin. Right? So this is the same model, same elements except to we are now connecting in parallel, the Maxwell model was in serious this is in parallel. Before we continue, I need to ask you a question. So is Maxwell model fluid? Okay? Think about it. Right?

So we come to that Kelvin Model. Here, we are connecting these two elements in parallel here. So we apply a stress here and we should be now able to find out what can what will be the response of this material. Now, since we have connected them in parallel, the strain response is the same for both the elements, but stresses are going to be completely different. So again without even getting into actually deriving the response of the material we will try and see whether one could find out how the response of the material is?

Let us say applied a load of this particular form and this is applied in particular way. So there is a spring and there is a dashpot. Both are connected in parallel. We subjected to this stress. If there was only this spring, there is going to be an instantaneous jump given by this for the spring alone. Okay? Now, if there was only a dashpot, what it is going to be. It is going to be, if you are talking about the strain it is going to be something like this.

Right now, when we combine these two things what will really happen, one will get something of this particular form. Right? What, why is this happening because the spring may try to instantaneously extent, but since the dashpot cannot do that, dashpot will try and restrained it. Therefore, know what will really happen so do not worry about this particular thing as of know. What will really happen if we apply this stress forever? Right?

Now what can happen for any given σ_0, the spring can extend only up to σ_0/R. So, that is given here. So this strain will reach this particular portion, and then what will really happen. The spring will refuse to extent, and the dashpot has to remain as it is. It cannot move because the spring will not restrain the motion. Okay, so that is how we have to really understand it.

Some of the people, who work on the foundations of mechanics, do not really like these kinds of illustration of the viscoelastic principle using the dashpot, but for undergraduate and graduate students for being exposed to these kind of concepts, at the very preliminary level, such kind of illustrations helps us to understand how the material response is. Okay?

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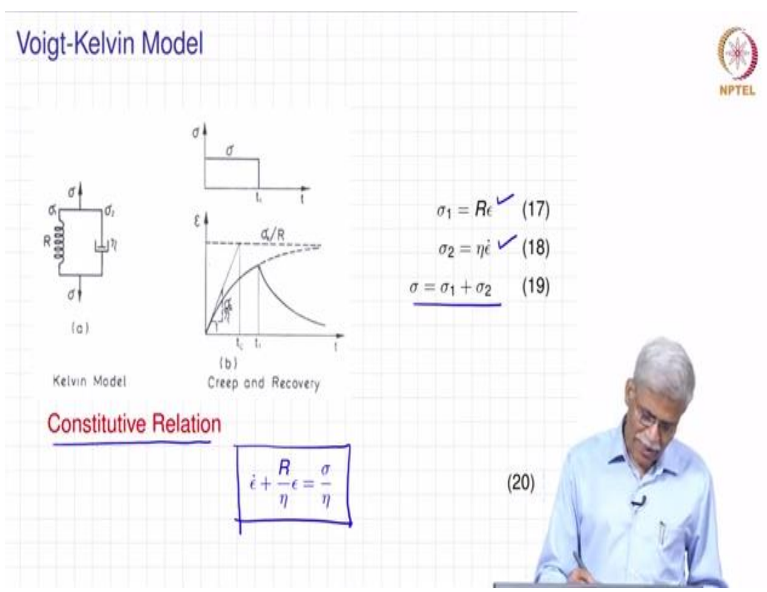

So the same way in which we discussed for maximal model. We are going to write $\sigma_1 = R\varepsilon$, $\sigma_2 = \eta\dot{\varepsilon}$ and the σ is equal to $\sigma_1 + \sigma_2$. So please do this simple algebra. So you need to write the expression for σ here, expressions for σ_1, expression for σ_2 and rearrange the term, you are going to get a model of this particular structure. So again, this is a constitutive relation.

Again, it is not an explicit expression, it is going to implicit. So we need to solve this expression to find out what exactly is the manner in which this is going to be.

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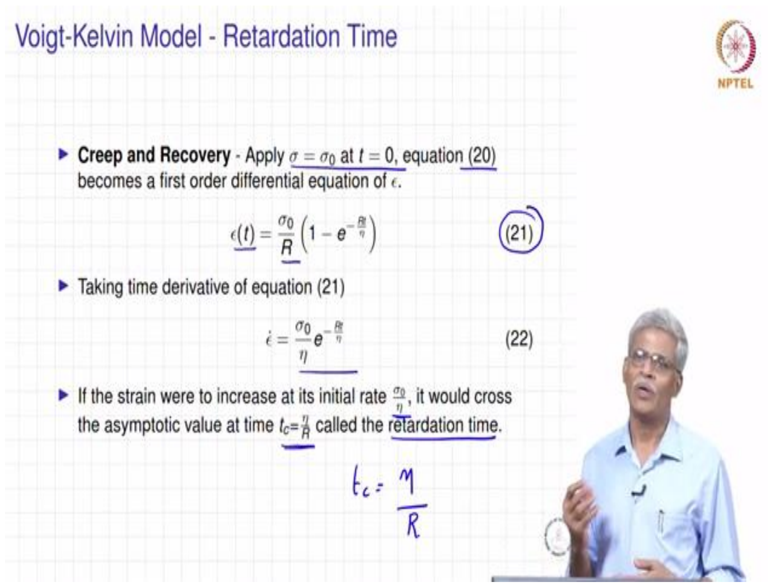

So, how we do this, we do creep and recover test, applying σ equal to σ_0 at t equal to zero. And when we solve this particular equation 20 is given here, you are going to get a solution of the following form. Again I need to emphasis here, that we are not solving this in the most rigorous way, we are solving it in a very simple way. So this is the expression that you are going to get; $\varepsilon(t) = (\sigma_0/ R) (1 - \exp(-Rt / \eta))$.

Now please focus your attention on σ_0/R if you go to the previous expression what can really happen. So when the total strain will reach σ_0/R. It will reach as t goes to infinity, in which case, this portion will vanish, and you are going to get σ_0/R here. Now there is an interesting analysis, one can do similar to what we did in earlier thing. We want to also find out how that time rates of change of the strain various here.

We want to find that out because what we really want to do is, what is the fastest way in which this strain could start evolving in time and so to find that out we are going to take the derivative of equation 21. This is what we will get here. So, if the strain is going to increase at its initial rate of σ_0/η. It would cross the asymptotic value at the Tc given by, so this is the parameter Tc and this is called as the retardation time.

If you write this carefully this is going to be η/R. The interesting thing is the expression here looks exactly similar to the stress relaxation time that we discussed earlier. So for the Maxwell model we mentioned about the relaxation time and for the Kelvin model we are talking about the retardation time. We need to understand that we are using this world of retardation; we are talking in terms of the strain and when you are talking about the relaxation time, we are talking in terms of the stress. Okay?

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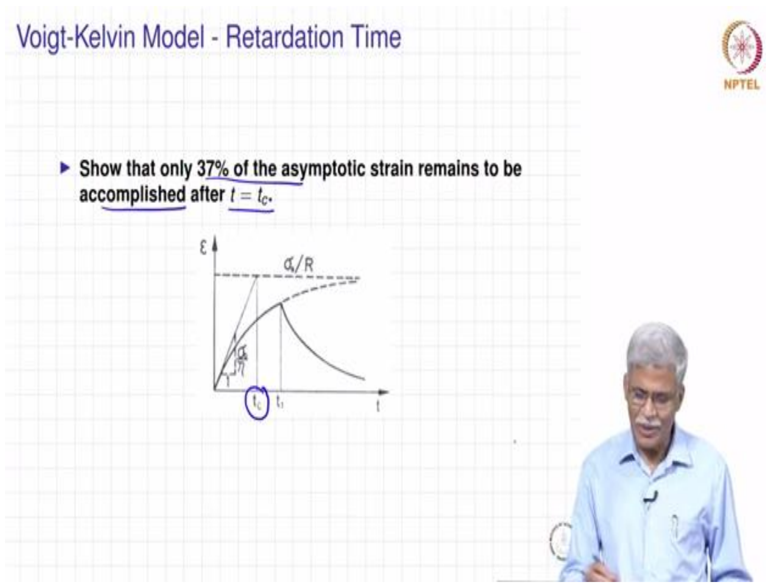

So, let us understand that subtlety very carefully. Again, similar to what we think what you did, as the exercise in the earlier one, you can always find out. This is the total strain at t is equal to τ_c. Those see what will be the remaining strain that needs to be accomplished and you should be able to find out that it is only 37%, the solution for this is available in the material in the book by Findley, Lai and Onaran.

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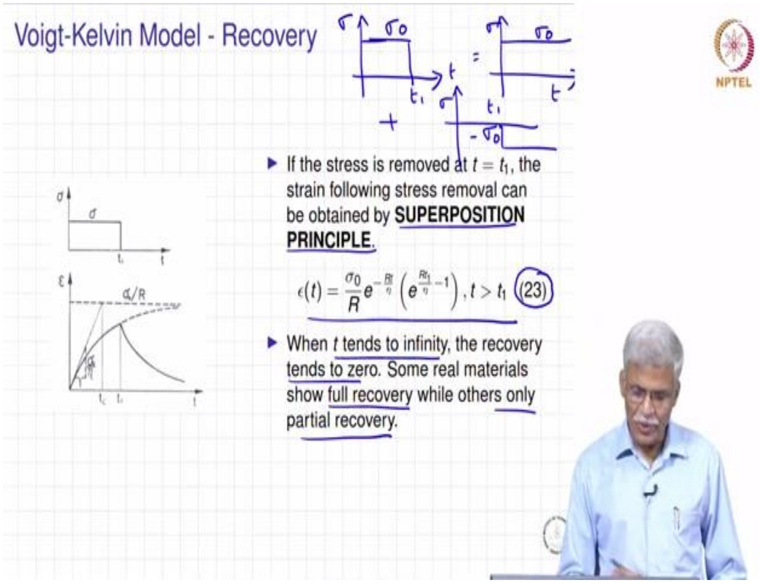

Right now, what we need to do is we need to understand how to solve this for the creep and recovery. Because what we did here is we are applying a load, something like this. Okay, whatever solutions that we have written here for ε(t). It is just that the time keeps running and σ_0 is here. So how do we really do this? What we are going to do is we are going to appeal to the superposition principle.

So what is the superposition principle. If you recollect what we did earlier, that if you want to apply something like this t1, t, this is σ_0. We are going to break it into two portions. One in which σ keeps running forever and another in which at t equal to t1, you are going to a minus σ_0. Go here, so let us take this particular case here, (σ_0/ R) (1 - exp (-Rt / η)).

So, if we are to write this for the particular case for the loading case, it is going to be (σ_0/ R) (1 - exp (-Rt / η)) plus you are going to apply (minus σ_0/R) (1 - exp (-R(t-t1) / η)). So this is the expression that you have to solve so if you simplify all this, you are going to get this particular form, and this is going to be full form for t greater than t1.

So t tends to infinity the recovery tends to zero. And while some real material will show full recovery some of them will show only a partial recovery. I suggest that the students who are taking this course. Try and see whether they could get this particular expression by doing this simple algebra that I illustrated in the earlier slide.

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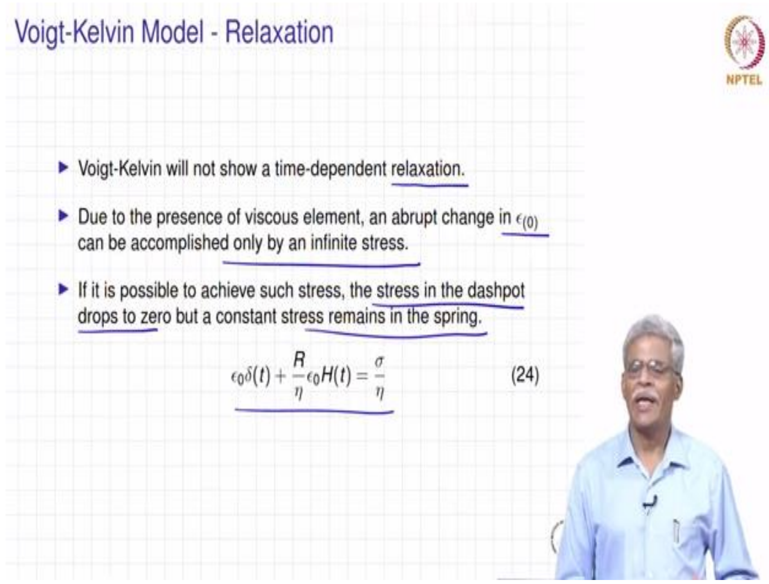

Now, similar to whatever we discussed as far as the dashpot is concerned, in which showed very clearly that one cannot do a stress relaxation experiment in a Kelvin model, one really cannot do a stress relaxation experiment. Since there is a presence of viscous element which is connected in parallel to a spring it will not be easy to achieve an instantaneous strain ε_0 and one can do it only by means of infinite stress.

And if you could that this is how it is going to be but what will really happen the stress in the dashpot will immediately drop to zero, and there will be a constant strain that will remain in the spring, for this particular model will not exhibit the stress relaxation part.

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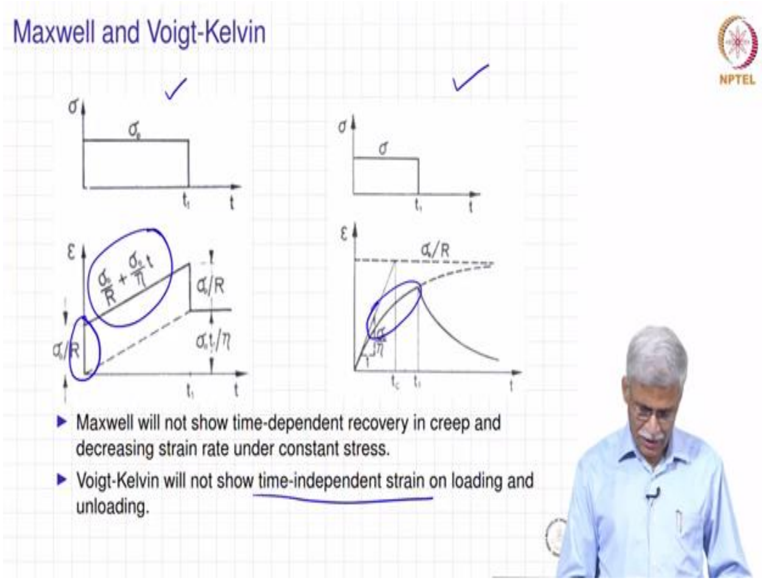

So let us now look at both of them together. So this is your Maxwell model for creep and recovery. This is the Kelvin model for creep and recovery, some of the points that we wrote what a models do. We wanted a specific time dependent creep, as well as a time dependent recovery, but that that is something that it will not do whereas. This particular model will do that but you will also notice that.

While the Maxwell model will show you a nice instantaneous strain or a time independent strain the Kelvin model will not show this time independent strain on loading and unloading condition.

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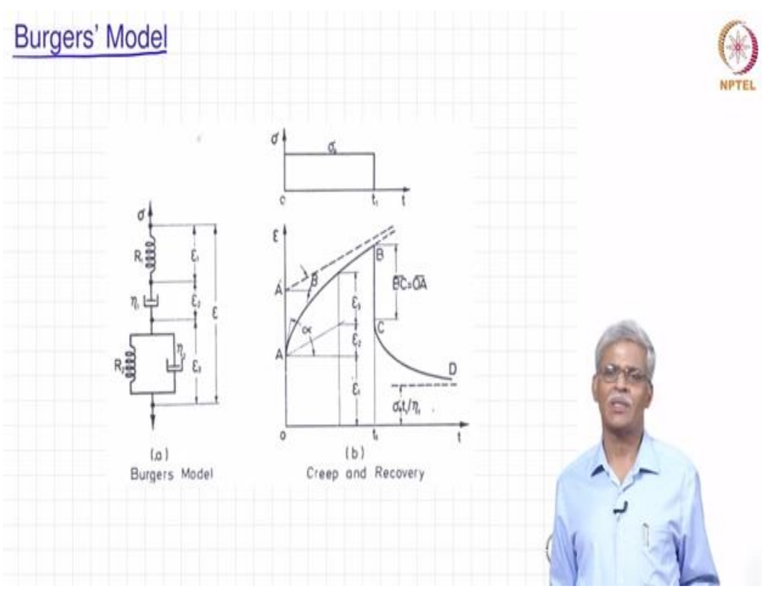

So what we need to do is not use any of this model, because while one model can do be a very good job as well as the stress relaxation is concerned, and another model can do some decent thing about explaining the creep and recovery both of them are really not useful modeling the response of the creep and recovery as well as the stress relaxation. For doing that, we need to use what is really called as a Burgers model, and that we will discuss in next section.

So, please go back, look at all the derivations that have been simple derivation we discussed in this particular lecture and try and see whether you can re derive all their own. Also please answer that whether this is a solid model, or whether this is a fluid model? So we are talking about viscoelastic solid and viscoelastic fluid. Try and relate based on the discussion that we had in our earlier classes about the solid as well as the fluid response of the material. Thank you very much and in the next class will be discussing the Burgers Model.