

Design of Masonry Structures
Prof. Arun Menon
Department of Civil Engineering
Indian Institute of Technology, Madras

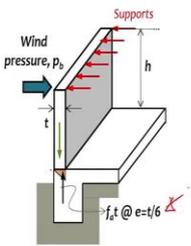
Module - 03
Lecture - 15
Strength and Behaviour of Masonry Part - V

Good morning, we will continue our lecture on the Behavior of Masonry under the action of out of plane loads, bending of a masonry wall and capacity under out of plane bending.

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- **Conventional bending analysis**
 - Wall on concrete foundation and laterally supported at top
 - **Stage (a):**
 - Tensile stresses from bending suppressed by compressive stresses due to wall weight (f_c)
 - **Stage (b):**
 - Tension at wall base on the windward side (limiting condition: triangular distribution)
 - Wall acts as a propped cantilever: $M_b = \frac{p_s h^2}{8}$
 - Tensile stress at base: $f_t = \frac{p_s h^2}{8S} - f_c$
 - For zero tensile strength, limiting wind pressure for cracking: $p_s = \left(\frac{8S}{h^2}\right) f_c$




So, we were looking at the conditions under which out of plane bending becomes essential and critical as a resisting mechanism of masonry and we are examining unreinforced masonry still. So, the case of requirement of resistance when you have vertical bending, when you have horizontal bending and when you have a combination of vertical and horizontal bending is the context in which we are going to be examining the behavior in bending. It is in the presence of gravity forces, so it is actually a combination of bending and compression induced by the gravity forces.

So, to begin with, let us examine how from conventional bending analysis, assuming linear elastic behavior of the material, we can put limits on the strengths of the material and arrive at possible estimate of the lateral load that the wall can carry at critical points

of the behavior. So, that is the first thing that we will examine, and of course, move on to a possible non-linear stress strain relationship that can be assumed in the cross section and if that becomes our basis, the force displacement coming out of the analysis can be different, ok. So, we are looking at a wall which has lateral support at the top and it is sitting on a concrete foundation, it has a foundation and it is a different material, assume you have a plinth beam there.

So, you will have a different material between your brick masonry and the foundation material and therefore, it is possible that you can have a rotation at the base if a crack is already formed at the base. But, let us assume that the crack is not formed now at the base, and under the condition of just gravity forces acting on the wall you should have uniform compression at the base of the wall itself with zero eccentricity of the resultant. So, this is the initiation of the initial condition of loading in the wall and as the lateral forces act on the wall, you will have a combination of the effects due to the lateral force and that of the gravity forces.

So, in the first stage if you have slight lateral force acting on the wall, the wall is in uniform compression, because of the gravity forces and the tensile stresses that can be induced at the base because of the action of lateral forces get nullified by the uniform compressive stress due to the weight of the wall itself. So, in the first stage we are looking at the compressive stress due to the weight of the wall, f_a nullifying any tensile stresses that can come due to the lateral forces acting on the wall, wind forces in this case.

So, we are assuming that the bottom is fixed at this stage and the top there is a lateral support, right. Therefore, you are looking at structural situation where it is a propped cantilever. You can idealize the system as a propped cantilever and look at the analysis, elastic analysis using this assumption. Then as the wind forces keep increasing, you can start getting tension at the base, you get a non-uniform distribution of the stresses, because of the bending plus the axial compression in the wall. We have seen this limiting case earlier, a limiting case due to the combination leading to zero stresses at the windward end and maximum stresses assuming a triangular distribution of stresses in compression at the other end or the leeward end of the wall.

So, this, if you consider this as one of the limiting cases, the first limiting case, we are interested in knowing what wind pressure acting on the wall under the condition of the wall being loaded in gravity by its self-weight or any superimposed load at what wind pressure would you reach this limiting triangle distribution of stresses. So, as I was mentioning, in this condition we are really looking at the wall being idealized as a propped cantilever.

Implying your maximum bending stresses are going to occur at the fixed base, at the bottom and of course, there is positive bending occurring somewhere around mid-height below that. And therefore, since the maximum negative moment is now acting at the base, if we assume that there is no tensile strength in the material under this condition, we should start seeing the initiation of cracking at the base of the wall itself. So, with the assumption that the wall is a propped cantilever, the bending moment corresponding to this situation, which be the maximum bending moment in the wall is nothing but the distributed force here, the wind load $p_b h^2/8$.

The tensile stress at the base itself is now going to be the bending stress minus the compressive stress that you have and we said that this being a limiting case the tensile stress is equal to 0. So, f_t in this case is nothing but the bending stress $p_b h^2/8 - f_a$; that is already acting at the wall and this is the limiting condition and we have seen this limiting condition where the eccentricity is at $t/6$ with respect to the thickness of the wall.

So, if you use tensile strength to be 0, from this we can estimate at what level of lateral pressure should you get cracking in the wall of you reach this first limiting condition here S is the section modulus of the cross section.

$$P_b = \left(\frac{8S}{h^2} \right) f_a$$

The wind pressure continues to increase and move forward you have partial section cracked beyond this point and compressive stress is increasing in the cross section, we could then look at a limiting case where the entire section is cracked, the whole section is capable of rotating and it is through the hinge that the resultant compression is passing. So, you have now moved from a propped cantilever kind of situation to a situation where the wall is pinned at both the ends. So, the boundary conditions change because of the increasing wind pressure and the maximum bending moment now moves to mid height

of the wall, and when that approaches tensile strength of the material you can get initiation of cracking at mid height.

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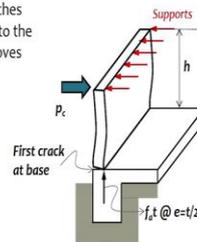
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Conventional bending analysis

Stage (c):

- Crack opens at the base when f_t reaches the flexural tensile strength normal to the bed joint of the; upward reaction moves closer to leeward side.



So, proceeding further stage c is where the crack at the base has now occurred, the resultant in compression gets pushed towards the leeward edge and it is now acting almost like an ideal hinge, boundary conditions have started changing to pin from earlier propped cantilever condition. And now we are interested in looking at maximum bending stress due to the combination of wind force and the gravity, but now this is going to be somewhere at the mid height of the wall itself.

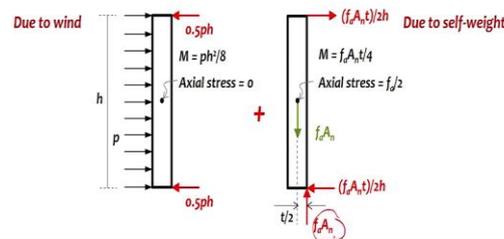
So, basically you see a slow progression here. Of course, we have idealized a situation where the resultant is very close to the leeward edge, but for the entire cross section to crack, a slow increase in the lateral force is essential. So, this is stage c as the wind load increases from the limiting condition p_b to the situation p_c .

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Conventional bending analysis

- Self-weight at mid-height produces a counteracting moment with a lever arm $t/2$.
- Second crack occurs at load p_d when f_t reaches f_{tr} .



Further continue in this configuration where now the wall is pinned at both its ends and we are now interested to estimate at what level would you get the mid height crack, which is what will initiate its failure, the system failure itself.

So, it is essential at this stage to understand that because of the eccentricities of the self-weight acting at the centroid of the wall segments and the hinges which are at the leeward edges. There will be an eccentricity due to the self-weight itself which actually comes into your resistance calculation and additional load calculations.

So, in this condition you have due to the wind let us take the situation of just the wind force acting on the wall and then combine it with the situation where the axial load equilibrium can be considered in the wall and superpose them because you are working in the regime of elastic analysis. So, due to wind you have now a situation where the wall is acting like a pinned-pinned structure with the bending moment of $M = ph^2/8$; uniformly distributed load on a simply supported system.

In this case it is just due to wind we are assuming that the effect of self will be considered separately. So, add to this you look at the effect of the equilibrium for the gravity forces. Now due to self weight, because the of the pin-pin situation and the migration of the hinge towards the leeward edge, there is going to be an eccentricity that has caused at the bottom and eccentricity which is equal to $t/2$. Because we are assuming

that the wall is now pinned at its leeward end and so, this causes an additional bending moment and it is essential to consider this additional bending moment.

The additional bending moment here is f_a into so, the compressive stress acting at that point into the net area of cross section available into $(t/2)/2$. So, into $t/4$ and you can see that the reactions of the top and the bottom; the differences in the reaction the top and the bottom is because of the eccentricity, considered at the base with respect to the hinge which is where $f_a \times A_n$ is actually acting.

So, this additional component will come into our calculations and it is really a combination of the bending moment due to the lateral forces and the bending moment cause due to the eccentricity of the self-weight resultant that is now going to be used to estimate what is the wind pressure at which you will get the ultimate state in the system. And the ultimate state in the system that we are interested in is when the load corresponds to a second crack which will form that is the location where the maximum bending moment is occurring at the mid height of the cross section, and you get a tensile crack. And we are again looking at the tensile strength of the material normal to the bed joint f_{tn} which is something that we have examined earlier looking at the material strengths.

Your question is as cracking propagates at the base of the wall; is it essential to consider the direction of the movement of the hinge? Yes of course, it is essential, because in this case we are looking at one side being the windward side and the other side being the leeward side. So, the natural movement of the hinges towards the leeward edge, right. So, when you are doing your analysis; of course, there is symmetry in the section, the symmetry in the loading, but you have to be careful that you know which is the windward side, the outside part of the wall and the inside part of the wall and so, it is based on that assumption that we are looking at a migration of the hinge towards the inner edge. So, it is not something, the axial stress resultant is not acting at the middle of the section, its acting at the leeward edge. What we are actually doing is, for the deflected shape, what is the point at which the in the deflected form where is the gravity force acting and at what eccentricities it with respect to the base cross section, inner edge of the base cross section. We are actually considering that there is a finite displacement and it is that deflected shape which is actually leading us to an additional moment coming because of the migration of the hinge towards the leeward side ok.

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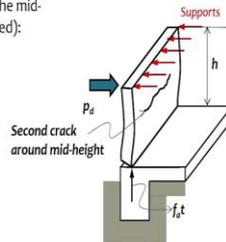
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Conventional bending analysis

Stage (d):

- Second crack initiates at mid-height
- Maximum tensile strength (f_t) at the mid-height of the wall (simply supported):

$$f_t = \frac{ph^2}{8S} - f_a \left(\frac{A_c t}{S} \right) - \frac{f_a}{2}$$



So, moving forward at this stage the crack initiates at the mid height under this condition of deflection of the wall due to the lateral force acting on it, the mid height crack is formed and this is when the tensile strength of the wall at the mid height is reached. So, the tensile strength at that cross section is reached, this is the tensile strength normal to the bed joint itself. The maximum moment is assumed to be acting at that cross section itself.

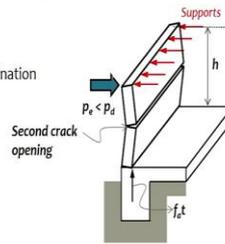
So, you now have the condition for which we can expect cracking to form where you have the stress distribution caused by the wind load, the distribution of stresses due to the eccentricity in the axial force, the second component and this is at mid height of the wall. And therefore, the total load that you are considering the self way the reconsidering is one half of the weight of the wall and therefore, the compressive stress is taken as $f_a/2$.

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▪ **Conventional bending analysis**

- Stage (e):
 - Second crack occurs at critical combination of load and capacity
 - Not always at mid-height



So, from this you can then estimate at what p is, at what value of the lateral force is this condition reached. And in the final condition when you have collapse, we are assuming that the displacement at mid height increases with increasing wind load and wall is taken towards instability.

So, the critical points are formation of the base crack, increase in lateral force, formation of the mid height crack and with that the strength capacity is reached. Beyond that it is actually some additional displacement of the wall that you will get and the wall would then lose its stability itself. So, once the mid height crack has formed, it is almost like the wall is composed of two rigid blocks rotating about the crack that is formed at the mid height and the crack that is originally formed at the base itself.

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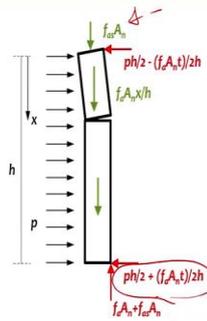
▪ **Conventional bending analysis**

- **Stage (e):**
- If lateral displacement at O is negligible, stability of top section of wall about O:

$$\left[p_e \left(\frac{h}{2} \right) - \frac{f_c A_c t}{2h} \right] x = p_e \frac{x^2}{2} + \frac{f_c A_c t x}{2h} \quad p_e = \frac{2f_c A_c t}{(h-x)h}$$

- If crack occurs at mid-height:

$$p_e = \frac{4f_c A_c t}{h^2}$$



So, to write down the final expression, if the crack were not going to form at mid height and the fact is the; this is dependent on several factors; the distribution of load. And therefore, it is possible that it depends on the distribution of the compression in the wall itself due to the superimposed loads. So, you could have superimposed loads, you could have high super impose loads and this actually alters the location of the second crack that you get. So, it is not always going to be at the mid height, it could vary. So, it is useful to write this expression assuming that the second crack will form at a height x from the top, x of h from the top.

So, in this particular case you have a superimposed load acting on the wall and then the self weight of the wall, self weight of the wall above the crack as the compressive stress into the net area that is being considered into x/h and the part below which is the rest of the wall itself. So, you again have the bending moment from the lateral force, the bending moment due to the eccentricity of the resultant in compression.

And at this stage last stage of loading whether resistance is reached, we could write down the equilibrium in the sections;

$$p_e = \frac{2f_a A_n t}{(h-x)h}$$

For p_e as a function of x and therefore you get the maximum load that the wall can resist at this critical stage of loading itself. Of course, if the crack were to occur at mid height you can simplify the expression x can be replaced as $h/2$ and that will simplify as

$$P_e = \frac{4f_a A_n t}{h^2}$$

So, you get by simple elastic bending analysis and expression that will tell you what the load, the lateral load would be at a critical points of loading till the maximum load is reached. So, simple framework considering just the equilibrium of the problem at different stages of loading can help you arrive at a forced displacement relationship right.

Now, of course, we have assumed linear elastic behavior and typically out of plane bending capacities are not very significant; we have seen that right at the beginning how the in plane bending capacity and the out of plane bending capacity is so different in an unreinforced masonry wall. And therefore, a more accurate estimate of the force displacement behavior and the capacity of the wall in the out of plane direction is essential, because considering that these are low values in terms of capacity, it will be a low value in terms of capacity anything that contributes to a more accurate estimate is better. So, there are two things that we will be looking at; one is to be able to estimate considering the formation of non-linearity in a cross section.

So, here we have been assuming that the distribution of stresses and compression is always triangular and there is no plastification even at ultimate conditions. So, that is one important aspect that, does contribute to additional load carrying capacity and the second aspect is what we had, what I had mentioned at the beginning which is additional resistance available due to the clamping at its ends which is called arching action and that comes, because the wall starts deflecting and gets clamped at the supports. So, that is the second effect. So, we will examine the first effect and try to modify our expressions and then examine the second effect and see how these expressions can be modified.

So, that is primarily what we will be doing. Now we did this for vertical bending analysis, the same set of calculations are valid for horizontal bending analysis where the wall is subjected primarily to; the lateral force, the axial stresses do not play a role there, because your cracking is actually vertical crack right. So, if you primarily remove the axial load; the axial stresses due you to the self weight and any super impose load from

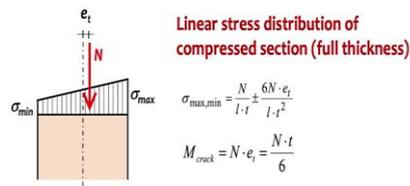
these expressions, you should be able to get the loads corresponding to horizontal bending and crack formation under horizontal bending.

Of course, in that case what matters as material strength is not f_{tn} , the bending strength flexible bending strength normal to the bed joint, but f_{tp} the flexural tensile strength parallel to the bed joints. So, that is the fundamental difference between the vertical bending and the horizontal bending that we will have to assume ok.

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Mechanical Behaviour of Masonry in Out-of-Plane Bending 50

- Out-of-plane bending
 - Section analysis considering non-linear stress-strain in compression



So, therefore, let us first look at examining the same problem, but under the assumption of a non-linear stress strain relationship for the material in compression at the critical section. So, some section analysis basis is essential. So, let us just discuss that; it is something that we have seen earlier and then incorporate that within a framework that will help us get a force displacement, load deflection curves under out of plane bending. So, if we were to assume that you have a non-linear stress strain curve in compression then let us examine three critical stages of loading and the distribution of compression in the cross section.

So, initially when you have low eccentricities, then when you have zero eccentricity you have the stress being uniform across the cross section.. But as eccentricity of loading increases, then you have the distribution of stress, compressive stress is not uniform- you have minimum compressive stress occurring at one edge of the cross section, maximum compressive stress occurring at the other end of the cross section. Here the eccentricity

that we are talking about is the combined effect of the lateral force plus the gravity force. So, in this condition we still have linear distribution of stresses.

We can assume that it is low eccentricity, the compressive stresses are still linear, the material is still acting in a linear elastic manner and the entire section is compressed and basically to account for the effect of the eccentricity. We can look at an estimate of the maximum compressive stress and the minimum compressive stress in the cross section considering the effect of the bending caused by the eccentricity.

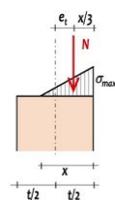
So, $N \times e_t$ here is nothing, but the bending moment caused due to the eccentricity divided by the section modulus, $It^2/6$, and so it is just N/t . The axial stress due to the load N plus or minus moment by the section modulus which is the second part of the expression. From this we can then take it to the limiting case, where we said that the limiting case is when you have tension cropping into the cross section.

So, that would occur when you have the load acting at $t/6$. Therefore, we can get an expression for M crack limiting the tensile strength of the material to 0 and saying that M crack is going to occur when the load has an eccentricity $t/6$. So, $Nt/6$ will be the cracking moment in the cross section, where your σ_{\min} goes to 0 and σ_{\max} is now the maximum stress in a triangular distribution, ok.

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- **Out-of-plane bending**
 - Section analysis considering non-linear stress-strain in compression



Linear stress distribution in a partial section (neglecting region in tension)

$$x = 3 \left(\frac{t}{2} - e_t \right) \quad N = \frac{\sigma_{\max} \cdot x \cdot t}{2}$$

$$\sigma_{\max} = \frac{2}{3} \cdot \frac{N}{t \left(\frac{t}{2} - e_t \right)}$$



Continue the increase in eccentricity which is the resultant eccentricity of the loading the lateral force and the gravity force acting on the wall, as this increases more cross section goes into tension and we are neglecting that and we consider only the partial length. So, the partial length here is, the partial compressed zone is of width x , t being the total thickness of the cross section and since we are still assuming that the material is linear elastic, the triangle distribution is acceptable. So, the triangular distribution is continued at this stage triangle distribution of stresses is still acceptable, because we are still assuming that the material is linear elastic and from that assumption, we are right in assuming that the stress resultant, the force is now acting at the centroid of that triangular distribution and hence we get into our expressions the distance from the edge of the resultant is going to be $x/3$.

So, with this condition for our equilibrium we are neglecting the portion which is basically cracked and we are working on the partial section and in this partial section from the triangle- the geometry of the triangle, we get an expression of what is the compressed length of the wall in terms of the total thickness of the wall and the eccentricity of the load itself. With that it is now possible to look at what will be the moment at this situation; the equilibrium in the axial force can be written down as N is equal to σ_{\max} into half into base into height of that triangular distribution into the length.

So, $\sigma_{\max} x/2$ into length will be the force equilibrium and since now we have expression for x and the force equilibrium plugging in x into this expression can give us an expression for σ_{\max} . So, now, we have an estimate of σ_{\max} where all that is done is x is substituted into this expression for N and I write it in terms σ_{\max} . So, now we have again linear elastic, but we had the first stage limiting where we had zero tensile strength in the cross section full section compressed, second stage is partial section compressed and we have an estimate of the maximum compressive stress in both these cases, its linear elastic behavior of the material.

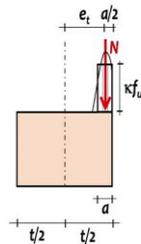
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Mechanical Behaviour of Masonry in Out-of-Plane Bending

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Out-of-plane bending

- Section analysis considering non-linear stress-strain in compression



At ultimate condition:

$$M_u = N \left(\frac{t-a}{2} \right) = \frac{N \cdot t}{2} \left(1 - \frac{N}{k f_u t} \right)$$



But as we progress, we now are going to be looking at increase in the compressive stress in the cross section, it is here that it is unrealistic to assume that the material remains elastic. So, the edge compressive stress will be significantly high and under that situation you should start getting plastification of the section. So, as discussed earlier it is possible to assume that the distribution of stresses here at this edge is really parabolic in nature.

Of course, you can use a parabolic stress strain curve and make your calculations. To simplify things, the rectangular stress block is something that is acceptable for which you need the equivalent stress block dimensions and here the equivalent stress block is a factor k , that is multiplied into the ultimate strength in compression; f_u is the ultimate strength and compression of the masonry, it is again the assembly, masonry composite.

So, the parabolic stress distribution is equivalent to rectangular stress block of dimension a , which is the width of the stress block and k times f_u as the height of the stress block is now going to be used in the expression. Therefore, at ultimate conditions that is what is going to give you the moment equilibrium in the section and therefore, it is nothing but the axial load into the eccentricity and now the eccentricity is $(t/2) - (a/2)$ coming from where the resultant is actually sitting at $a/2$, half of the rectangular stress block.

So, rewriting this in terms of the stresses you can bring in the force, the force equilibrium knowing the stress which is $k f_u$ and the stress block itself as a , and so here we are basically bringing in the stresses and writing it for getting an expression in terms of M_u

at the ultimate compressive stress condition in the wall itself. So, this is a framework that we can use instead of just assuming that the cross section is in a linear elastic condition.

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Mechanical Behaviour of Masonry in Out-of-Plane Bending 53

▪ **Out-of-Plane Bending Capacity Considering Non-Linearity**

From moment equilibrium about the base reaction: $H_{top} = w \cdot \frac{h}{2} + \frac{W \cdot \Delta}{2 \cdot h}$

From moment equilibrium about O:

$$R \cdot x = N \cdot \Delta + \frac{W \cdot \Delta}{2} + \left(\frac{w \cdot h}{2} + \frac{W \cdot \Delta}{2 \cdot h} \right) \cdot \frac{h}{2} - \frac{w \cdot h^2}{8}$$

$$= N \cdot \Delta + W \cdot \frac{\Delta}{2} + \frac{w \cdot h^2}{8}$$

Static instability occurs when $\Delta = x$

Priestley, 1985



Let us look at creating a framework that will help us calculate the force displacement right from initial loading, lateral force increasing and taking it to failure, but now considering the section analysis that we have examined you know in the last few slides.

So, I am looking at a wall which is loaded with the self weight and some superimposed load; N being the superimposed load here and the self weight of the wall W, the lateral force that is acting on the wall. We could do these calculations irrespective of whether we are looking at wind forces or inertial forces, right. So, if you are examining wind forces as wind pressure acting on the wall, you could examine the capacity; where earthquake forces then you could examine the problem as the inertial force acting on the wall as the face loaded forces.

So, the pressure is nothing but out of plane forces, distributed forces due to the self weight of the wall itself. So, that is why $M \times a$ (mass into acceleration) is being brought in. So, the inertial force acting on the wall is, what is the out of plane load acting on the wall in the presence of the gravity force in the wall itself.

So, in this particular example we have h as the total height of the wall and this assumption that the wall cracks at mid height is something that we can use to simplify

the problem or continue by assuming that the crack can occur at any other location, but in this particular derivation, let us assume that the mid height is where the crack is going to occur.

We also commence by assuming that the top and the bottom are free to rotate already, which means the initial condition of causing the base crack has already occurred, because we are really interested in looking at the behavior around the peak condition where the mid height reaches it is capacity; mid height section reaches it is capacity. So, we are starting with the assumption that the two ends are pinned already in this particular case.

So, if you are looking at one half of the cross section, you have the lateral force acting on the wall, lateral force acting on the wall is W which is coming from the inertial mass in the mass into the acceleration and in this particular case, we are really looking at a condition where the mid height crack is also propagated and we want to understand at what level of lateral load is the failure in the section going to occur. So, what we are really examining is a condition where the triangular distribution of stresses in compression in the wall has already been reached in the mid height of the wall and then the load keeps increasing and at what stage do you get the section failure right. So, we are already assuming that the wall is in the form of two rigid blocks hinged at the mid height crack itself, right.

So, the section is now partial already you have the resultant which is acting close to the leeward side as you can see; R being the reaction close to the leeward side. And the position of the reaction with respect to the center of the cross section is x and at the mid height of the wall the displacement, the maximum lateral deflection of the wall is Δ and we are assuming that the two blocks are moving in a rigid manner. And therefore, we can use the geometry of the triangle here and estimate the displacement at any other location along this half block that we are considering for equilibrium.

So, we have the moment equilibrium about the base reaction in this case; the lateral force, because of the wind force or the earthquake force acting on the wall as,

$$H_{\text{top}} = w \cdot \left(\frac{h}{2} \right) + \frac{W}{2} \cdot \left(\frac{\Delta}{h} \right)$$

Now, if we were to examine this enlarge block here and look at the point O and write down the moment equilibrium about point o,

$$R \cdot x = N \cdot \Delta + W \cdot \frac{\Delta}{2} + \frac{wh^2}{8}$$

So, this expression links Rx into what is on the right hand side, I can rewrite this in terms of w. So, what I really require is an expression for w as a function of the geometry and the deflection that is occurring in the wall. So, now you have this expression, the expression for the lateral load in terms of the geometry of the wall and the you have it in terms of x and with shifting locations of the reaction you can then get different points on the force displacement curve.

As you can see when delta becomes equal to x, you will get 0 there for w the expression gives you the possibility of estimating up to instability what the force displacement capacity of the wall can be. So, this is an expression; I mean so, this is we are basically following the following a simplified procedure for estimating out of plane capacities- seismic out of plane capacities of wall proposed by Priestley in 1985. So, that is the basis of the expressions here.

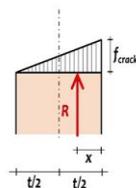
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Force-deflection curve (case of constant N and varying w):

- First, calculate curvature at mid-height section and the associated moment, then evaluate displacement at mid-height assuming a given curvature distribution. Assume opening of crack will happen when $x = t/6$



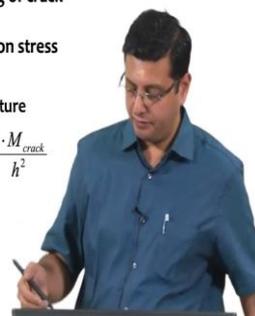
$M_{crack} = Rt/6$ is the moment at opening of crack

$f_{crack} = 2R/t$ is the maximum compression stress

$\psi_{crack} = f_{crack}/Et$ is the corresponding curvature

$$x = t/6 \quad M_{crack} = \frac{w_{crack} \cdot h^2}{8} \Rightarrow w_{crack} = \frac{8 \cdot M_{crack}}{h^2}$$

$$\text{and } \Delta_{crack} = \frac{5 \cdot w_{crack} \cdot h^4}{384 \cdot E \cdot J}$$



Now, we are going to be examining this case we need the entire forced deflection curve. So, we basically looking at different situations under increasing or varying value of w. So, here we need an estimate of the curvatures, we need an estimate of the curvatures

again, we are looking at a cracked condition and uncracked condition. So, similar to what we had done in the earlier case. So, if we can calculate the curvature of the mid height section and then estimate the moments from the curvature and the displacement at mid height with certain assumptions on the curvature distribution we can then get all stages up to failure.

Now, we assume that the crack loading progresses crack opening happens and up to that it is still with the assumption of the linear elastic distribution, linear distribution of stresses. So, the condition of crack opening occurring at t/x is equal to $t/6$ still holds. So, at this stage the compressive stress in the wall is f_{crack} at which with 0 tensile strength of the material.

The first crack is formed at the mid height the moment at the stage is the reaction into the eccentricity $t/6$ at the point when the initiation of crack opening occurs; the stress at this stage is nothing but $2R/t$. And we can estimate the curvature of the section at this time point as the strain over the compressed length, ϵ over the compressed length and the strain written down as f_{crack}/E as we had done in the P-delta problem earlier and this is at a condition of x is equal to $t/6$.

So, from this we basically have an expression for what is the moment carrying capacity when the first crack is forming in the wall. So, $w_{crack} h^2/8$ and from which we now, have again as an expression of w_{crack} from this itself, which could be one of the critical stages before the section starts going into plastification. At this stage we are assuming that the wall is simply supported with a uniformly distributed load. So, the mid span deflection from early this is again linear elastic distribution.

So, we can assume the elastic analysis and get the maximum deflection and the maximum deflection at mid height at the point when cracking is expected to initiate as

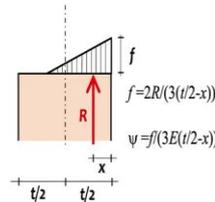
$$\frac{5wh^2}{384EI}; \text{ so, you now have the load and the deflection at this critical point ok.}$$

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Post-cracking:

- Behaviour is non-linear and the following relationships are used for a given value of deformation/stress at the right edge of the section, from which the corresponding value of x can be found:



Conservatively, it can be assumed that displacement is proportional to curvature at midheight:

$$\Delta = \frac{\psi}{\psi_{crack}} \cdot \Delta_{crack}$$



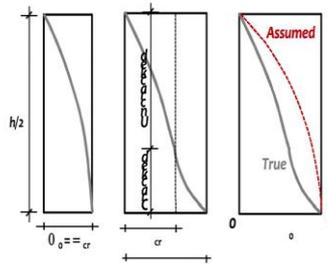
So, this from this stage on we start looking at reduced cross section available in compression till the non-linear distribution of compressive stress in the cross section. So, if we ask we start assuming that the cross section is going to be replaced as w increases, we then have the possibility of writing down the compressive stress at the edge section and the corresponding curvature for the remainder of the section.

So, with that we are basically going to be able to write down what is the moment in the cross section, supported by the cross section itself. So, in this case again, because the curvature can be different at different segments, because cracking is not happening uniformly in all sections. It will be useful to have one estimate of the curvature as a simplification and we need an estimate of the deflection and therefore, we assume that the deflection the displacement is proportional to the curvature at the mid height and use that for the entire height of the wall itself. So, we are using the estimate of the curvature at the cracked stage when the initiation of crack occurs; use that to proportion the curvature in the wall itself.

So, this is an assumption which is airing on the conservative side and so, is an acceptable assumption itself.

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Therefore, the assumption $\Delta = \frac{\psi}{\psi_{crack}} \cdot \Delta_{crack}$ is conservative....



So, if you really look at the wall looking at one half of the height of the wall and at mid height, you actually have the cracking that has occurred at the mid height. So, the estimate of curvature is at the mid height considering the crack condition; however, the curvature is going to be different in all the other sections, because those are uncracked sections. So, with this being the reality if we were to assume that the whole height of the wall has a curvature which is equal to the, which is proportional to the cracked curvature. You see that the assumed curvature is greater than the true curvature and this is therefore, an assumption that will make us err on the conservative side.

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- **At ultimate conditions:**
 - A rectangular stress block can be assumed and corresponding curvature is evaluated, from which displacement at mid-height is estimated.
 - Having determined displacement Δ at mid-height corresponding to a given value of x , from equation:

$$w = \frac{8}{h^3} \cdot R \cdot (x - \Delta)$$
 - ...corresponding value of distributed horizontal force is determined, and the non-linear force-displacement curve is determined point by point.
 - Static instability may occur before the attainment of the ultimate stress. The behaviour is elastic non-linear, i.e. the wall will load and unload along the same curve.

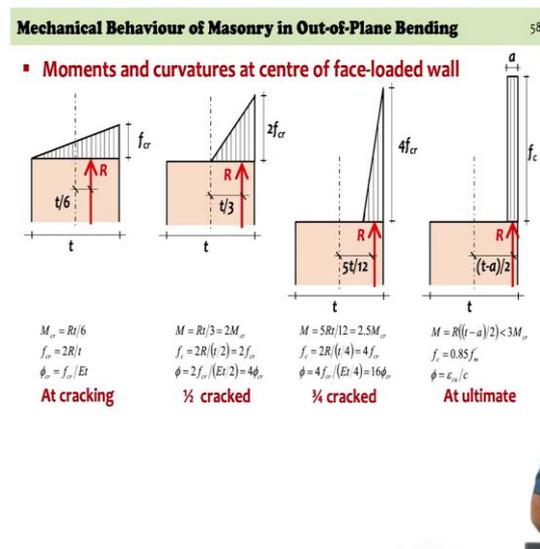


So, this assumption is conservative and at ultimate conditions we can then replace the stress distribution with the stress block and use the stress block to rewrite the moment and displacement expression at the failure condition of the wall itself. So, the w corresponding to a condition, where you have a compression in non-linear, non-linearity of compression in the cross section is written and the corresponding displacement is estimated.

So, at different stages, the first expression that we had, is written in terms of the reaction force and as reaction force depends on where this reaction force is sitting that will determine what the compressed length is. And so, we get the entire distribution post cracking up to a condition where $x = \Delta$ and you get 0 resistance in the system. So, you get this non-linear curve, which will then give you the force displacement behavior on the wall itself. It is true that depending on the geometry of the wall, you can also have instability before this condition is actually occurring in the wall and fundamentally.

If you have dynamic loading, we are considering this behavior on the static loading; you can have conditions in dynamic loading, where the displacement may go to a state corresponding to the limiting condition that this expression gives, but then can spring back and continue to support the lateral forces.

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So, we will examine how the force displacement curve finally, looks, but then basically we are looking at distribution at cracking in the cross section to estimate the moment and

from the moment the force and then when let us say one half of the wall is cracked or three quarters of the wall is cracked and finally, we look at the condition where the rectangular stress block is used. And so, the moment at each stage from the geometry is available to us, which to then be useful to plug in the corresponding values of x and the corresponding value of the reaction force gives us different points on the force displacement curve.

So, this is one format that we could use to estimate the lateral force capacity of a masonry wall, but considering the non-linear behavior of the section that is critical ok. So, we will continue in the next lecture on this and then look at the effect of clamping in the section itself due to arching action.