Mechanics of Material
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Pressure vessels and failure theories
Lecture – 93
Vonmises condition

Welcome to 33 lecture in mechanics of materials, the last lecture we began looking at failure theories. We saw that the failure theory should be based on the stress more rather than the strain and then we saw that there are two kinds of materials, one which are hydrostatic pressure insensitive to failure which are hydrostatic pressure sensitive to failure ok. The materials are hydrostatic pressure insensitive to failure or undergo usually a ductile kind of a failure and we were looking at failure theories for these kind of materials.

We looked at Tresca criteria; which says that the maximum shear stress should be less than a particular value before the material begans to yield or fail ok. So, in particular we saw that the failure envelop for Tresca criteria is are shown here in the form of an hexagon is are shown here in the form of an hexagon ok.

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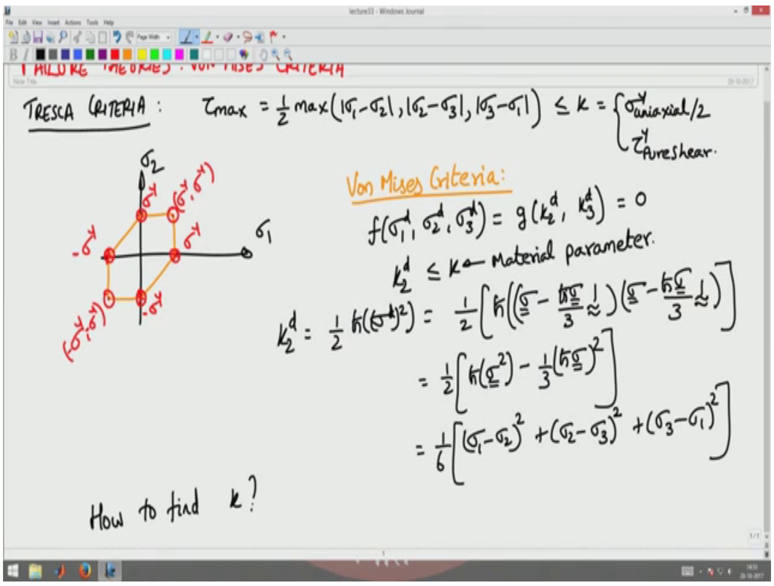

And then we saw that this kappa which is the which is like a material parameter can be determined from a uniaxial test or from a pure shear test. In particular if it is determined from a uniaxial test it is sigma y by 2, and if it is determined from a pure shear test it is tau y ok.

So, we saw this in the last class, in today's class we are going to look at another failure criteria which is called as Von Mises criteria ok. I going to look at Von Mises criteria what this says is, we saw in the last class that a general form for failure surface or yield surface specification as a function of the form f as a function of sigma 1 d, sigma 2 d, sigma 3 d for hydrostatic pressure in sensitive materials or a function of k 2 d, k 3 d, where k 2 d and k 3 d are the invariance of the deviatoric stress right ok.

What the Von Mises criteria says is, this k 2 d should be less than or equal to kappa where this is again a material parameter. From our lecture 9, we know that k 2 d is nothing, but 1 half trace of sigma d squared which is nothing, but 1 by 2 into trace of sigma d is sigma minus trace sigma by 3 identity multiplied by sigma minus trace sigma by 3 multiplied by identity which is nothing, but 1 half trace of sigma square minus 1 by 3 trace sigma whole square ok.

This expression in particular for can be written as 1 by 6 into sigma 1 minus sigma 2 the whole square, plus sigma 2 minus sigma 3 the whole square plus sigma 3 minus sigma 1 the whole square when I represent sigma in terms of is principal values ok. So, the this is k 2 d ok. Now how do you find kappa that is the question that we have to address next, how to find kappa you will follow the same thing that we did for Tresca criteria.

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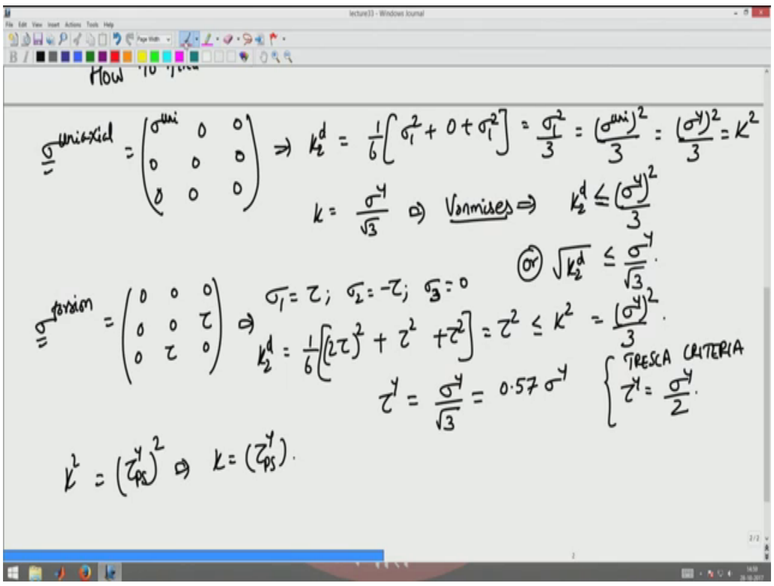

We will take a uniaxial state of stress axial state of stress for which it is sigma uniaxial 0 0 0 0 0 and for this you can compute k 2 d to be 1 by 6 sigma 1 square plus 0 plus sigma 1 square which is sigma 1 square by 3 ok.

So, basically to make dimensionally consistent let me call this as kappa square rather than kappa. So, k 2 d must be less than or equal to kappa square. So, this is sigma 1 square by 3. So, ok so, sigma 1 now is sigma uniaxial squared by 3, that is a sigma 1 principal stress now I know that when this stress reaches sigma y there might less begun to yield. So, it is sigma y by square by 3 must be equal to the kappa that is a limiting value of the stress for which a kappa square as a limiting value of stress for which the material began to yield in other words kappa is sigma y by root 3 sigma y by root 3.

Now, so, the Von Mises criteria reduces to this implies, Von Mises criteria implies k 2 d is equal to sigma y must be less than or equal to sigma y square by 3 or I can rewrite this as square root of k 2 d must be less than or equal to sigma y by root 3 ok. Now let us do what we did before, let us say I found this kappa from a uniaxial test and I want to predict what happens for a torsion experiment ok. So, sigma for torsion is just like what we did for Tresca criteria same arguments, 0 0 0 I am assuming only sigma yz to exist at a particular point so that will be tau tau ok.

Now, for this I know sigma 1 is equal to tau, I know sigma 2 is minus tau, and sigma 3 is 0 right ok. Now what happens then k 2 d becomes 1 by 6, 2 tau whole square plus tau square plus tau square which will be tau square ok. Now this has to be less than or equal to kappa square which is sigma y square by 3 ok. So, the limiting value of the shear stress, they or the yielding value of shear stress would be sigma y by root 3 which will be 0.5 it will be 0.57 sigma y.

Compare this with what we got for Tresca, from Tresca criteria Tresca criteria gave tau y to be sigma y by 2 ok. That is even though we agree on the uniaxial yield stress we will would not agree on the shear pure shear yield stress ok. Now let us do the reverse just like what we did for Tresca criteria, let us assume that we find kappa from the shear stress let us assume you found kappa from the shear stress. So, kappa square tau y pure shear square must be equal to kappa square in other words kappa should be tau y pure shear ok.

Now, I want to predict what is it for uniaxial test for uniaxial test I found k k 2 d to be sigma 1 uniaxial whole square by 3, for uniaxial test I found k 2 d to be uniaxial square stress divided by 3 ok.

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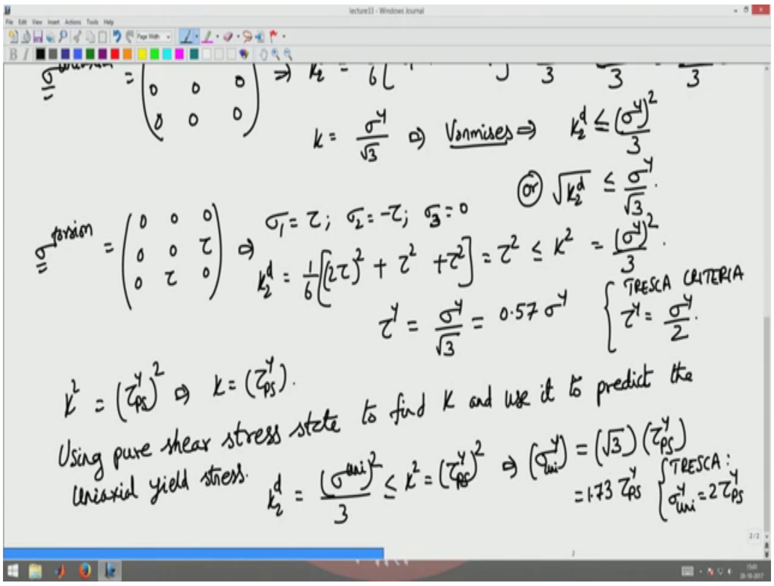

So, basically now, I will have now what I am doing is I am using shear pure shear stress state to find kappa and use it to predict the uniaxial yield stress ok. So, k 2 d for uniaxial yield stress is sigma uniaxial whole square by 3 from what we computed above, this should be lesser than or equal to kappa square which is sigma pure shear y the whole square ok.

So, this implies uniaxial yield stress should be equal to square root of 3 times tau y pure shear ok. Contrast this with Tresca criteria and Tresca criteria you got it as 2 times this is nothing, but 1.73 times tau psy ok. In Tresca criteria you got it as sigma y uniaxial to be 2 times tau y pure shear ok. So, basically there is a difference between Von Mises and Tresca, it might agree at some points, but there will be deviation at some other points ok.

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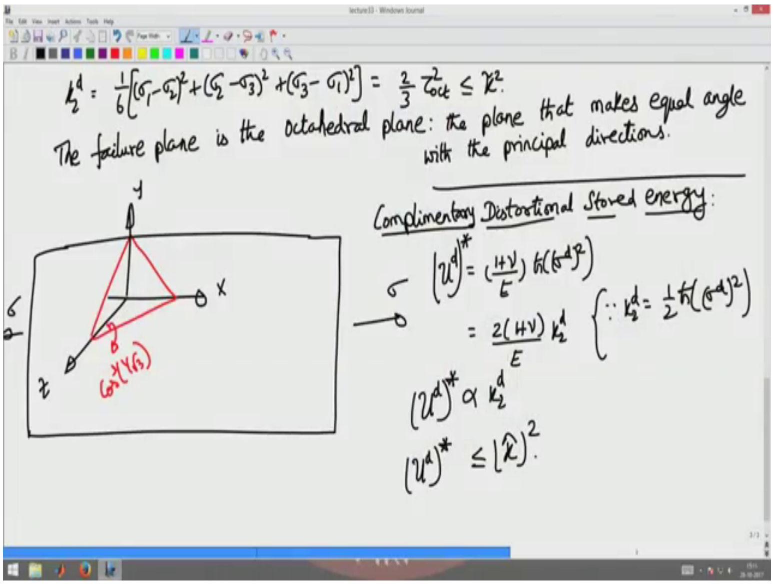

Next let us understand what happens what are the other interpretations of Von Mises criteria ok. We saw that k 2 d was given by 1 by 6 sigma 1 minus sigma 2 whole square plus sigma 2 minus sigma 3 whole square plus sigma 1 whole square. In lecture 9 we saw that this is nothing, but 2 by 3 times octahedral shear stress square ok. Octahedral shear stress is the shear stress that occurs in the octahedral plane, there is a plane that makes equal angles with the principal axis principal directions ok. So, we saw all those things in lecture 9 ok.

So, now,. So, I can interpret the Von Mises criteria as the octahedral shear stress being limited to a particular value of kappa square kappa tilde square, this kappa and the previous kappa different because I have factor 2 by 3 coming in the for your octahedral shear stress ok. So, I can view it as some octahedral shear stress being limited to a particular value this defines the plane in which the body will fail the body when it obeys Von Mises criteria will fail in a plane which makes equal angle with the principal directions whereas in Tresca criteria it will fail in a plane where the maximum shear stress occurs ok.

So, there is a difference. So, the plane of failure in Von Mises and plane of failure in Tresca would be different ok. So, if I interpret it like this the failure plane is the octahedral plane, hedral plane by octal plane is the plane that makes equal angle with the principle directions ok. Now let us see what the octahedral plane is for uniaxial state of stress ok. For uniaxial state of stress I know the prescribed coordinate system is the principal direction also because state of stress is sigma 0 0 0 0 0 0 0 0 I know that my prescribed x y z axis to represent the problem is the principal axis ok.

Now, a plane which makes an equal angle with the all this would be this plane which makes an angle cos inverse of 1 by root 3 with these axis it is this plane are about which the failure would occur wherein all this angles are cos inverse 1 by root 3 ok. So, there will be 8 such planes, 1 on this direction 1 in that direction and you can imagine that there are 8 plane the hence it is called as a octahedral plane. So, any one of those eight planes it might slip a slide in those planes ok.

Now,. So, we saw for Tresca criteria this failure plane was 45 degrees to the x axis right. If I am applying if my body was like this if my rod was like this and if I applied a uniaxial stress sigma here, then the failure plane now would be this octahedral plane, which makes equal angle with the all the xy and z axis whereas, then Tresca criteria it will be a plane which made 45 degrees with respect to the x axis ok. So, that is the difference between the failure planes of octahedral and Tresca criteria ok. Now there is one more interpretation of this octahedral shear stresses that is Von Mises criteria can be interpreted as the limiting value of the distortions stored energy ok.

Now, let us define another quantity called distortional stored energy that is this is the energy caused by the deviatoric stress. So, this energy we will denoted by us of d, I will define the complementary strain energy complementary distortion stored energy would be more specific ok. So, this will be nothing, but 1 plus nu by E into trace sigma d whole square ok. Now you know that k 2 d is nothing, but 1 half of sigma d the whole square. So, this is nothing, but 2 times 1 plus nu by E k 2 d.

Since k 2 d is 1 half trace of sigma d square trace of sigma d squared ok. So, you find that a distortion complementary distortion stored energy is proportional to k 2 d ok. So, complement is distortional energy is proportional to k 2 d and the hence Von Mises criteria can be interpreted as a criteria of shear stress when the complimentary distortional stored energy reaches a particular value the material fields why do you want to interpret it as complementary distortional stored energy?

The reason is you know that to form new surfaces when a crack or when a sliding occurs, you need energy to form a new surface. So, that energy only when it is available a new surface on a crack would form ok. So, which means the energy strain energy is the one which should govern when the crack forms ok. So, in that prospective Von Mises criteria is also interpreted as a criteria of iron the complimentary distortional strain energy reaches a particular value the failure begins to occur ok.

So, you can say that when a ud star is less than or equal to some kappa hat squared the failure occurs ok. So, one criteria you have a three different of explanations for the same criteria, one it is just comes from the invariant theory of these stresses, second it is interpreted as the octahedral shear stress not exceeding a particular value, third the same criteria is interpreted as the distortion stored energy not exceeding a particular value ok. So, these are the three different interpretations of the same Von Mises criteria ok. Next let us go ahead and construct the failure surface for Von Mises criteria.

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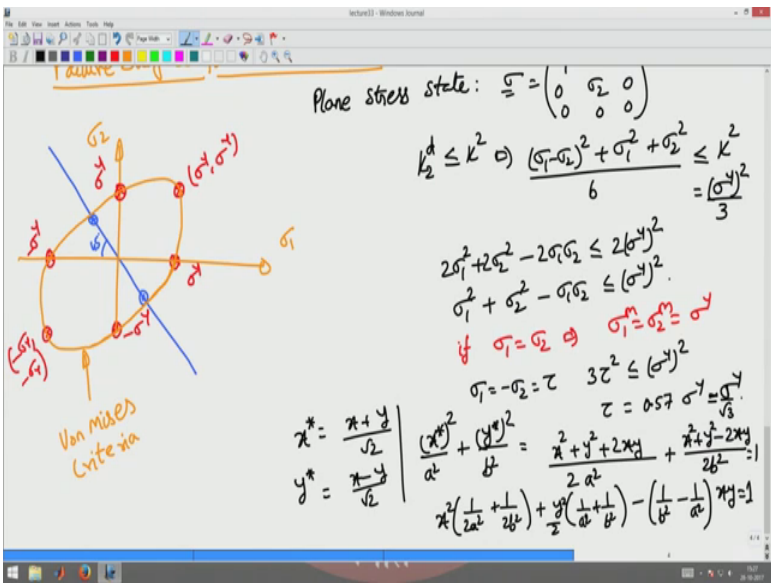

Next what we are going to do is, construct failure surface for Von Mises criteria ok. Now as before I am going to look at a plane set of stress plane stress state, which we saw can be represented in terms of sigma 1 0 0 0 sigma 2 0 0 0 0 and hence I am going to retain the first usage of the Von Mises criteria is k 2 d must be less than or equal to kappa square which will imply that, sigma 1 minus sigma 2 whole square plus sigma 1 square plus sigma 2 square by 6 must be less than or equal to kappa square which we will write it as sigma y whole square by 3, in other words I have 2 sigma 1 square plus sigma 2 square minus 2 sigma 1 sigma 2 must be lesser than or equal to 2 times sigma y square ok.

So, this in other words boils down to sigma 1 square plus sigma 2 square minus sigma 1 sigma 2 must be less than or equal to sigma y the whole square ok. Now what does this equation represent? This equation represents that of an ellipse inclined about an axis not about sigma 1 sigma 2 axis, but inclined 2 sigma 1 sigma 2 axis let us see how to construct that ellipse. Let us say this is sigma 2 axis, this sigma 1 axis I know that when sigma 2 is 0 sigma 1 would be sigma y and when sigma 1 is 0 sigma 2 will be sigma y the uniaxial state of stress points I know this point and I know this point, this should be sigma y and this should be sigma y. I mean sigma 1 equal to sigma 2 if sigma 1 is equal to sigma 2, I know that the above equation reduces to sigma 1 equal to sigma 2 equal to sigma y for the maximum stress states ok.

Sigma 1 equal to sigma 2; this sigma 2 square and sigma 1 sigma 2 gets cancelled, sigma 1 has to be equal to sigma y ok. So, I have sigma y sigma y point which I have to pass through these are the 3 points I know of which are on the ellipse ok. Similarly I have three points in compression tension or compression does not make a difference just like in Tresca criteria, and hence what happens is in compression also I have minus sigma y minus sigma y and sigma 1 equal to sigma 2 equal to minus sigma, it will give me this minus sigma y again minus sigma y comma minus sigma y these are the 6 points by which t curve has to pass through ok.

Next let us consider a case wherein sigma 1 is equal to minus sigma 2 equal to tau, then I will get it as 3 tau square must be less than or equal to sigma y whole square or tau must be equal to 0.57 times sigma y. That is 1 by root 3 times sigma y ok. So, I I can sigma 1 equal to minus sigma 2 equal to tau is a 45 degree line, this is a 45 degree line here and I know the point on this line and I know the point on this line ok. So, now, the ellipse has to pass through all these points ok. So, this is the yield surface this is the yield surface for Von Mises criteria. It is in the form of an ellipse with sigma 1 equal to sigma 2 and sigma 1 minus sigma 2 45 degree rotated ellipse ok.

So, let us see how it is 45 degree is rotated ellipse let us do sigma 1 let us rotate let us say x star is x plus y by root 2 and y star is x minus y by root 2 then x star squared by a square, plus y star square by b squared would be would be x square plus y square plus 2 xy by 2 a square plus x square plus y square minus 2 xy by 2 b square equal to 1 ok.

So, now you adding this you get x square by 2 a square. So, 1 by 2 b square plus y square into by 2 1 by a square, plus 1 by b square minus 1 by b square minus 1 by a square xy equal to 1 ok. Now comparing the coefficients of sigma 1 square, sigma 2 square and sigma 1 and sigma 2 we want 1 by 2, 1 by a square plus 1 by b square to be 1 by sigma y the whole square and we want 1 by b square minus 1 by a square to be 1 by sigma y square ok.

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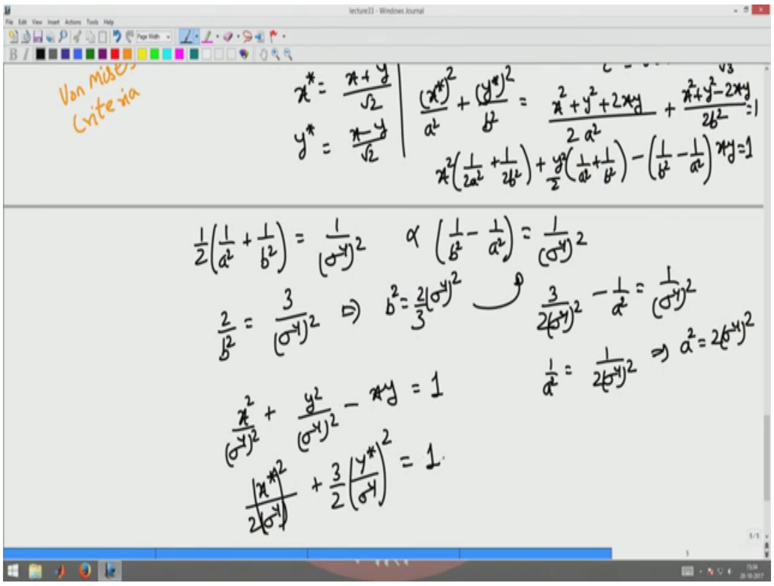

So, we got these two equation by comparing the coefficients of x square, y square, xy with sigma 1 square sigma 2 square and sigma 1 sigma 2 ok. Now let us add these 2 equations 2 by beta square should be equal to 2 by 3 by sigma y the whole square right ok. So, that will imply b is should be 2 by 3 sigma y the whole square ok. Now substituting this b square is 2 by 3 sigma y whole square now substituting this in here you will get it as 3 by 2 sigma y whole square minus 1 by a square, equal to 1 by sigma y square. In other words 1 by a square is 1 by 2 sigma y whole square or this implies a is a square is 2 sigma y square 2 by 3 sigma y whole square and a square is that.

So, substituting that back in our equation you get x square by sigma y square plus y square by sigma y squared minus xy is equal to 1 or in terms of x star y star squared it will be, x star by 2 sigma y whole squared plus y star by 3 by 2 y star a sigma y the whole squared must be equal to 1 ok. So, that gives us the equation of this ellipse.