Mechanics of Material
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Stresses and deflection in beams not loaded about principal axis
Lecture – 73
Bending equation for bending about principal axis

Welcome to the 26th lecture in mechanics of materials. The last lecture, we saw how to compute the moment of inertia put an arbitrary oriented axis, in particular, we saw that if y and z axis lies in a plane of z cross section, then I by y given by integral z squared dax I zz given by integral of y square dax I yz given by integral yz dax.

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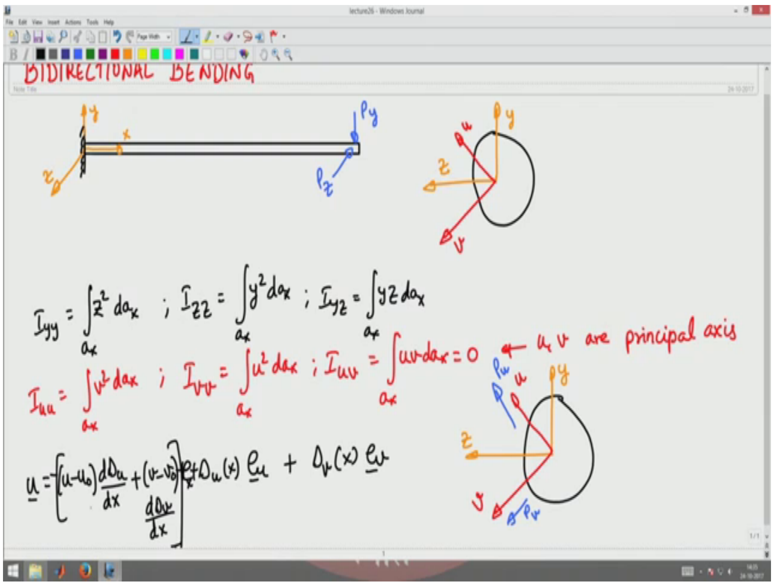

We saw how to compute these moment of inertias and then we said that since the sign of the cross movement of inertia I yz flip sign on the coordinate system rotates by 90 degrees, they will exists a orientation of iron it will go to 0 and we defined such oriented axis where in the cross moment of inertia is 0 as the principal axis for the cross section similar to the principal plane principal directions that we are for these stresses and strains.

So, we saw that and then we said in the last lecture that we were interested in looking at a beam subjected to loads in two directions.

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In particular, we will now analyze, what happens in a beam is subjected to a loading along its principal direction, this is a rectangular cross section and the y and z parallel to the sides is a principal axis for this cross section. So, we are interested in studying, what happens when I apply a lower like this it bends in this manner and similarly, if I apply a load perpendicular to this in this direction, you can see that it is bending like this.

So, we are interested in analyzing, what will be the displacement field and what will be these stresses that will develop in this cross section due to loads operate both vertically and perpendicular to the plane of the beam cross section. So, we are interested in studying this displacement coupled with this displacement in this lecture.

So, now, what we are interested is in the scenario where in the load is separate along the principal direction both the principal axis u and v axis, then we are interested in writing, what will be the displacement field the displacement, field u would be there will be a delta u as a function of x eu plus delta v function of x ev plus along the x direction that will be a displacement negative u minus u naught d delta u by dx plus v minus v naught d delta v by dx into ex plus.

Let us see, how does the displacement field arise when you are bending due to load separate both along the both the principal axis, it comes from plane sections remain plane hypothesis the same hypothesis that we used when the beam was bending along one direction.

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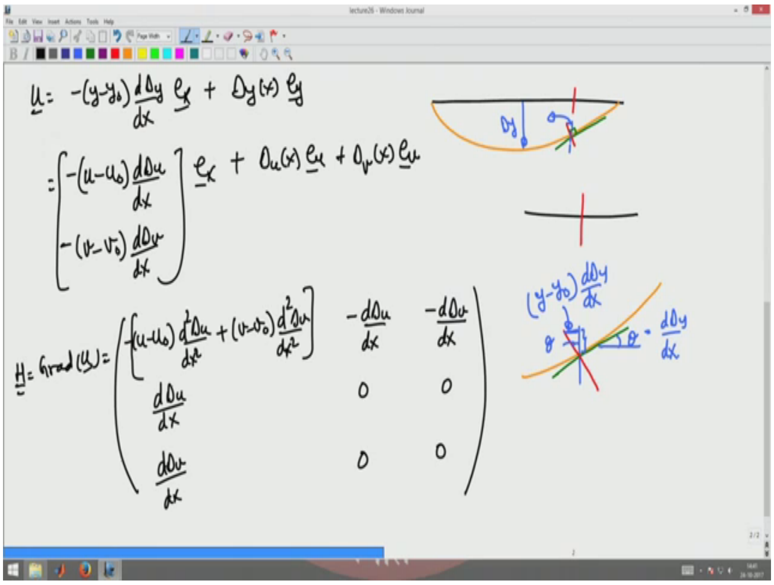

So, now when the beam was bending along one direction you add u given by minus y minus y naught d delta y by dx ex plus delta y as a function of x ey, right because what we said was if this is a neutral axis of the beam, it will deform into a shape something like this due to bending, then a section which is perpendicular to the axis of the beam initially will remain perpendicular and will remain plane.

So, basically what happens is the tangent line to this neutral axis and this makes 90 degrees. So, this vertical line which now when extended down extended down see say displacement which is given by this distance and this distance was this distance was I will magnify this figure. So, basically you add the neutral axis initially like that and then it deformed into something like this and this straight line curve be in a straight line like that, but he deformed into some line like this such that it is perpendicular to the neutral axis at that point. So, it would have been something like that.

And we are interested now in finding what this displacement is we are interested in finding this displacement this displacement would be; if I assume small rotations, this distance times; this angle theta and that angle theta will be related to the slope of the tangent to the neutral axis at location theta this is nothing, but d delta y by dx if this is delta y that deflection is delta y that will be d delta y by dx and then this will be y minus y naught d delta y by dx, right.

Similarly, now what is happening is its bending like this, but instead of delta y there is a delta v coming in there. So, you have u minus u naught d delta u by dx ex plus delta u. So, function of x eu because the cross sectional axis in the plenary cross section is u and v and then what happens is this straight line. Again, now instead of bending in the plane of the board here bends like this outside the plane of the whiteboard here.

Since it bends like that that will result in a displacement field of delta v of x ev delta v of x ev and this delta v similar to the arguments that we used here, similar to the arguments used here will produce a displacement in the x direction which is which will be now instead of y, it will be z in this case, in other case will be v minus v naught into delta v by dx. So, that will produce a displacement minus v minus v naught d delta v by dx, this entire thing is the x component of the displacement that is how we got that.

The remaining steps are the same, now I want to find this displacement gradient H which is gradient of u which would be minus u minus u naught d square delta u by dx square plus v minus v naught d square delta v by dx square dx square and then it is going to be minus d delta u by dx minus d delta v by dx, here it is going to be d delta u by d x d delta v by dx 0 0 0 0 that is a greater our displacement field.

Now, I am interested in finding the strain. So, basically now I am interested in finding the strain epsilon.

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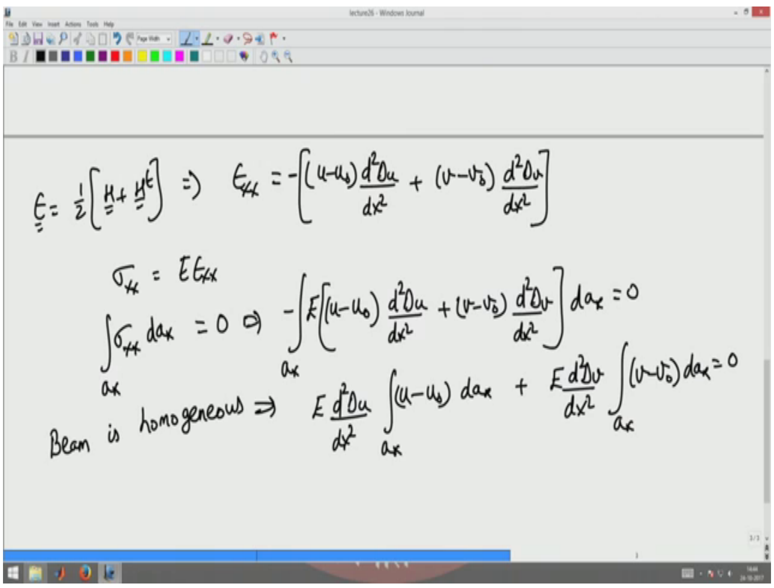

Which is one half H plus H transpose that will give me this will imply that epsilon x x strain alone is nonzero which will be u minus u naught with a negative sign d square delta u by dx square plus v minus v naught d square delta v by dx square.

Now, I have to substitute this in the expression for the actual force net actual stress sigma xx. Now from here, I will get sigma xx as e times epsilon xx using a one dimensional constitute relation, then I am interested in finding sigma xx d ax ax that is a net actual force any cross section this we said 0 for beam bending problems. So, this has to be 0, this will imply that integral e times u minus del u minus u minus u naught d square delta u by dx square plus v minus v naught v square delta v by dx square dax has to be equal to 0.

Now, what happens; I assume the beam to be beam is homogeneous this will imply I can pull out the e d square delta u by dx square integral u minus u naught dax ax plus negative sign, I can leave it out because this equal to 0 dav dx square integral v minus v naught dax ax as to be 0 ok.

Now you have one equation, but you have 2 unknowns u naught and v naught which are different from the one equation what the argument, we give here is each of these individual terms this has to be 0.

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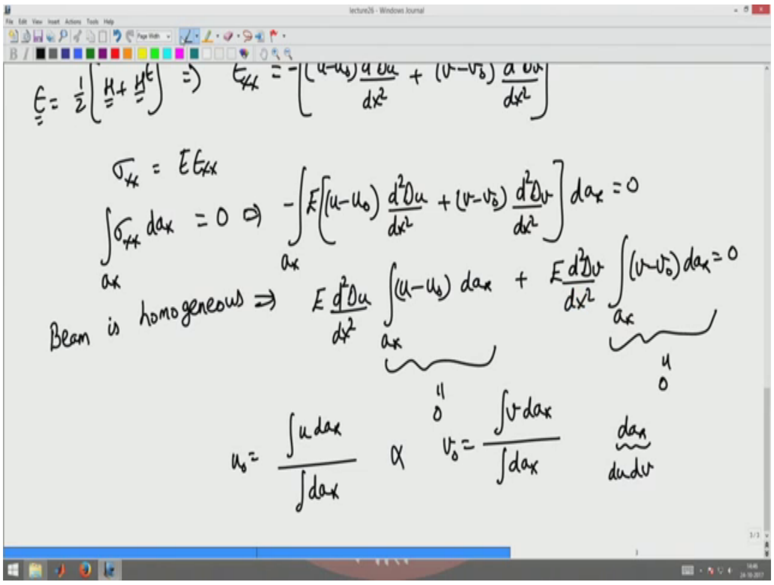

For that old and this has to be 0 for above equation old because delta u and delta v are independent I can have delta u displacement, but no delta v displacement, I can have delta v displacement and no delta u displacement and still this equation as the old.

Hence you require the individual components of this additive equation to be 0 from where you get u naught as integral u dax by integral dax and you get v naught as integral v dax divided by integral dax where dax here means du dv, I am integrating in the principal axis plane du dv.

So, basically now this is the expression for u naught and v naught next the expression is next I have to find d square delta u by dx square next I have to find d square delta u by dx square d square delta v by dx square that comes from the moment equations the moment equations are I have M v moment given by integral u minus u naught sigma xx dax.

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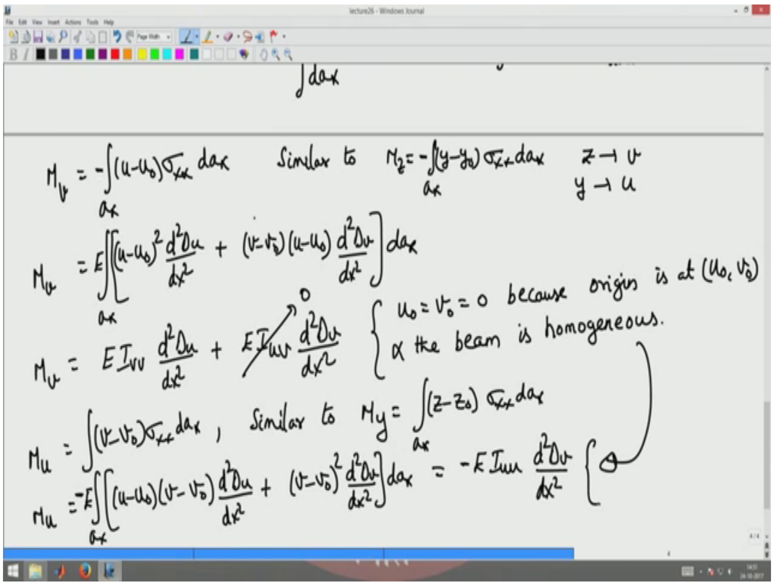

This is similar to M similar to M z moment being given by integral y minus y naught sigma xx dax ax where I replaced z with v and y with u.

So, basically I have change from xyz axis to the xuv axis hence this change, now then what will I get? I got sigma xx has this equation in here epsilon xx substitute in here with hence more less gave me the sigma xx equation. So, I will substitute that equation back in here for sigma xx to get M v as integral u minus u naught square d square delta u by dx square times e, again I assume the homogeneous cross section of the beam plus v minus v naught into u minus u naught d square delta v by dx square dax ax.

Now we find that u naught and v naught are nothing, but the center of the cross section from the expression that we got here you understand that u naught and v naught are the center of the cross section. So, if the origin of the coordinate system showed at a center of the cross section u naught and v naught would be 0 and hence you this equation will simplified to M v would be e times I vv d square delta u by dx square plus e times I uv d square delta v by dx square , but this by a deflection of uv axis is 0.

Now here I use the fact that u naught v naught is equal to 0 because origin is at u naught v naught and I use the fact that and the beam is homogeneous. So, that is the first equation.

The second equation similarly will be the moment for the mu direction which will be integral v minus v naught sigma xx dax. So, this is similar to M y moment which is integral z minus z naught into sigma xx dax ax.

Now, this moment is if I substitute for sigma xx I will get this as mu moment as integral e again I assume the beams homogeneous ax u minus u naught into v minus v naught d square delta u by dx square plus v minus v naught whole square d square delta v by dx square dax this is simplify to e times I uv is again 0. So, this will be I uu d square delta v by dx square for sigma xx there will be a negative sign in there because epsilon also negative sign in there . So, this will be this.

So, now from for same reasons as this you got this equation same reason here.

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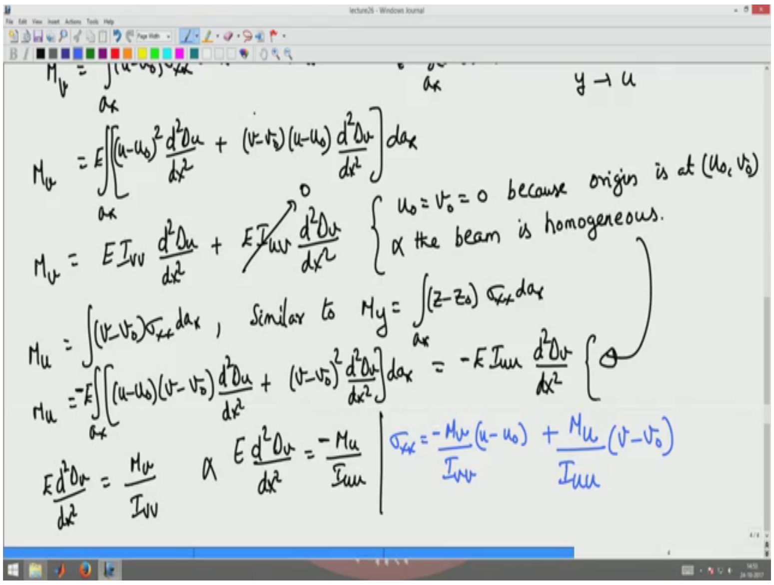

Now you have e d square delta v by dx square equal to M v by ivv and solving this equations e d square delta v by dx square equal to mu by I uu with negative sign in.

Substituting now, how I know M v and mu ways as a function of x. So, I use this equation to find delta u and delta v, then I substitute this entire thing into the stress expression to get my sigma xx as M v by I vv into u minus u naught plus mu divided I uu into v minus v naught will be the stress in the member.