Mechanics of Material
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Constitutive relation, strain energy and potential
Lecture – 41
Bulk Modulus

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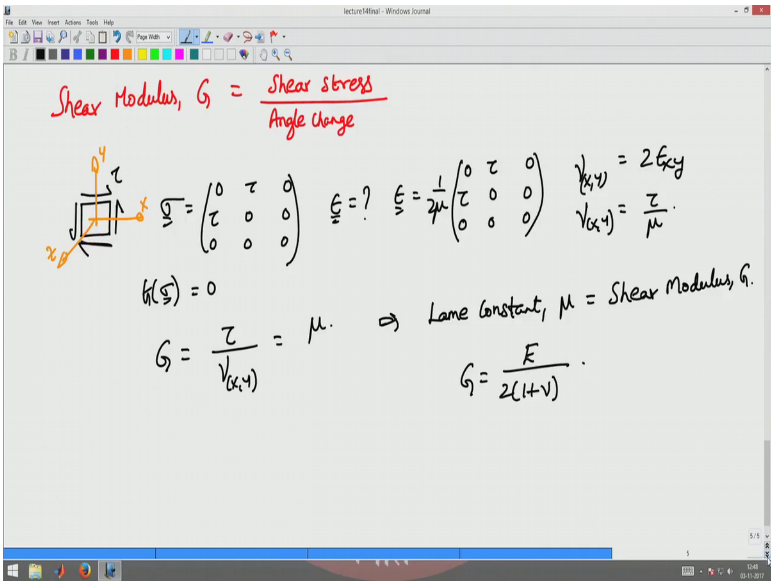

The next model is that we want define is what is called as the Bulk Modulus K.

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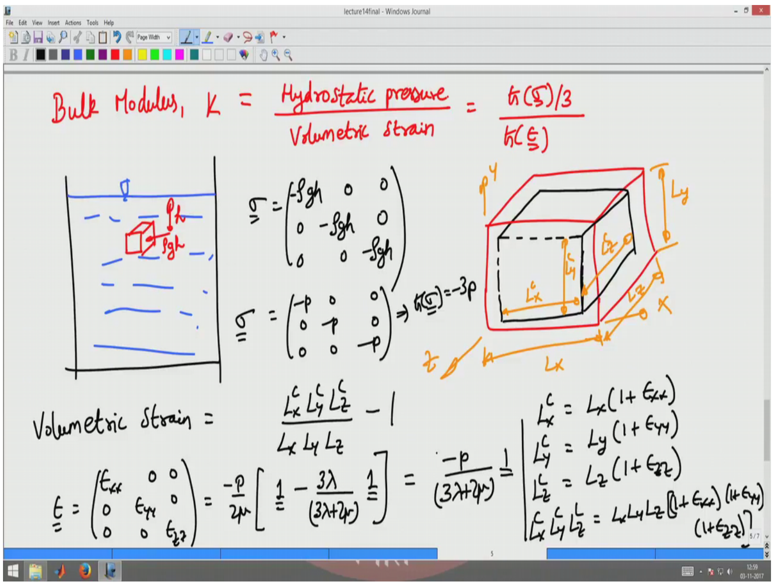

This is denoted by K, which is defined as the hydrostatic pressure divided by volumetric strain. We saw before that hydro static pressures a pressure state where in there exists no shear stresses in any plane, it is like immersing a material in water or in a fluid where in there is only hydro static pressure acting on that material.

So, in other words the hydrostatic pressure can be related to stress of sigma by 3 this is nothing, but stress of sigma by 3, and we will now see that the volumetric strain is nothing, but stress of epsilon. So now, what we have is some fluid in which a block is immersed block of a material is immersed, it sees an hydrostatic pressure rho gh this is h this pressure is rho times g of h, where rho is the density of the fluid.

So, the stress state for this state of this experiment is given by rho gh 0 0 with a negative sign, 0 minus rho gh 0 0 0 minus rho gh ok. Now, the definition that will undergo is a volumetric change, basically this cube is going to now shrink isotropically to some cube like this. And the some cube like that isotropically that is going to shrink that is if I choose my coordinate system this should be x y and z, I have the length along x direction where to be Lx, Ly, Lg and Ly and if my deform lengths were to be Lcx, Lcz and Lcy, then volumetric strain is defined as 1 minus Lcx, Lcy, Lcz divided by Lx, Ly, Lz ok.

Now, for the assumed set of stress I will rewrite this as sum minus P 0 0 0 minus P 0 0 0 minus P. From the strain expression you will get that this stress corresponds to this stress uses a strain, which is Epsilon x x 0 0 0 Epsilon yy 0 0 0 Epsilon zz, which is 1 by 2 mu minus P into identity minus lambda by 3 lambda plus 2 mu, identity into 3 because stress of sigma is from here you get that stress of sigma to be minus 3 P.

So, I substitute in there I get that. Now this one will simplify to minus P by 3 lambda plus 2 mu into identity, from where I will get that the relationship between the strain and the operator hydrostatic pressure is given by minus p by 3 lambda plus 2 nu times identity ok. So, what does this mean, this means that Lxc is given by from our definition of the strain it is Lx into 1 plus Epsilon xx.

Similarly, Lyc is given by Ly 2 1 plus Epsilon yy and Lzc will be Lg into 1 plus Epsilon zz ok.

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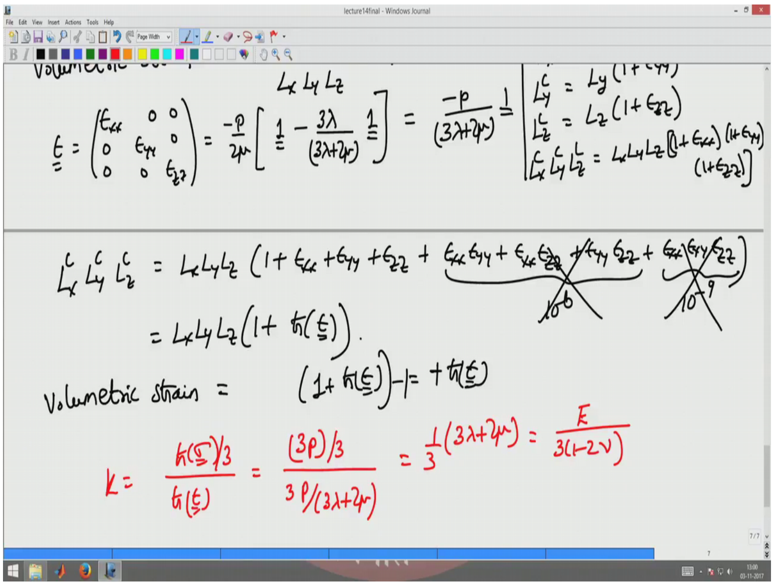

So, now Lxc into Lyc to Lgc, would be Lx Ly Lz into 1 plus Epsilon xx into 1 plus Epsilon yy into 1 plus Epsilon zz. Now expanding this multiplication to find that Lxc, Lyc, lzc is nothing, but Lx, Ly, Lz into 1 plus Epsilon xx plus Epsilon yy plus Epsilon zz plus terms like Epsilon xx Epsilon yy plus Epsilon xx Epsilon z z plus Epsilon yy Epsilon z z plus Epsilon xx Epsilon yy Epsilon zz. Now we ignored the irr terms before the displacement grading saying that it is small same thing holds.

Now, so this will be of the order 10 power minus 6 Epsilon is 10 powers minus 3, and this will be of the order 10 power minus 9. So, I ignore these terms and then, I write it as Lx, Ly, Lz is 1 plus there is nothing, but stress of Epsilon 1 plus stress of Epsilon. So, I get that as 1 plus stress of epsilon. So, now volumetric strain would be, strain is 1 minus 1 plus stress Epsilon, which gives us that this is nothing, but stress Epsilon minus stress Epsilon ok. Basically, you get the negative (Refer Time: 08:28) in because I am assuming compressive stresses.

So, there will be negative volume otherwise, I should have defined this as lambda x minus 1 in which case this would be minus 1 then this will be plus Epsilon plus j Epsilon. It is the volume at exchange will be plus j Epsilon, going back to the definition of bulk modules; let us just a sigma by 3 by stress Epsilon which was the volumetric scene which is what I showed right now ok.

So, bulk modulus K now would be stress of sigma by 3 divided stress of Epsilon sigma was stress of sigma was 3 P divided by 3 and stress of sigma stress of Epsilon is 3 times P divided by 3 lambda plus 2 mu. From here you get the bulk modulus as 3 lambda plus 2 mu into 1 by 3. So, this in terms of young's modulus poissons ratio will boil down to E by 3 to 1 minus 2 mu, a by 3 into 1 minus 2 mu. Now to summarize what I have seen today is we have seen that there are six different constants that we can define for a material isotropic material.

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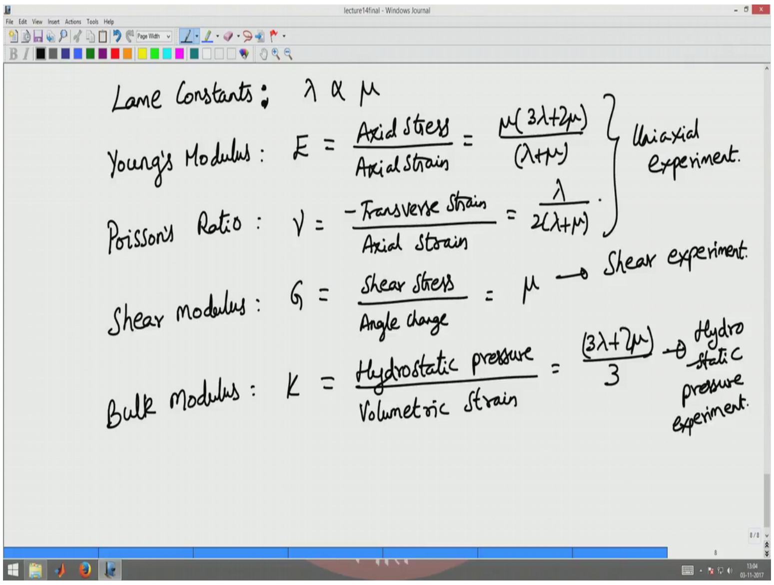

One is purely based on mathematical arguments to get lame constants, which is lambda and mu is called as lambda mu, which we denote by lambda mu and then we got from a uniaxial experiment what is called as the young's modulus.

Which is denoted by E, which is defined as the axial stress divided by the axial strain which is nothing, but mu times 3 lambda plus 2 mu by lambda plus mu, and then we define another parameter called poissons ratio nu which is nothing, but negative of transverse strain to axial strain.

Which in terms of lame constants was lambda by 2 times lambda plus mu, these 2 we got it from a uniaxial experiment then we defined another model is called as shear modulus which we denoted by G, which was shear stress divided by angle change, which we found in terms of lame constants to be mu this came from a shear experiment then we defined bulk modulus.

Which we denoted by K, which was hydrostatic pressure divided by volumetric strain, this we found was 3 lambda plus 2 mu divided by 3 this came from now hydro static pressure experiment. The next class we will see what are the restrictions on these parameters and we will go on from there.

Thank you.