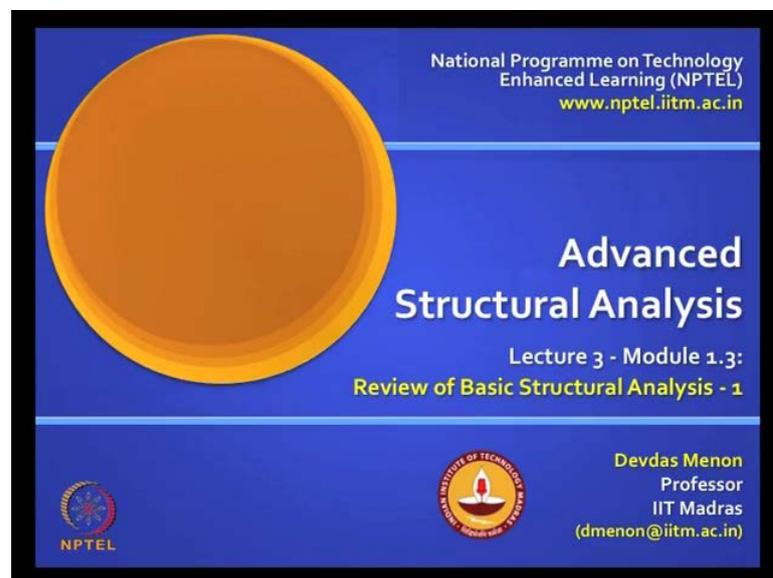


Advanced Structural Analysis
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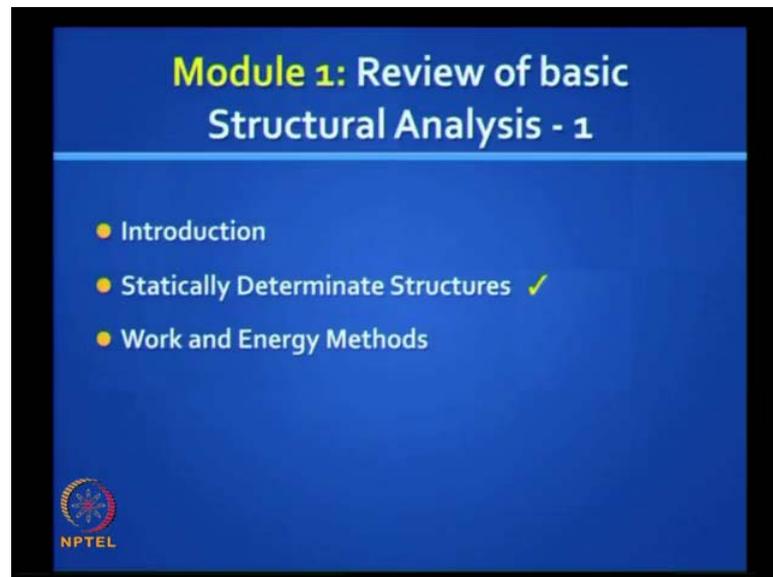
Module No. # 1.3
Lecture No. # 03
Review of Basic Structural Analysis -1

We are now doing the third lecture on the first module - Review of Basic Structural Analysis.

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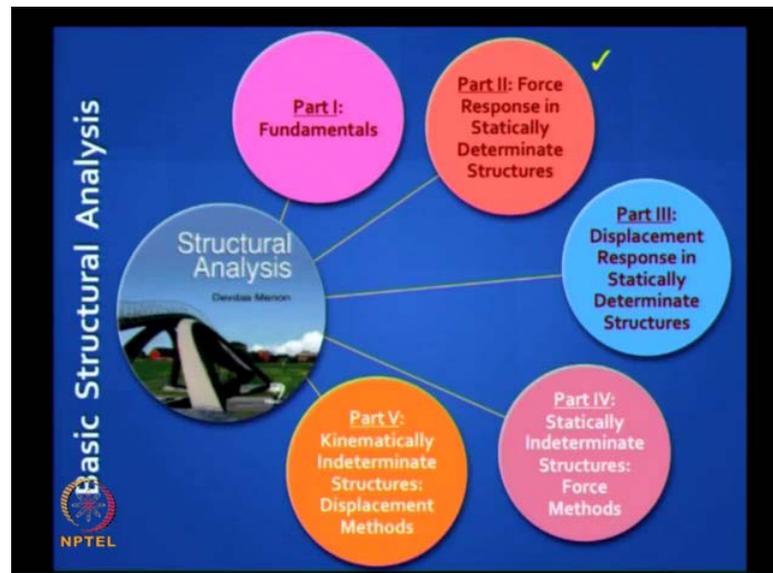


If you recall, in the last session we covered a bit about statically determinate structures.
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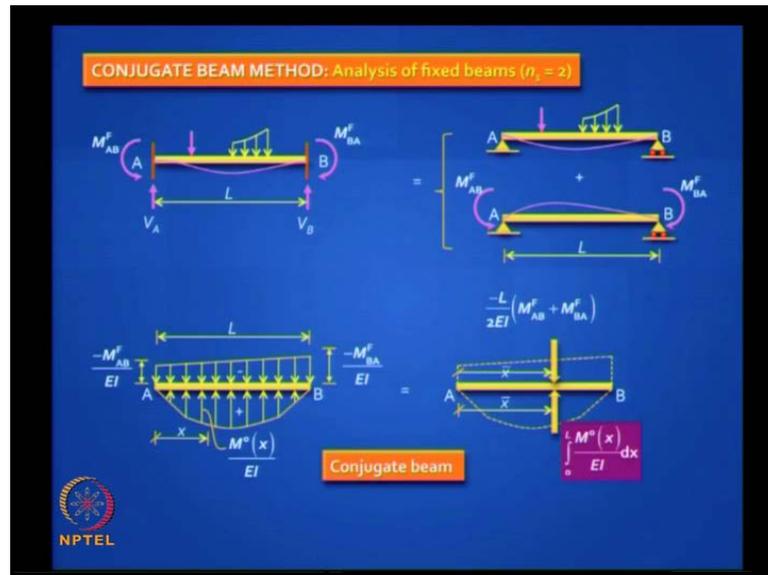
I think we were looking at beams and we stopped with the conjugate beam method. In this session, I hope to finish that topic and so that we can do work and energy methods in lecture four.

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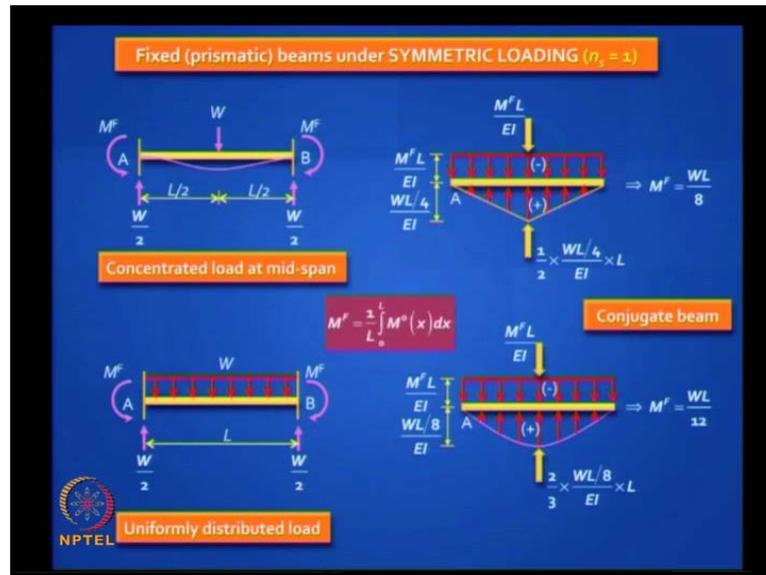
So, just to remind you, the book we are following is Structural Analysis and you need to refer to the book to look at problems and the solved examples.

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We were looking at the conjugate beam method. Here, you will see how it can be applied to solving fixed beams. This is a fixed beam AB with arbitrary loading, gravity loading. You can release the fixity and so you get a statically determinate structure which is a simply supported beam. You can divide into two parts as you can see there, the one - where you have the loading, and the other - where you apply the fixed end moments which are hogging moments. M_{AB}^f and M_{BA}^f are the two ends and you can now look at the conjugate beam, which will be now free free because, the original beam is fixed fixed and you will find that on the top. You will be putting the curvature diagram caused by the negative moments. So, that is a straight line. At the bottom you will have the free moment diagram $M^o(x)$ by EI. These two must self-equilibrate. So, you will find that the resultant force must be equal and the lines of action must also be equal. What is interesting here is that, if we equate the two areas, you will find that the bottom areas $\int_0^L M^o(x) dx$ by EI and there are some advantages you get when the loading is symmetric.

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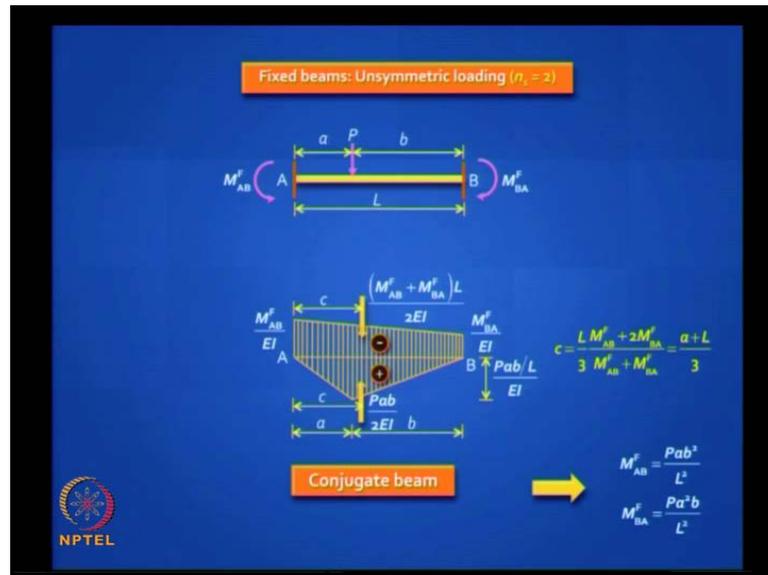


So, let us look at those special cases, where you have symmetric loading and I will show you here, two pictures of symmetric loading- the first is a standard concentrated load W acting at mid span and the second is a uniformly distributed load of total magnitude W acting at the mid span. The support reactions are W by 2, W by 2 and the two fixed moments at the ends will be actually equal and opposite to each other. So, we can call it just M^f instead of M^f_{AB} and M^f_{BA} . So, if you were to draw the conjugate beams, you will find that finding the fixed end moment is quite easy.

The area on the top is the area of that diagram $M^f L$ divided by EI and that must be equal to the area of the free moment diagram divided by EI . So, if you equate the two, you have this interesting equation, which suggests that the fixed end movement is nothing but the average value of the free moment diagram because $\frac{1}{L} \int_0^L M^a dx$ actually means that it is an average value and so, it is easy to calculate.

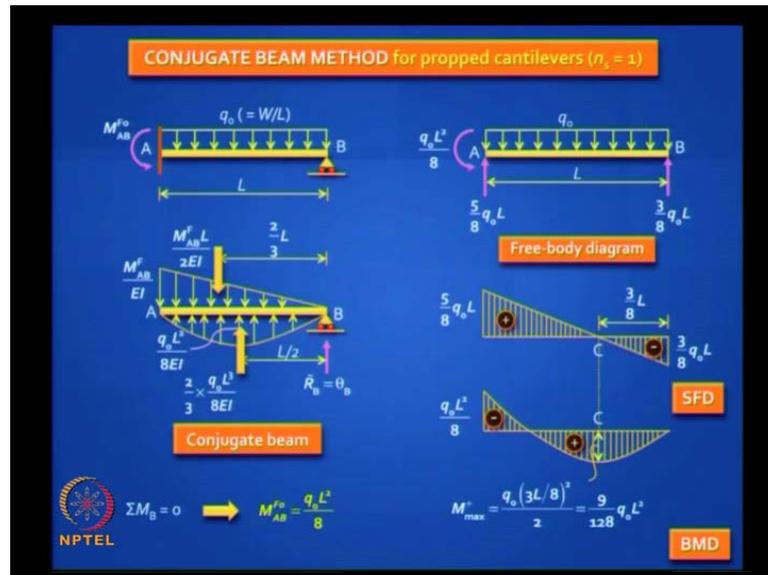
If you take the first case, it is a triangle; so, the area is easy to calculate. The second case, it is a parabola; the area is two-thirds of equivalent rectangle. You can easily find the answers. The answers are: In the first case, WL by 8, M^f is equal to WL by 8 and the second case, it is WL by 12. So, you will find that, for symmetric loading it is extremely easy to do. It will take you just about a minute or so to crack the problem. So, the conjugate beam method is a very useful method to find fixed end moments when the loading is symmetric.

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When it is not symmetric, then you need two equations. This is another example that needs a little more work but you will still get the results. Here, you see that if you have load P acting not in the middle, then you will have two moments, which will not be equal to each other. The larger moment will always be nearer to the load. So, in this case, if A is less than B , M_{AB}^f will be numerically more than M_{BA}^f . You have to equate the area of a trapezium with that of a triangle and you have to find their lines of action; this can be done. We will not go through this, but these are the formulas that you need to recall; especially, when you do displacement method. The fixed end moment formula here turns out to be P into a into b square by L square for the moment M_{AB}^f , and P into a square into b by L square for the moment M_{BA}^f . The previous case where you had a concentrate load in the mid span is obviously a special case of this span, A is equal to B is equal to L by 2 .

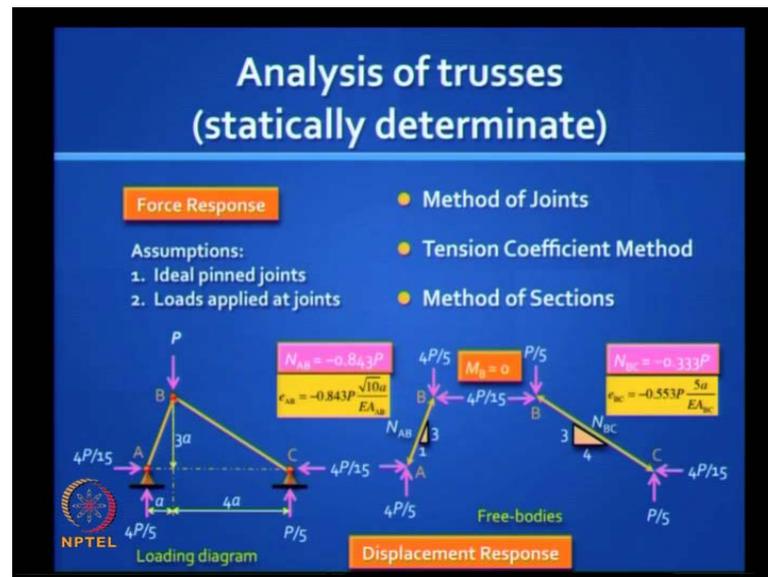
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Conjugate beam method can also be used to find fixed end moments in propped cantilevers beams. Here, you have only one unknown fixed end moment and here is a convention that we will consistently follow. You will find that we are not using M_{AB} , here, but we are putting small naught there in M_{f0AB} to remind ourselves that the far end B is a hinge and the small round that you see there is a reminder, and this also can be done in exactly the same way.

Here, you will find that the conjugate beam is not free free; it is free-hinged. So, the support reaction in the conjugate beam will take some reaction and that reaction will be equal to the slope of the beam at B. Again, you equate the forces and you can get the solutions. In this case, it works out to be $q_0 L$ square by 8 for a uniformly distributed loading. So, you can draw the shear force diagram, bending moment diagram once you get the fixed end moment because, once you get the fixed end moment, the beam is now statically determinate. You know how to draw shear force and bending moment diagrams for a statically determinate beam. The maximum bending moment will be where the shear force changes sign. You are quite familiar with all this.

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We are done with the topic of beams. We looked at how to draw shear force and bending moment diagrams. We also did some interesting cases, where the bending moment diagram was given to you and you have to figure out, what loading cause that bending moment. We also looked at how to calculate deflections and slopes using direct equilibrium method.

We now quickly move on to the topic of trusses. The basic assumption made in trusses is that the actual forces are the only forces that act in the various members. Bending moments and shear forces are assumed to be negligible and this is valid, if we assume ideal pin joints at the junctions of various bars and if the loads are applied exactly at the joints. Now, we know that the first assumption is strictly not true because you need to have just need to one rivet or one pin for free relative movement between the members, but in practice you have more than one bolt in trusses.

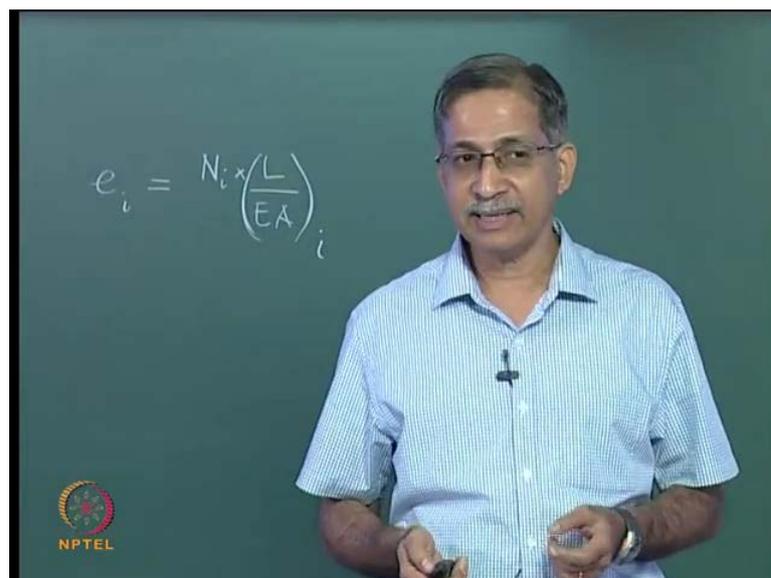
In bridge trusses, you may have about thirty to forty bolts in a junction. If the elements are welded, then it is actually rigid joining connection, but that makes truss highly statically indeterminate. Even if you were to do a very rigorous analysis, you will find that the departure from the assumption of ideal pin joints is not very significant. You get moments, but they are called secondary moments. For a first order analysis, we can normally ignore them. In design practice, if it is a heavy bridge truss, we try to account for it indirectly by assuming a 10 percent increase in the trusses. So, please note the

whole idea of doing first order structural analysis is to get quick results, which are reasonably accurate. You are familiar with this conventional method of joints and the tension coefficient method which is a generalized method of joints that can handle any type of statically determinate truss with any loading. Then, you have the method of section which should be applied intelligently to save time.

Here, is an example of two bar truss and you can see that the calculations are very easy to work out. We need to take advantage of the fact, that there is an internal hinge at B and there is no bending moment transferred from one member to the other. You only have actual forces in the two members and it is very easy to compute. So, let us move ahead, once you have the actual force in the two bars, then you also have the actual deformations because, we know that there is a relationship between actual force and actual deformation. We use a word elongation to refer to the actual deformation, where elongation is strictly appropriate, if the actual force is tensile and this is what we assume when the force is positive.

If the force is negative, then it is compressive and the elongation turns out to be a contraction. What is the relationship between the action force and the elongation? Elongation is equal to what? [Noise] (Refer Slide Time: 10:56).

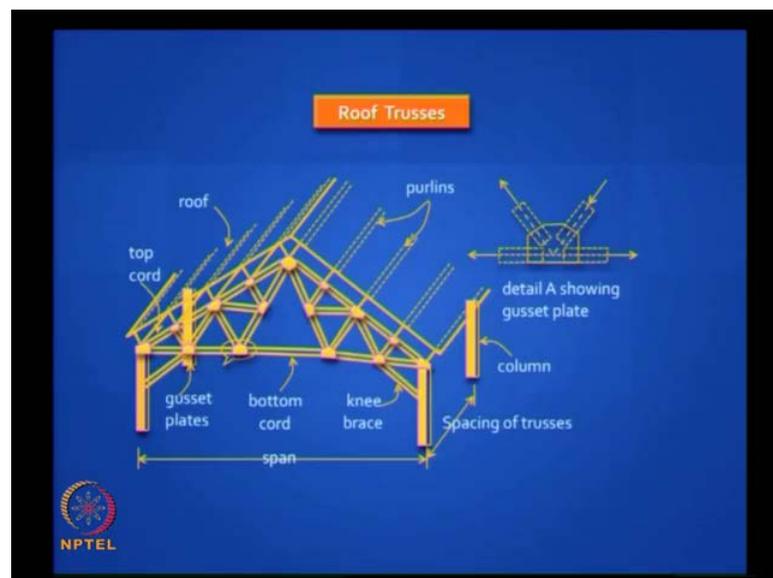
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The elongation in the i th bar would be equal to the actual force in that bar divided by length and the actual rigidity of that part. Therefore, this follows directly from Hooke's law. So, we can do that. Once, you got the bar elongation, you can locate the displaced position of the joint B and you can do it geometrically.

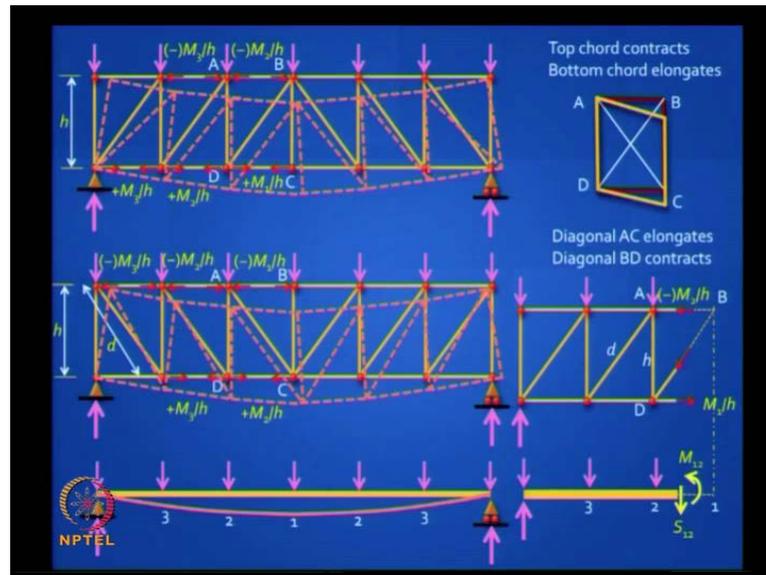
I have shown a picture here, where AB has reduced in length and BC has reduced in length, but if they were to be shown in this picture, then they do not meet but we need to make them join. So, the only way to do that is to rotate AB about A and CB about B till they meet. You can draw arcs or tangents to do that and that is how you locate geometrically. That is how you construct and you can actually get the deflection. We will find there are better ways of finding deflections.

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Trusses are commonly used in roofs and they are also used commonly in bridges. (Refer Slide Time: 12:26) Bridge trusses are referred to either through type trusses when the deck is below and the traffic goes where you can see the two trusses on either side of you carriageway, or they are called deck trusses when the truss is below the deck and there are other intermediate types called semi through and so on.

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Now, we first studied beams and then we looked at trusses. So, it is good to see the equivalence between the truss and the beam. Here, I have shown two pictures of what are called as N trusses because they look like the letter N. One of them does and the other is a mirror image. Let me ask you a question - for the loading shown here, which of the two is better? Which is most structurally efficient; the top one or the lower one lower one? [Noise – not audible] (Refer Slide Time: 13:20) Can one of you raise your hands and I will ask you? [Noise] Lower one. Top one. Answer is top one. Why did you say the top? [Noise] The numbers take tension (Refer Slide Time: 13:34).

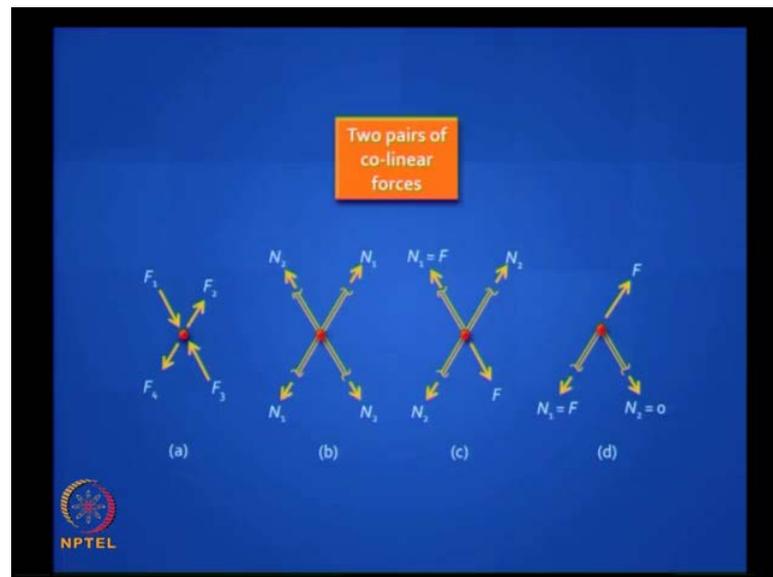
Let us talk about the top chord in both the members. Don't they both take roughly the same order of actual compression? [Noise – Bottom part] (Refer Slide Time: 13:34). What is so special about the bottom? When you cut the beam at any place the reaction at the left side support is upward and so, the shear acting when you cut will be downward. So, the member there is aligned downward and it will be having a tension. So, in the second truss, the diagonal members will be in tension and in the first truss diagonal members will be in compression that you get from the equivalents within the beam, when you do this method of section. So, we should prefer the bars going into tension. Why? [Noise – not audible] (Refer Slide Time: 13:34) Because you have a problem associated with buckling, the size of the members has to be larger and so on. So, these are the important points to note. So, the only difference between the two is the diagonals; the alignment of the diagonals. You see, overall, the truss deflects like a beam and if your

right brain is very good, you can actually see the contraction that is taking place in the top diagonals and the elongation that is taking place in the bottom diagonals.

This is an equivalent beam (Refer Slide Time: 15:03) and you will find that the method of section essentially establishes a relationship between the actual forces in the various truss members with bending moment and shear force values in the equivalent beam over the same span. So, if you cut a section, here for example, (Refer Slide Time: 15:03) if you take moments about the point B, you can find out the bend. The actual force in the top chord in the second truss and the bottom chord in the first truss is given by that bending moment at that location three, where B is divided by the height of the truss. So, this is a quick way and those are the maximum values of forces in the truss.

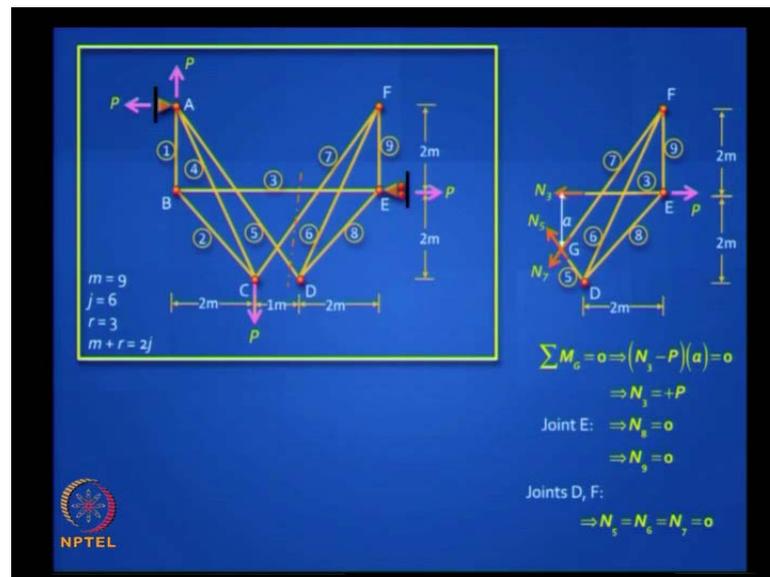
If you had to design a truss quickly, you can do it very fast; find out the equilibrium bending moment in a beam and divide it by the height. So, this is a quick way of doing it and you will find that, this is how it can work through the different actual forces in the top chord and bottom chord. The actual forces will be the maximum in the middle just like the bending moment is in middle region and slowly dies down and sometimes your end members carry zero force. You see that picture (Refer Slide Time: 16:22) where the rectangle becomes a rhombus. One of the diagonals increases in length and the other diagonals decrease in length. So, that gives you a visible idea of why one diagonal goes into compression and the other goes into tension and you can calculate those shear forces.

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The other simple thing that you can take advantage of is a fact that, whenever you have two sets of lines, they need not be orthogonal; they could be inclined and the aligned in the way which is shown here. Then the forces must be equivalent and opposite in any one line. So, for example, I have shown you four pictures there and each of those lines could be a bar. So, if it is a bar, then you are talking about an actual force in that bar. So, you can take advantage of these pictures. If you look at the picture d, you will find that if say, one of those bars has zero force. Sorry, if we do not have the opposing force at that joint, then you can take that force to be zero. So, you must develop the skill of identifying which are the zero forces in a truss by inspection and also the ability to identify whether the force that you get is tension or compression. So, there are many problems and we will quickly see some examples.

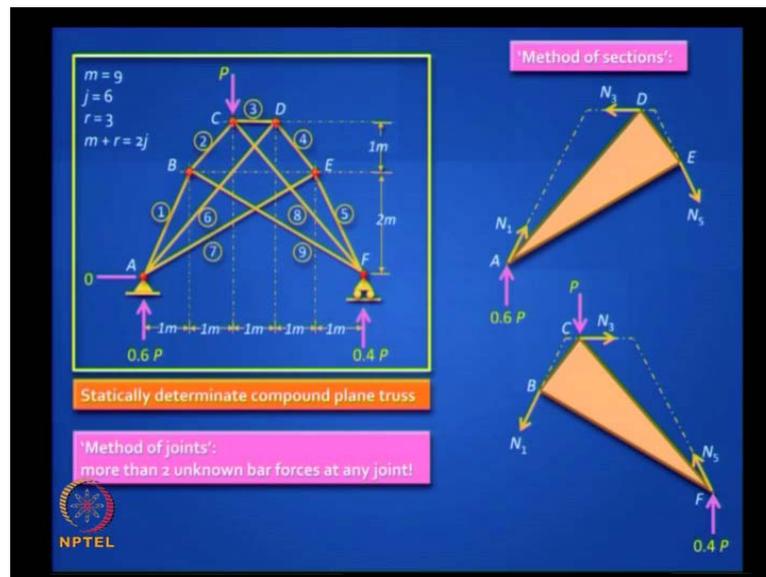
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Here is a truss which looks little complicated because you have larger number of members and you can do a quick check and you find that m plus r is equal to $2j$. So, it is a simple truss, but you do not know where to start because of so many forces, but perhaps some of them are zero and you need some skill to figure out. We have done this problem earlier and so, I will go through it quickly. The first thing to recognize is if your load is P acting downwards, those are your reactions and the reactions are statically determinate. They all turned out to be P , then if you cut a section say, where I have shown and you blow up that part you will find...

What can you make out from this picture of the section? [Noise – not audible] (Refer Slide Time: 18:42) N_3 is equal to P - that follows from the fact that if you take moments about G , then you can prove N_3 is equal to p . Then, once N_3 is equal to P , you go to the joint D . You will find that necessarily there are many forces which are zero and you will find that actually you can remove those bars once they are zero and your truss reduces to this problem. (Refer Slide Time: 19:12). So, it is a much simpler truss to analyze and this can be done easily. You can analyze by method of joint triangle of forces.

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This is an easy problem and is an extremely interesting problem. This actually was posed by **Timoshenko** in his book on Structural Analysis and the normal way of cutting sections does not work here. Wherever you cut a section you have far too many unknowns and you know that when you deal with the plane truss, you can at the most have three unknowns, cut sections. If you are doing method of joints, you can have at most two unknowns.

You will find that traditional techniques do not work here. Here is a clever way of doing it and this again is a compound truss, but statically determined. What you can do is - you know to view that segment as a triangle and separate the triangle, which means using a pair of metal scissors and snipping through those connecting bars. You expose those bars and here you have one look at this picture (Refer Slide Time: 20:36) and you know that there are only three unknown forces and you have three equations equilibrium available. You can solve it or you can take the other triangle and solve it. Once you get the idea, it is easy to calculate because you can use any three equations and solve it.

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'Tension Coefficient' Method

$F_x = F_{ax}i + F_{ay}j + F_{az}k$

$T_i = N_i / L_i$

Direction Cosines:

$\sum F_x = 0 \Rightarrow F_{ax} + \sum_i N_i l_i = 0 \Rightarrow F_{ax} + \sum_i T_i (x_i - x_A) = 0$

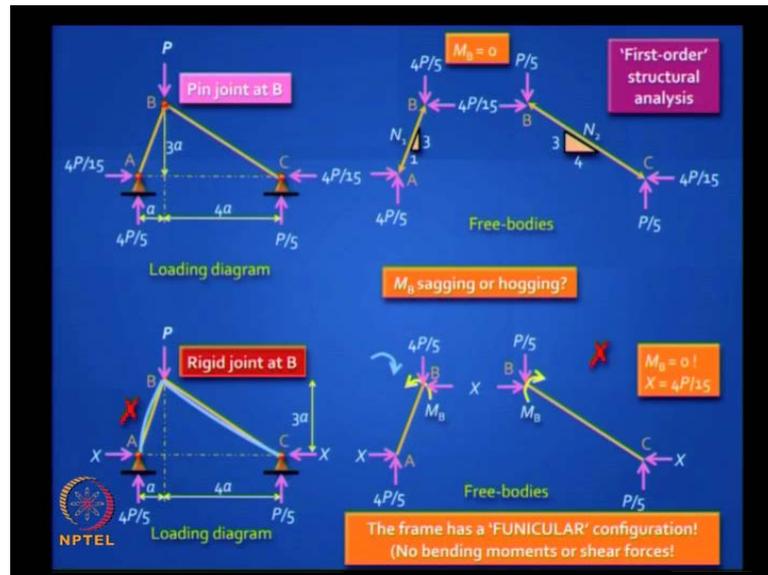
$\Rightarrow F_{ay} + \sum_i T_i (y_i - y_A) = 0$

$\Rightarrow F_{az} + \sum_i T_i (z_i - z_A) = 0$

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Finally, the robust method of cracking any problem is a tension coefficient method, which means you can take any joint and this could be useful, especially when you do space trusses. You write the coordinates of those joints and you define the tension coefficient as the actual force in any bar divided by the length of the bar. You can write a generalized equation of equilibrium along the x direction, along the y direction and along the z direction. So, you can systematically write this equation moving from joint to joint and solve them. This is the generalized way of doing it. This is a way, you would probably do if you had to do using computer to solve the problem because, you do not take any shortcuts, but you just go through it and solve series of simultaneous equations. You can make use of the direction cosines l, m, and n for each bar and this is a systematic way of doing it. We are done with trusses for the time being. (Refer Slide Time: 20:43).

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Now, we move on to the next topic and this is a good interface. Look at this problem, which we looked at earlier, you have a pin joint at B and what happens if you make that joint rigid? Strictly, what all can happen? You got actual forces in the first case. Now, what is the extra that you can get? You can get bending moment because now the joint B is rigid, which means you have a moment resisting joint. Now, if you get a moment and the question I wish to ask you is - well, your vertical reactions do not change; your horizontal reaction is your unknown; if you get a moment at B, is that moment going to be sagging or hogging? I have shown here, sagging. Will this work and why would not it work? No, that is not the reason. Why would not sagging moment work?

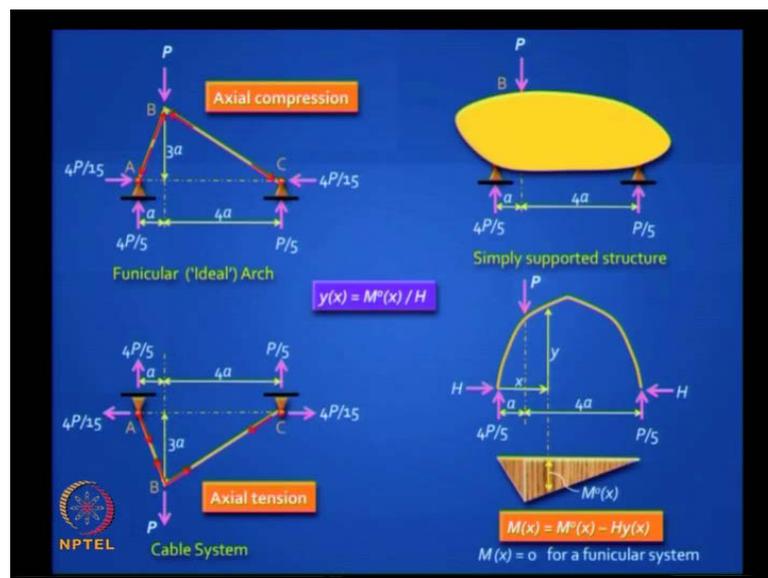
Now, to really understand this, try to visualize it. Visualize this by looking at deflected shape, the possible deflected shape. Now please note, we normally assume, when you deal with frames, this is a frame and not truss any more. We make the assumption and their actual deformations are negligible. So, B does not really move down. B will move down only if the two bars will reduce in length. So, you will have difficulty in drawing the deflected shape because if you draw a deflected shape like this (Refer Slide Time: 23:32) and you have to preserve the angle at B between the two bars because it is a rigid joint. If it were a hinged joint, you do not have to worry about that compatibility requirement. So, you will always find that if one of the elements is shown sagging, the other will be hogging but your arrows will not satisfy that and either you draw like this or you have to draw like this, which means you will never get both sagging and hogging.

The deflected shape will not look like that. (Refer Slide Time: 23:57) So, you should know unsuitably that something is wrong here.

What is the conclusion that you draw from this? Though the rigid joint has the potential to transmit a bending moment across the joint, in this particular instance, that bending moment cannot be transferred and it will not work out. The bending moment is actually going to be zero. So, it is like saying you have a savings bank account and the bank account can take in any amount of cash value, but being a student very often it turns out to be a zero. It just shows there is a capacity, but you need not have any value to that. So in this case, you do not have that. This is another reason for this.

Can you tell me why there is no bending moment? [Noise – not audible] perfect. This shape is funicular for that particular loading. How do you recognize it is a funicular shape? [Noise – not audible] (Refer Slide Time: 25:18) If you were to assume that there is a beam spanning across the support say, a simply supported beam and if you were to draw a bending moment diagram and if the shape of the frame that you erect between those supports resembles to some scale, the bending moment diagram, then you will get only actual forces in that structure. That is called the funicular configuration and often we miss this out because, once you recognize a funicular configuration, it is ideal.

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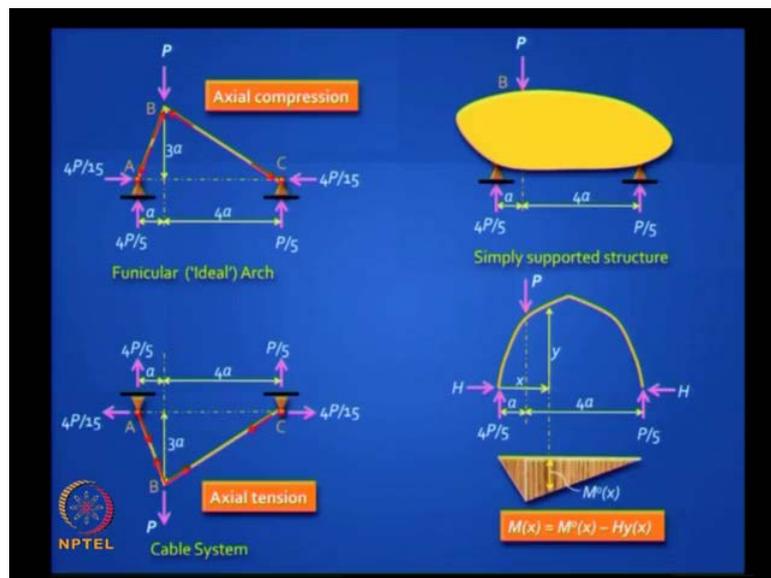


The word funicular comes from the word 'funicle', which means a string and it is the ideal shape. So, you will find that this shape is funicular. In this instance, AB and BC

will only have axial compression. If you flip it over, then it is like a cable and it will only have axial tension. What is the main difference between arch and the cable? [Noise – not audible] (Refer Slide Time: 26:23) Yeah, who is answering? Yeah, arch takes compression and cable has tension. Right, but there are some more differences between an arch and a cable. In addition to compression, the moment is always zero. [Noise – not audible] (Refer Slide Time: 26:45)

You see a cable and by definition is very flexible. An arch is rigid, is it not? It does not move around and by virtue of its rigidity, it is capable of resisting; in addition to axial compression, bending moment and shear force. A cable just does not have that ability. So, a cable will always assume a funicular shape for a given loading, but an arch is something that you can make at the most funicular for one loading, but when some other loads act on that arch, it will be non-funicular and there will be bending moments in shear forces in addition to the actual forces. This is important to understand.

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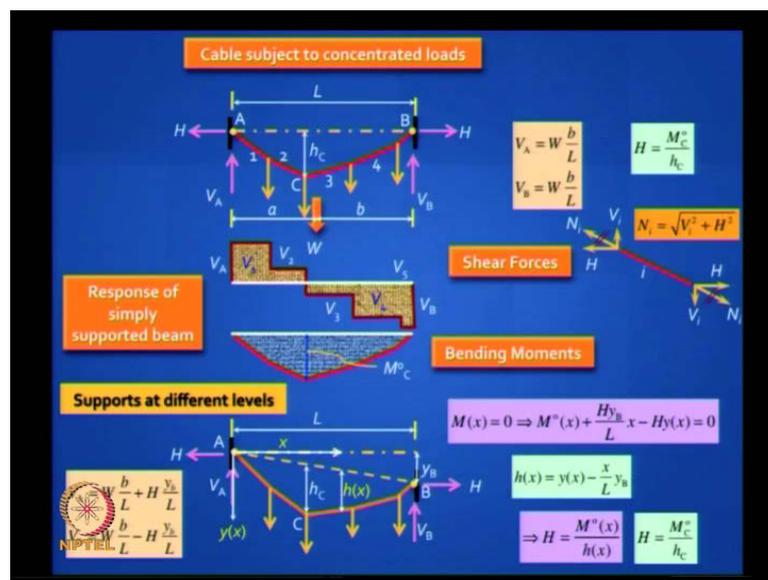


To fully understand that bending moment relationship, look at that bolder on top; it is simply supported and there is no horizontal reaction. Look at the frame, which is also simply supported. In both incidents, the distribution of bending moment along the x- axis will resemble the triangle, if you have a concentrative load.

If you are able to generate a horizontal reaction at the two supports, then that has a tremendously beneficial effect because at any section you get a hogging moment, which

is that horizontal reaction times a height y and your net moment at any section is now reduced to this free moment or the sagging moment $M_0 \times$ minus hogging moment H into y . A Funicular shape is one, where M_0 is equal to H into y or rather you can say y is equal to M_0 by H and if H is a constant. So, it clearly follows the shape of y , the ordinate of your frame at any location resembles your bending moment diagram, then you get a funicular shape. So, it is very easy to appreciate this point.

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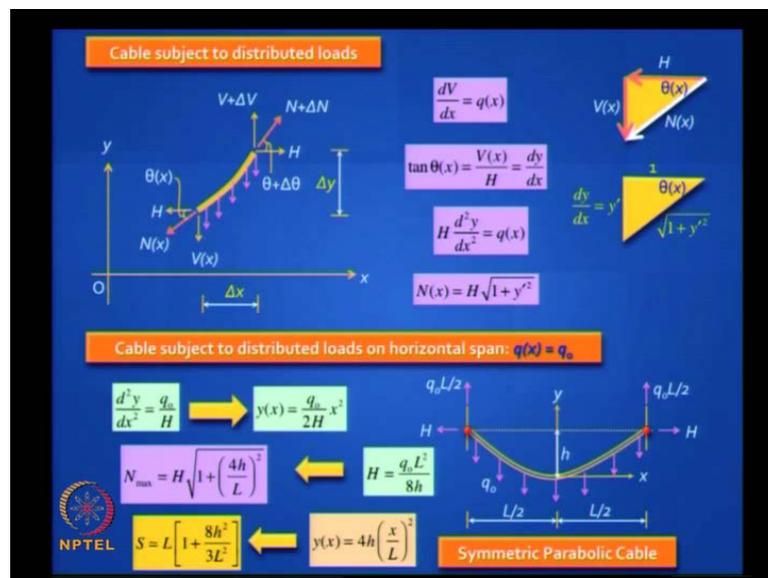
So, here is something on cables. Cables are very interesting to analyze. Again, you can compare with a simply supported beam. You get shear forces in the beam, you get bending moments in the cable. The cable will be made up of a series of straight lines, if the loads are acting on a series of concentrative loads, then the vertical reactions are very easy to calculate as you would in a simply supported beam. Let us say the total load is W and it is located at the distance a from the left end and b from the right end. Then V_A and V_B are very easy to calculate.

The horizontal reaction is actually obtained by the bending moment, the free moment in the equivalent beam divided by the height at that location. So, you can work out the heights at different locations, once you know the bending moment. All you need is a height at any one location and usually you know the height at maximum dip of the cable and that height is called sag. Here, I have shown h_c and so you can you can get the horizontal reactions. If a cable is inclined as shown, then the axial force in that cable is

related to the shear force in the beam, as well as the horizontal force at the two ends. Now, if you cut a section anywhere, you find that the horizontal component is constant and some times, it is referred to as horizontal tension in a cable. In an arch, it is referred to as horizontal thrust. So, you will find that H is constant for all cables and for all segments. V is changing depending on the shear force and the actual force is the resultant of H and V as shown there. (Refer Slide Time: 31:04)

If the support is at two different levels, then you can relate it to the first case, you will find that if the arrangement of loads is same the horizontal force does not change. We can prove that it is a vertical force that changes, you now get a couple, the 2H is separated by a level difference and is self-equilibrated by reduction and an increase in the vertical forces. So, this can be proved. We will move ahead. The difference between the first case, where the two supports are at same elevation, and the second case, where the two supports are at different elevation is only in the vertical reactions. The horizontal force is the same. V_A increases to some extent and V_B reduces.

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If the cable is subjected to distributed loads, then it is not going to be made up of straight lines, it will be curved and you can actually generate an equation which relates geometry of the cable to the forces in the cable. If you take vertical equilibrium of a segment of the cable, you get this beautiful differential equation, which is similar to the equation that we derived for a beam. Is it not? So, q here is a load intensity and it is related to the shape of

the cable and if you take the slope of the cable as y' , then you have a direct relation between θ and the cable forces; $\tan \theta = V/H$. V can change depending on the location of x , but H is a constant if you have only gravity loads, and that is equal to dy/dx . If you substitute this value in the first equation, you get another equation where y or x comes into play and this is a basic differential equation of a cable.

The actual force can be easily generated from the triangular forces and you can use this equation to work out what is the profile of a cable for any given distribution and loading? The most common type of distribution loading is uniformly distributed load and this can be done in two ways: one is uniformly distributed along the horizontal span, the other is uniformly distributed along the curved span. When do you encounter these two cases? Well, self-weight is uniformly distributed along the curved span and the shape of that cable is called catenary from the word 'catena', which means chain, but this is not that common in practice.

In practice, we use cables to support structures, to support roofs, to support bridge. Cable Suspended bridges are of two types: you have the cable straight bridges, where the cables are straight and they have force attention. Each cable will have a different force and you have the classic suspension bridges. What is the shape of a cable in suspension bridges? [Noise] It is parabolic (Refer Slide Time: 34:31). Why is it parabolic? Uniform loading, it is parabolic with the idea that the parabola is the ideal shape, the funicular shape, when you have a uniformly distributed load. So, you need to ensure that it will remain in parabola even when vehicles move on the bridge. How do you do that?

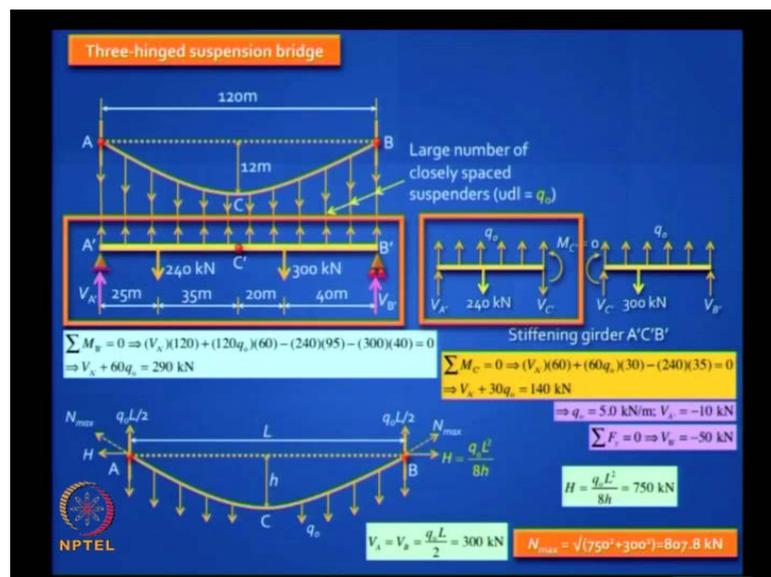
You have to make the deck stiff and these are the equations (Refer Slide Time: 34:59) this relationship is very simple. If the sag in the cable is H in the middle, then the horizontal tension in the cable is nothing but the bending moment, which is $q_0 l^2 / 8H$. So, it is a very simple formula and you can work out the rest of the equation very simply.

(Refer Slide Time: 35:30)



Here, is an example of one of the most famous suspension bridges way back in 1937, this bridge was designed, analyzed by using simple manual calculations, the kind of formulas that we saw in the previous slide, but today’s engineers and today’s students would rather use a computer to solve this problem, but at least for a first order analysis, I think it is important to know the simple calculations for a parabola.

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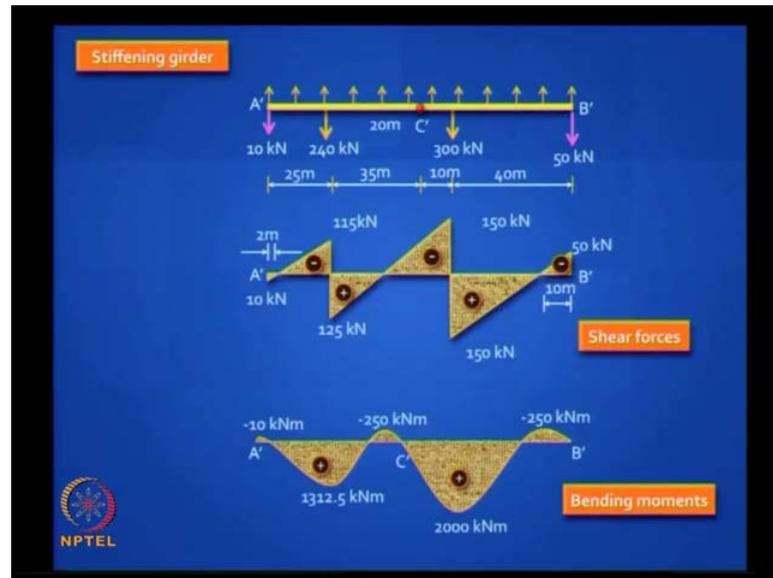
Here, is an example of cable suspended bridge of the girder that you saw in the previous slide at the bottom is called the stiffening girder. It has to be fairly deep and rigid

because, the idea is it should not allow the cable on top to change its shape. All the load that goes on the deck is actually transmitted to the parabolic cable through those vertical elements called suspenders. If the cable is parabolic, then you can safely assume suspenders carry a uniformly distributed load. This can be facilitated by providing a hinge at the center as shown here (Refer Slide Time: 36:38). Because, if you provide a hinge at the center and you make this assumption of uniformly distributed load, this problem becomes statically determinate; otherwise, it is indeterminate. If it is indeterminate, then actually the force in every cable can also be different and that is a real problem, but this is simplification.

If you isolate the cable, you will find that there is only one unknown - uniformly distributed load q_0 . If you know q_0 , you will know the rest of the problem. How do you solve for q_0 ? Well, if you take a free body of that stiffening girder and you analyze it, you take moments about one of the ends; you can find the relationship between the vertical reaction and q_0 . Next, you take that beam apart and you invoke the equation of the condition that, at the hinge there is no moment transfer and MC equal to 0. Then, you will find that you can solve this equation and get another relationship from which you can prove and establish the value of q_0 . Once you know q_0 , both systems can be separated out and the cable is easy to analyze and design.

In olden days, the real worry was about, what is the size of the cable to be used - the wire rope. So, you need to have an idea of what is a maximum tension in the cable. So, it is very simple. Find the total load that can come on the bridge and half the load will go to one end - on the cable - that is you need is value of q_0 and q_0 into L by 2 is a vertical reaction and H is $q_0 L$ square by $8H$ and the resultant of that is the actual force. If you know the allowable stress in that cable, you can work out backwards on what is the diameter you need. You apply some factor of safety and that how it needs to be done. It is a simple straightforward calculation.

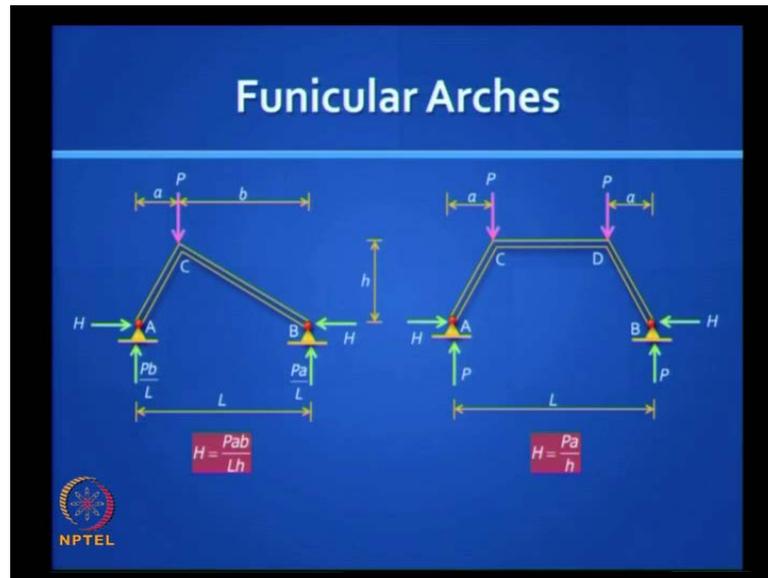
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Now, in this problem you can solve and work this out, you can get the results. Once you separate out the stiffening girder, then you can work out statically determinant. If you cut any section, you can work out shear force and bending moment diagrams.

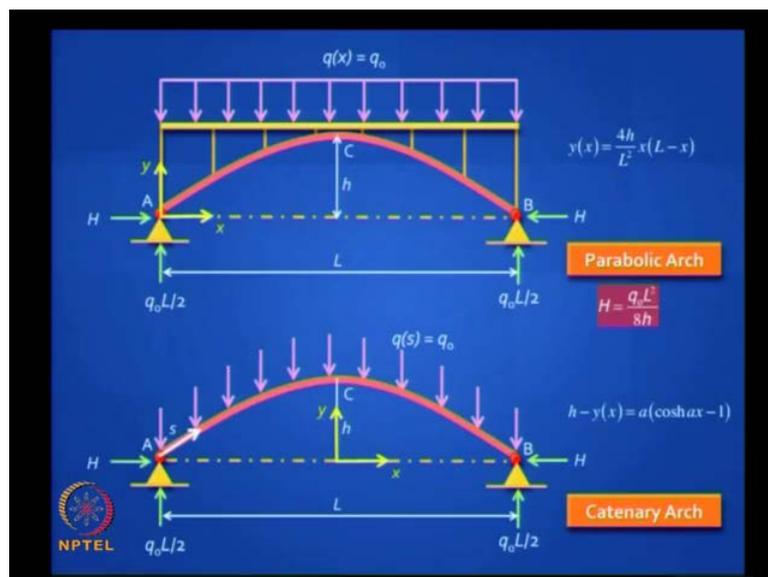
Here is the picture of something we did (Refer Slide Time: 38:58). I think in 2004 or so, your senior students built this bridge, made entirely of newspaper and rope. So, it is a cable state structure, where they use these simple analytical tools to do it. You will find that the two vertical **pilances** are made up of newspaper. Lot of testing was done in our laboratory, those ropes are manila, and they actually want to put this up in the Guinness book of world records. I do not think they succeeded, but it did come out as a journal paper in the ASCE journal for Professional Issues in Engineering Education and Practice. Anita Rao was a civil engineering student, who led that team of about 100 other students and you can look at this article to see the history of how this was done.(Refer Slide Time: 39:10)

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Funicular arches are ideal arches. If you have concentrated load, they have to be made up of straight lines. Sometimes, they are called linear arches and they take this shape. They are very easy to derive. Now, you can find the horizontal thrust by simply taking the bending moment at any point and dividing it by the height of that point. So, these are called linear arches.

(Refer Slide Time: 40:23)



This is a parabolic arch. Where do you encounter these arches? [Noise] Bridges. Yes, in bowstring girder bridges. You can also encounter them in building and I will show you picture, where we did this. (Refer Slide Time: 40:36).

Now, this is a two-hinged arch, which is funicular. The boundary conditions are not important. If you can have uniformly distributed load along the horizontal span, parabola is the ideal shape. If you have a curved, if you have a self-weight kind of loading, then the catenary arch is the ideal shape. You can build walls with this shape and traditionally this was done. Here, some work (Refer Slide Time: 41:10) that we did many years ago, where we built these catenary shapes vaulted structures made up of coconut shells, which can take lot of compressive load. The entire structure is made up of coconut shells with plaster and with a catenary shaped form work inside which can be dismantled when we use.

This is a favorite quiz question. (Refer Slide Time: 41:35) that you see in this picture, three bridges; iron arch bridges built during 1850-1900 over the Harlem river in New York. What is special about these three bridges? All of them refer to three stages, three historical advents in the history of structural analysis. What is so special about them? Here, the first is a three-hinged arch and you can barely see that hinge in the middle (Refer Slide Time: 42:09). The three-hinged arch is a statically determinate structure and as we discussed earlier, people were afraid of building indeterminate structures. They got bolder and they went for the two-hinged arch, and later, the fixed arch. We will study this in the next module.

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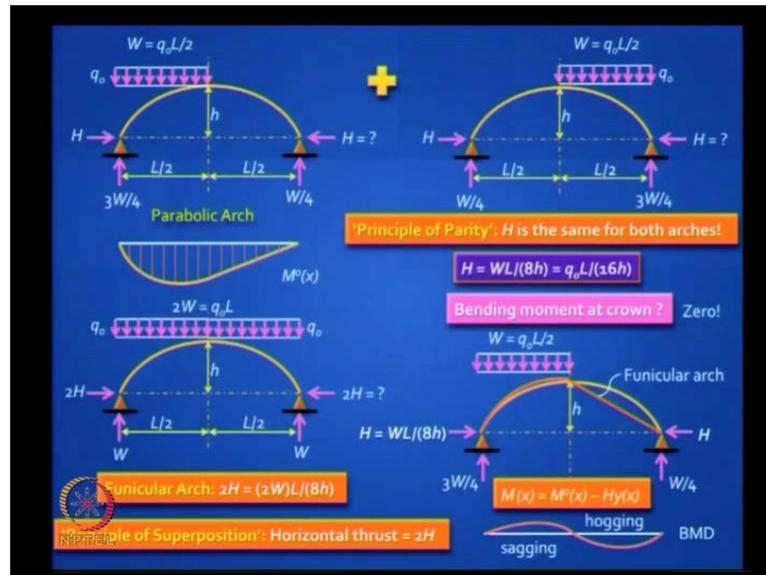
The slide, titled "Some useful principles", illustrates the relationship between beam deflections under different loading conditions. It features three diagrams of simply supported beams of length L . The first diagram shows a beam with a uniformly distributed load W over the left half ($L/2$), resulting in a mid-span deflection Δ_1 . The second diagram shows a beam with a uniformly distributed load W over the right half ($L/2$), resulting in a mid-span deflection Δ_2 . The third diagram shows a beam with a uniformly distributed load $2W$ over the entire length L , resulting in a mid-span deflection Δ . A formula box on the right states $\Delta = \frac{5(2W)L^4}{384EI}$. Below the diagrams, three orange boxes pose questions: "Mid-span deflection: $\Delta_s = \Delta/2$?", "Mid-span deflection: $\Delta_s = \Delta_1$?", and "Principle of Parity" $\Delta_s = \Delta_1$!. At the bottom, a pink box labeled "Principle of Superposition" shows $\Delta_1 + \Delta_2 = \Delta$ leading to $\Delta_s = \Delta/2$! The NPTEL logo is in the bottom left corner.

How to analyze indeterminate arches? Here are some useful principles. I think I have asked you this question earlier, but here goes again. Take a simply supported beam and let us say, half the beam is loaded and the deflection at the mid span is Δ_1 . Let us say the full beam is loaded, the deflection is Δ in that beam. Can you say that... you can get a formula for Δ , if you wish. Can we say that the mid span deflection Δ_1 is half of Δ ? Yes or No? Yes. [Noise] but if you remember, last time I asked you this question and all of you said no. so, why do we say yes? because of parity.

Now to understand parity, you take another beam with the other half-loaded and you apply the Principle of Superposition. You will find that the sum Δ_1 plus Δ_2 must be equal to Δ , but you can argue that Δ_1 and Δ_2 are the same. Because, let us say, we do an experiment and we load this half of the beam and we measure the deflection at the mid span. What you would draw is one, in which the right half is loaded. The picture that I will draw is the one with left half loaded, but we are actually dealing with the same beam and this is not symmetric, this is called parity and this is called the Principle of Parity. It all depends on how you look at the beam and the deflection is exactly the same.

If you invoke the principles of parity and superposition, you can prove that Δ_1 plus Δ_2 equal to Δ and Δ_1 is equal to Δ_2 , and so, it follows that Δ_1 is Δ by 2. This combination of parity and superposition, we can invoke in many cases.

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For example, in this two-hinged arch subjected to a distributed load on one-half, is this a funicular arch? No, because the bending moment diagram for this is not parabola, but still, can you say it is statically determinate? What is the horizontal reaction? Vertical reactions are known. What is a horizontal reaction? [Noise] (Refer Slide time: 44:54) Tell me the basis of how to find. What should we do? Do not cut the arch. You load the other half. We do not know H and you load the other half and when you add these two you end up with funicular arch, whose horizontal reaction is known. What is the value? It is $q_0 L^2$ by $8H$. How do you get the horizontal thrust in any one of these arches? It will be equal to half that value. That is it.

So, when you put them all together, you get two edge and you can easily work this out. If you go one step further, once your edge is statically determinant and you can draw the bending moment diagram and shear force diagram for this structure and you will find that the bending moment at the crown is zero. Why will the crown moment be zero? [Noise] Why will the crown moment be zero? Yes, it is like a three-hinged arch because if the moment there is zero, it is like you call a hinge at the crown. Why will it be zero? [Noise] (Refer Slide time: 46:31) One side it is hogging and one side it is sagging.

If we look from the other side if it is not zero, it should be either sagging or hogging, in which case, in the total arch should be two times that moment because, you are adding

and you are doing superposition but you will not be able to satisfy the relationships. The only answer is it has to be zero and you can actually draw the bending moment diagram.

Can you explain why the bending moments in the left half of the arch will be sagging and those in the right half will be hogging? Any sound argument? Yes.

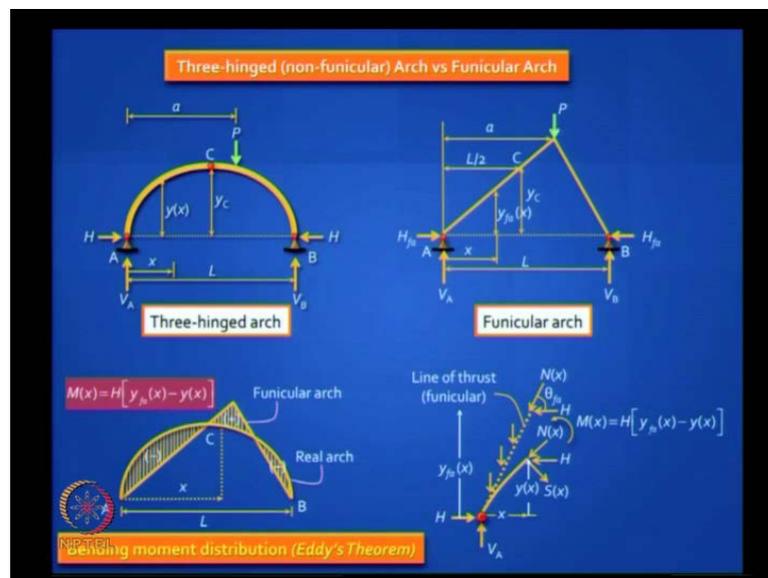
[Noise] (Refer Slide Time: 47:29) Balance the moments; when we take it individually and suppose we consider the loads in left side and it would be sagging, at the same time we consider the right side, it would be hogging and this is not possible at the same time.

No, that is a reason why the moment at the crown is zero. Why is left half is sagging and the right half hogging?

[Noise] (Refer Slide Time: 47:53) Because, it is the left half where the loads are applied.

No, intuitively you are right, but that is not good enough. Do you remember Eddie's theorem? So, you have to invoke Eddie's theorem; we will look at that soon.

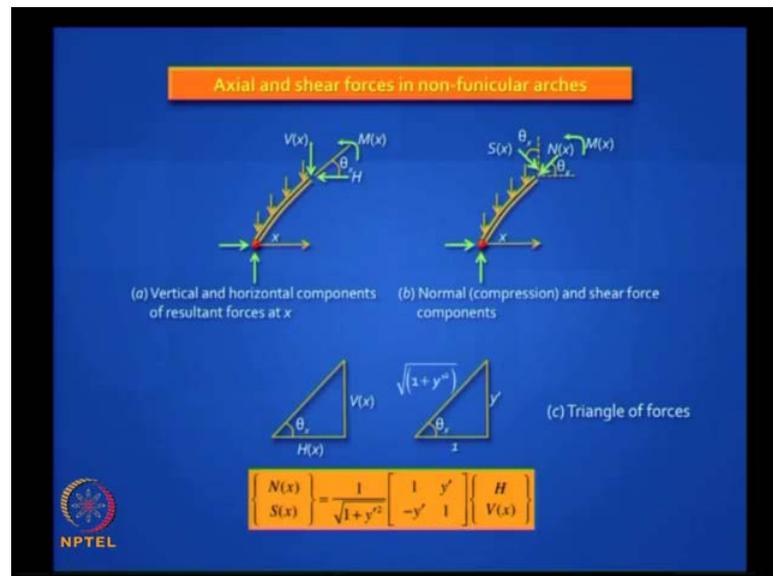
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Let us say, you have a load, concentrating load acting on a three-hinged arch like this. This is certainly not a funicular arch. Which would be a funicular arch? This would be a funicular arch. So, you superimpose both of them with the same height at the crown and you will find that the deviation from the funicular arch is actually a measure of your bending moment. So, that is what you have to do.

So, if you go back to the previous case, (Refer Slide Time: 48:52) you will find that the bending moment diagram would be as indicated there in the funicular arch. You gave more height than what is required for the funicular arch in the left half. If you take the equation, M_0 minus HY and minus HY is more than M_0 . so, you get hogging. On the other side, you give less Y than the required one, and so, it is sagging. It is a simple argument. Let us move ahead. The other thing you need to know is - you will also get shear forces and axial forces.

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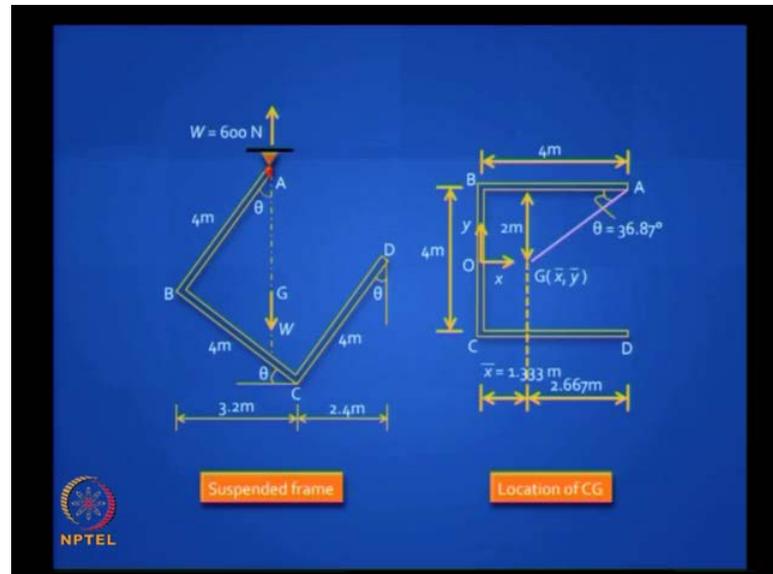


you can actually resolve the vertical force and the horizontal thrust into axial and shear forces using geometry of the arch, you have studied all this. These are simple examples that can be solved and these type of arches are often either segmental, which means part of a circle for which you can derive the equations or they are parabolic and we do not have time right now to solve a full example, but you studied this and it is pretty easy to solve and you can also invoke Eddie's theorem to draw shape of the diagram, you can write the expressions for bending moments and shear force and they will look like this (Refer Slide Time: 50:11).

You can also calculate the axial force and shear force, once you have axial force and the axial force based on the shear force based on V and H , that is, the vertical force and the horizontal thrust. So, you will find that there is not too much difference between the segmental arch and the parabolic arch because the shapes are more or less same. For this

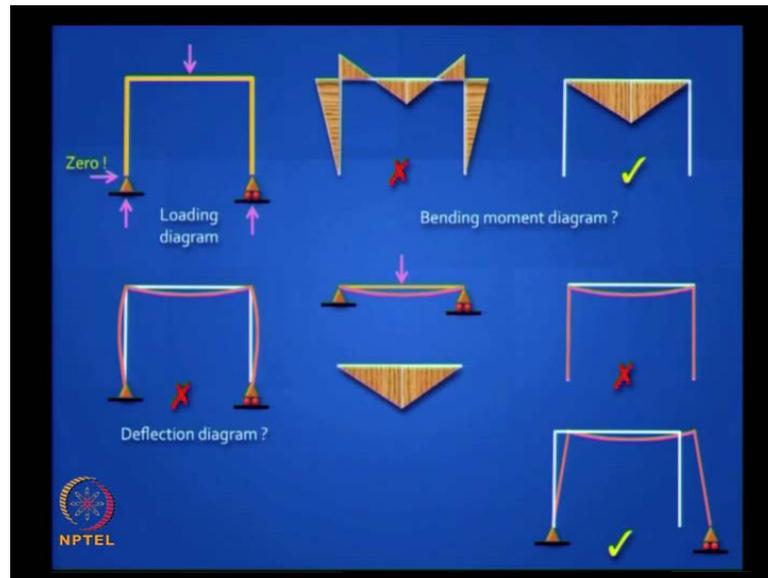
reason, you can even idealize the catenary arch as the parabolic arch which is easier to handle.

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We come to the end of this topic, this is a frame suspended by its own weight. So, it is an interesting question: can you draw the bending moment diagram for this frame? So, what you need to do is to see how it will look, if you hang that frame? You will find that the vertical line from A must pass through the center of gravity. You need to locate the center of gravity and then you can draw the free bodies of these ends. So, typically frames are like this (Refer Slide Time: 51:18). In a cantilever frame, it is wise to start from the free end of the cantilever and your bending moment diagram will look like this (Refer Slide Time: 51:24) and a rough shape of the deflected shape look like this. So, intuitively this must appeal to you. You must also note that this diagram is drawn on the tension side. So, the curvature must give you the feeling of tension on the right side.

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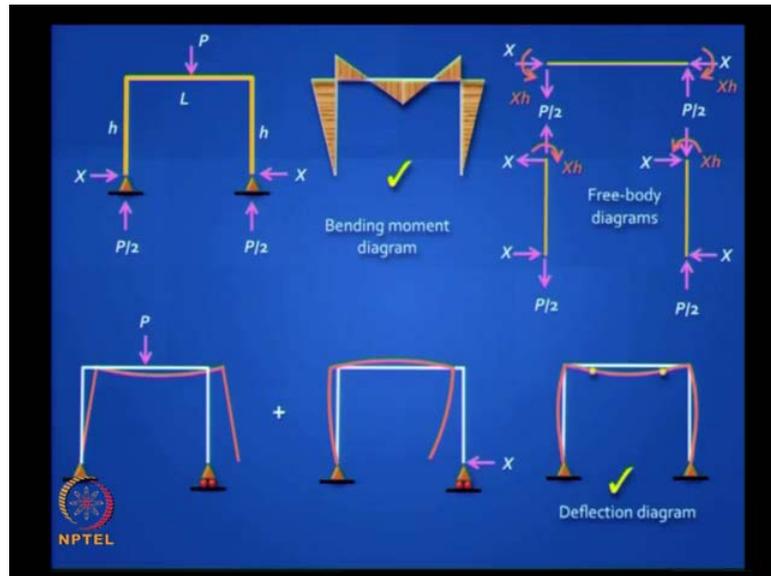


Some concluding points: this is the favorite question, which I can assure you, one in thousand engineers answers correctly, but since you studied this, you will probably give me the right answer. Can you draw the bending moment diagram? Most people will draw this. Is this right or wrong? [Noise] It is wrong. Why is it wrong? Because there is a vertical component and there is no bending moment. Yeah, there cannot be bending moments in the vertical components, that is, the columns. Why not? [Noise] Sir, only axial loads are there. Why only axial loads? Because, only vertical reactions are there and no horizontal reaction. You cannot have a horizontal reaction at the supports in the simply supported beams, subjected to only vertical loading. So, if the horizontal reaction is not there, you cannot get a bending moment in that vertical element. So, this is wrong. So, that structure will be a just like a simply supported beam and the bending moment diagram is just like a simply supported beam. Those two vertical elements are like some legs and props holding it together.

What about the deflected shape? That is a real challenge to draw. You will find that many people draw the diagram like this, (Refer Slide Time: 53:01) which is not correct. Why it is not correct? [Noise – not audible] (Refer Slide Time: 53:10) Because the vertical elements are seen to be bending here, but there is no bending moment. So, what should you do? Well, you first reproduce the simply supported beam drawing there. Why should you reproduce it? Because the curvatures are the same and the bending moment diagram is the same. What should you do for the vertical element? [Noise] 90 degree,

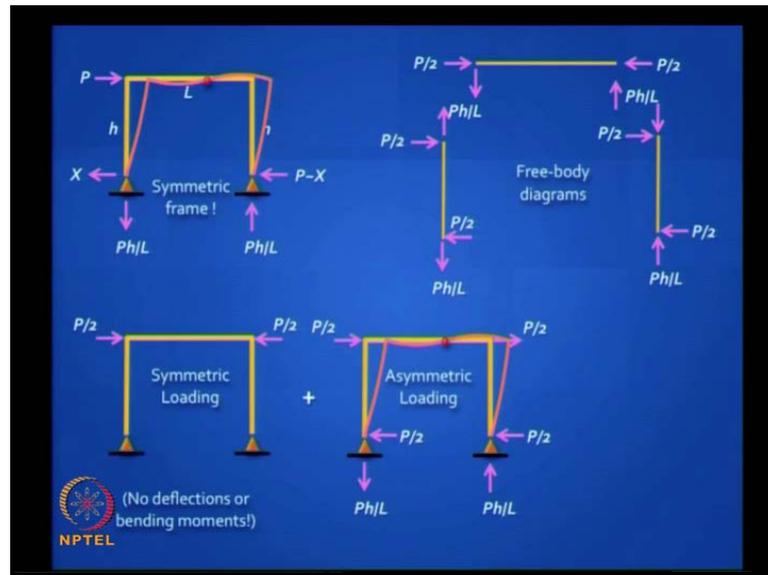
you preserve that. This is wrong and this would be violating the 90 degrees. So, you have to draw like this (Refer Slide Time: 53:42) and this is still feasible because, the roller support allows the moment. So, the original shape was this (Refer Slide Time: 53:49). So, a student who can answer this completely is a rare student.

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Now, we have drawn this picture earlier, you will find this bending moment diagram is correct, if there is no roller support and if both are hinged, then the picture is correct. Then how to find the reaction? It can be done from superposition. So, take the first case, where it is roller and that is a primary structure, you can move it to the right and so you need to apply that much force horizontally to eliminate that. That is how this can be proved.

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One last picture - this is the last topic. Is this structure statically determinate or indeterminate? - This frame. [Noise] It is indeterminate because you have four unknown reactions. So, how do you say it is determinate? [Noise] (Refer Slide Time: 54:48) By pariting; we separate it into two diagrams. Which diagram? Sir, in one diagram, it is P by 2 and P by 2. Very good. So, I will help you there. First of all, the vertical reactions are statically determinate, but the horizontal reactions are not. So, as you rightly say, you divide. This frame is symmetric; that is the reason, why it is going to be determinate. So, you divide two loading diagrams. One is the symmetric loading where you have p by 2, P by 2 and you have an asymmetric loading or anti-symmetric loading, where the p by 2 act in the opposite direction.

In the first diagram, if we assume the axial information is negligible, what is the deflected shape? There is no deflected shape. It will remain as it is because there is no change in shape. You have only an axial force. Whereas in the second one, if you invoke parity and superposition, you will find that you will have to have P by 2, P by 2 and so the deflected shape will look like that (Refer Slide Time: 55:56) and the deflection comes from the anti-symmetric loading part. So, once you say P is shared equally P by 2 and P by 2, the indeterminacy is gone. You can analyze this and you can draw the bending moment of the diagram. Thank you.