

Advanced Structural Analysis

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Module No. # 2.8

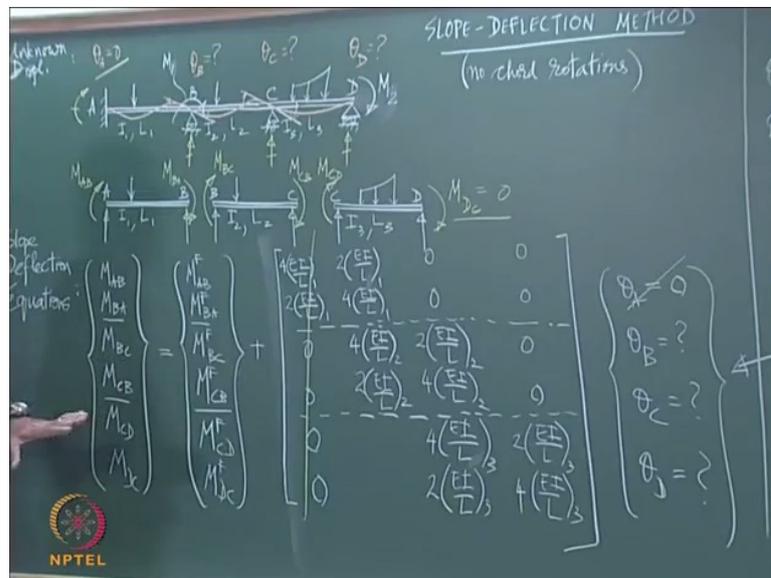
Lecture No. # 14

Review of Basic Structural Analysis - 2

Good morning. We are now doing lecture 14 in second module. The topic is still, Review of Basic Structural Analysis.

We will be looking more closely at this slope deflection method and the moment distribution method, which we introduced in the last class. This is described in part V of the book on Structural Analysis.

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Before we continue, I would like you to see this big picture on this board. Here, I have shown a typical continuous beam. In this particular example, you have three spans. All the beam segments are prismatic. They have constant EI. Our idea is to be able to draw the bending moment diagram and the shear force diagram.

To draw these diagrams, we need to know the end moments. If we know the end moments, then each beam can be treated as simply supportive. With known end moments, we can analyze each segment separately, get the reactions, and for each element, you can draw the shear force and bending moment diagram. Let us be very clear. Our real objective is to be able to draw the bending moment diagram and the shear force diagram.

Now, in the displacement method of analysis, we try to get to know these moments by relating them to displacements. That is why it is called a displacement method. Now, what are the unknown displacements? We need to worry only about the displacements at the joints.

There are four joints in this example: A, B, C, and D. None of those supports translate this example. So, there are no chord rotations; they are making it simple. So, we have four rotations at the four joints: θ_A , θ_B , θ_C , and θ_D . In this example, end A is fixed; so, θ_A is 0. You may get a problem as we did yesterday where, θ_A is not 0, but it is known. We have a known rotational slip. It does not matter.

Now, in terms of θ_A , θ_B , θ_C , and θ_D , you can write expressions called slope deflection equations for all the end moments. Each beam segment has two end moments - one at the left end and one at the right end. We assume them to be clockwise positive. So, the equations look like this (Refer Slide Time: 03:28). I have written them out and they are very easy to write down. M_{AB} is $M F_{AB}$ plus 4 EI by L_1 into θ_A plus 2 EI by L_1 into θ_B . In this example, there is no chord rotations; so, you can leave out the minus 6 EI by L into 5 AB. Similarly, you can write for M_{BA} .

Now, you notice that M_{AB} and M_{BA} - the end moments in this segment do not depend on rotations that happen far away. What happens at θ_C is not reflected in these equations. That is the beauty of it. In each segment, the expressions for the end moments depend only on the end rotations of that segment. So, when you write in a matrix form, you will find that these values are 0. M_{AB} is $M F_{AB}$ plus 4 EI by L_1 into θ_A plus 2 EI by L_1 into θ_B plus 0 into θ_C and 0 into θ_D .

Similarly, you can write for the other two segments - for the segment BC and for the segment CD. For the segment BC, you have dependence only on θ_B and θ_C , the second and third columns. For the segment CD, you have dependence only on the third

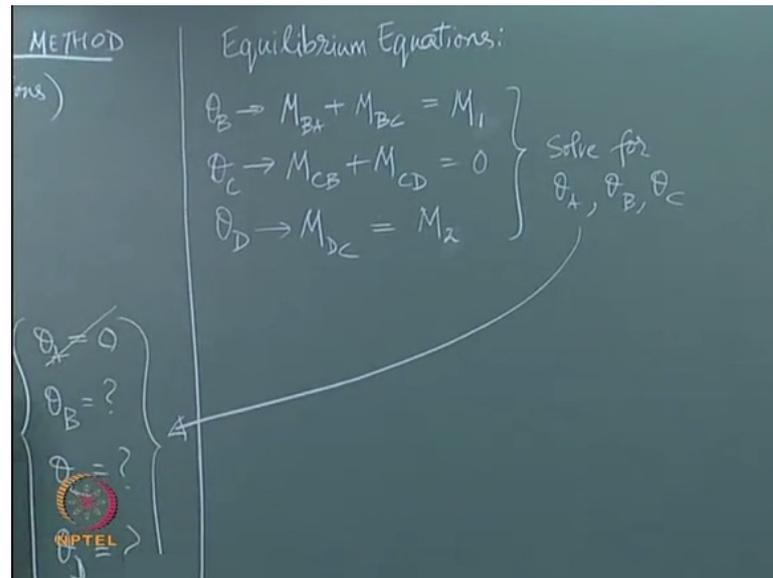
and fourth columns, which reflects θ_C and θ_D . It is very simple. It is not difficult. Now, with the help of these slope deflection equations, you can find the end moments and thereby, draw the bending moment diagram and shear force diagram provided you know these **unknown displacements / unknown rotations**.

How do you find the unknown rotations and displacement methods? By invoking equations of equilibrium. In this case, corresponding to each unknown rotation, you have an equilibrium equation. Now, since θ_A is 0, it is known. So, really speaking you need not include it in this unknown displacement vector. You can just knock it off because θ_A is known, which means, we need not have written the first column at all. Though it is there, if θ_A is not 0, you need to bring it into B, but since θ_A is 0, you do not need to write this first column, (Refer Slide Time: 06:07) or you can write it and delete it.

Now, you have three unknowns, you need three equations, and those equations are equations of equilibrium. So, obviously they must use the slope deflection equations, (Refer Slide Time: 06:31) if you can relate these two equations to this concentrated moment acting here. If this concentrated moment were not to act here, then these two moments should be equal and opposite. Why? You can say – to satisfy equilibrium, but you can also use Newton's third law, and say – every action has an equal and opposite reaction. When you join them together, there should be nothing left over. They should cancel each other unless you had a concentrated moment as you have in this case.

Solve the equation of equilibrium written algebraically correctly because all moments are assumed to be clockwise positive.

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It is M_{BA} plus M_{BC} is equal to whatever moment is acting there. In this case, it is M_1 . At the second joint C, M_{CB} plus M_{CD} is equal to 0 because there is no concentrated moment acting there. At this last support M_{DC} is equal to? In this case, the concentrated moment M_2 . So, you have three equations. You substitute these expressions in those equations. You got three simultaneous equations. You can solve for them, find the unknown rotations, plug them back into these equations, and get your end moments. After you get end moments, you draw these free bodies, draw your reactions, and draw your shear force diagram and bending moment diagram. Is it clear? It is beautiful. It is simple. It is clean.

In this method, you do not have to worry about fixing the unknown rotations. All that you can do mechanically. In fact, this is how all the books teach you, but we are trying to take some more advantages. What is the additional advantage we take? We reduce the degree of kinematic indeterminacy to how much? We reduced from 3 to 2.

How do we do that?

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We realize that it is a waste of time to bring this equation because you know the end answer. This is not an unknown. So, we do not waste our time trying to calculate θ_D even though it is unknown because M_{DC} is known. I do not need to write this in terms of

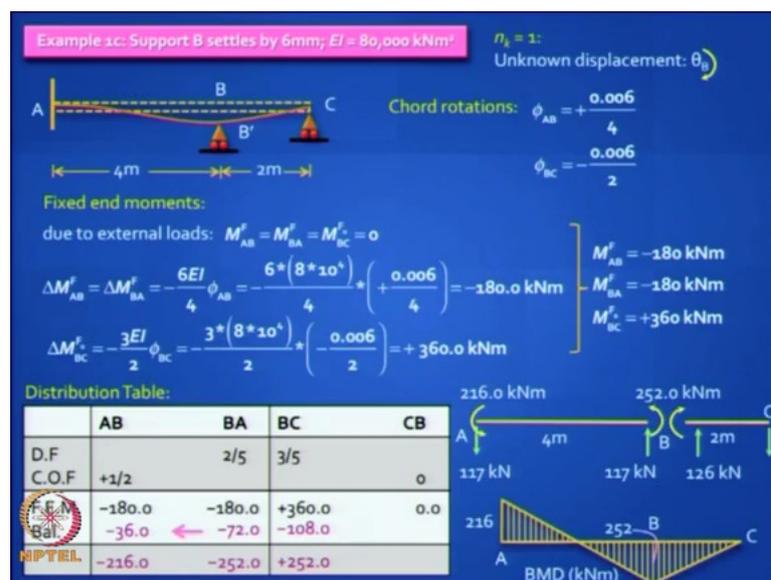
an unknown θ_C and θ_D . I do not waste my time. So, I take advantage of that and I say I do not need to know this. So, right in the beginning, I make that statement - I do not need to know, which means I leave it as it is; I do not arrest it even in this picture.

Even while drawing my fixed end moments, I left θ_D operate, which means that I am dealing with the propped cantilever when I draw this fixed end moments. In which case, I write here this is equal to M_2 (Refer Slide Time: 09:48). I do not even include it here and here, I do not do this calculation instead, I do this calculation. That is, I find out the fixed end moment for a propped cantilever, which I distinguish in notations by adding a naught out there. $M F_0 CD$. Is it clear? So, I do not need to write this equation. (Refer Slide Time: 10:10).

It is not required and I need to? I eliminate this column. It is not there. So, I just have two columns. I just have θ_B and θ_C , but there is a correction I need to do. I have to... Since I removed this, I have to replace this as 3. That is all. Don't you think this saves you tremendous time? You will find that solving two simultaneous equations is much easier than solving three, and you know the amount of work involved as your order goes up is extremely non-linear. Have you got the hang of this? I hope this clarifies some of the doubts you had yesterday.

Now, let us go ahead.

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Let us look at this problem where you have a support settlement at B. What is the degree of indeterminacy here?

2.

It is 2 in the old fashion conventional sense because θ_B is unknown and θ_C is unknown, but if you do it in the smart way, it is 1.

We are talking of, by the way, kinematic indeterminacy. Perhaps when you say 2, you refer to static indeterminacy. You forget all about static indeterminacy. While you are doing displacement methods, only think about kinematic indeterminacy. Is it clear? θ_B .

How do we do this problem?

We have another method called moment distribution method. (Refer Slide Time: 11:40) See, the big difference between moment distribution and slope deflection is - In slope deflection, you have to explicitly find θ_B and θ_C , and you substitute in these equations, but you are really not interested in θ_B and θ_C . Moment distribution method recognizes that. It says do not waste your time calculating all these, but get these moments directly by doing a distribution of whatever moment you get at the artificial fixity point. That is a clever iterative method; both options are there. This will give you an exact solution the other may or may not.

So, in these problems, you do not highlight any fixed moments due to external loads, but in moment distribution method, unless you have some fixed moments, you cannot do the balancing and so on. So, what we do is, we arrest that θ_B . We say that θ_B is 0. Incidentally, you get chord rotations known. That is, if the deflection is 6 mm, it is 0.006 by 4 for AB and it is minus 0.006 by 2 for BC. Why is it minus?

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Because you join ABC with a straight line. In this case, A B dash C and if the chord rotates clockwise, you give it a positive sign; if it rotates anti clockwise, negative sign. Clearly, BC is having a negative sign. So, you have these additional fixed end moments. That is a language we use in moment distribution method where, for AB, when you arrest B both ends are fixed. So, you know that the end moments you develop is minus 6

EI by L into phi. Remember, last class, but for BC, it will be minus 3 EI by L into phi because you are dealing with the propped cantilever when you are fixing B. Is it clear? Remember – last class. Are you comfortable with these two equations? You get them from the... This is a known sway problem. The chord rotations are known. Difficulty comes when you have unknown chord rotations, which we will see later.

(Refer Slide Time: 13:54) Once you have these moments, what should you do? You can distribute them. You can add them up. You have $M_{F_{AB}}$ as minus 180, $M_{F_{BA}}$ also as minus 180 – all coming from chord rotations. In case you had some intermediate loads, you have to add those effects also and $M_{F_{0_{BC}}}$ is plus 360. Why is it plus? Because it is minus of minus. You understand? It is minus 3 EI by L into phi, but phi itself is minus because it is anti-clockwise. So, you have to get this correctly. Then, you can do the distribution table. Remember – we did this problem in the last class, we already calculated the distribution factors. You write them neatly in the third row. Fixed end moments are minus 180, minus 180, plus 360, and of course 0 at the propped end. What do you do now?

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How did you get these fixed moments by arresting θ_B ? You had no business arresting θ_B . So, you let go of it. You release that moment. You should have a net moment there equal to 0. So, the net moment there right now is, minus 180 plus 360, which is plus 180, which we have to balance. (()) It is called balancing. How do you do that?

[Noise] (Refer Slide Time: 15:15)

You distribute it in proportion to the relative stiffnesses. Just multiply it by 2 by 5, 3 by 5, you get those numbers. When you do it manually, you first put the minus sign. To remind you that both will have negative sign, then you fill in those numbers, but in this operation you should also do a carry over, but you carry over only to the left end A not to the right end B. This is because left end A only is fixed. Carry over factor is plus half. So, half of minus MD_2 is minus 36 and that is it. You have done the balancing. This is called a one cycle distribution. You got the exact results and many problems can be done very fast in this method. You add the total you got those moments. Then, you can draw the free bodies. You can draw the bending moment diagram. Is it clear?

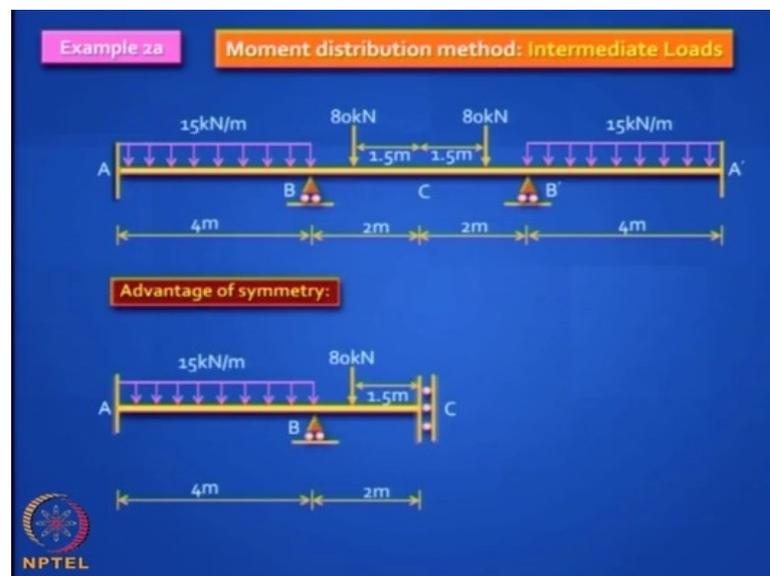
I have demonstrated in these last few problems - How to solve problems with known support moments, either they are support settlements or they are rotations.

You can... Now, let us do another problem with loading. This looks difficult, but actually there is a?

Symmetry

There is a symmetry, which you should take advantage about. There is a beautiful symmetry both for the structure and for the loading. Therefore, for the response with respect to C.

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So, you can cut that structure into half. You can either take the left half or you can take the right half. You have to reduce the difficulty in the problem. What is the degree of indeterminacy?

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True actually, but θ_B and δ_C , but you can avoid δ_C because you can modify your relationships. So, θ_B only is the unknown. In the first few problems, we do simple problems where you can reduce everything to one unknown.

Single unknown rotation θ_B (1) kinematic indeterminacy. What should you do now, if you are doing moment distribution method? You should find the fixed end moments.

How do you do that? You arrest only θ_B . Do not arrest δ_C . If you arrest δ_C , then your indeterminacy is 2. We are not doing that, but we are making it simple. So, how do you do this?

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Fixed End Moments: (with $\theta_B = 0$)

$$M_{AB}^f = \frac{-qL^2}{12} = -\frac{(15)(4^2)}{12} = -20.0 \text{ kNm}$$

$$M_{BA}^f = \frac{+qL^2}{12} = +\frac{(15)(4^2)}{12} = +20.0 \text{ kNm}$$

$$M_{BC}^f = -\left(\frac{Wab^2}{L^2} + \frac{Wa^2b}{L^2}\right) = -\frac{(80)(0.5)(3.5)}{4^2}(3.5+0.5) = -35.0 \text{ kNm}$$

$$M_{CB}^f = -[(80 \times 0.5) - 35.0] = -5.0 \text{ kNm (sagging moment)}$$

Distribution and Carry-over Factors:

$$k_{BA} : k_{BC} = \frac{4EI}{4} : \frac{EI}{2} = \frac{2}{1} : \frac{1}{2} = \frac{2}{3} : \frac{1}{3} \Rightarrow d_{BA} = \frac{2}{3}; d_{BC} = \frac{1}{3}$$

COF (B → A) = $+\frac{1}{2}$
COF (B → C) = -1

Distribution Table:

	AB	BA	BC	CB
D.F		2/3	1/3	
C.O.F	+1/2			-1
F.E.M	-20.0	+20.0	-35.0	-5.0
B.L.	+5.0	+10.0	+5.0	-5.0
	-15.0	+30.0	-30.0	-10.0

You arrest it. Your fixed end moment. I think by now you know the formulas. You can (()) get those values.

M_{AB} and M_{CB} are easy to calculate. You have both M_{AB} , M_{BA} and M_{BC} , M_{CB} . Is it clear? We have discussed this earlier.

I think you know how to do it. We have done this earlier. Please be careful though, when you are finding M for this span. It is best to visualize this with a double span with the load also acting on the right half and work out the fixed end moments for that. That is how you get M_{BC} .

You have to take the effect of not only this load and the double span, but also the other load which occurs here with the correct values of A and B . You need to do this correctly. I think we discussed this earlier.

As far as the middle moment is concerned, which is the sagging moment; you can take a free body and do it. It works out to do this.

You know how to calculate fixed end moments? Then, what do you do? What is the next step you need to know? You need to... First, before the distribution table, you need to write down the distribution factors. How are you going to distribute the moment at B? So, you have to do this and carry-over factors. So, you do it in terms of relative stiffnesses. k_{BA} is to k_{BC} is $4EI$ by L is to? by EI by L because you are dealing with the cantilever behavior for span BC. Remember those three magic numbers? Either $4EI$ by L or $3EI$ by L or EI by L . In this case, it is EI by L for the right span. Can you work out those ratios? It comes to 2 by 3 is to 1 by 3.

What about carry-over factors? Tell me. If you distribute at B, you have to carry over to A. How much do you carry over? Plus half. You also have to carry over to C. How much would you carry over? Plus or minus?

Plus

No, it is minus. It is a cantilever. Cantilever will always have equal and opposite moments. That is the only thing you need to remember.

Actually, I have only three numbers possible. Either it is 0 when you have a simple support or it is plus half when you have a fixed support, or it is minus 1 if you have a guided fixed support. Is it clear? Only three carry-over factors are possible. When it is 0, you do not mention it. So, it is either plus half or minus 1. Then, you draw the table. The table will look like this. Are you now comfortable with this?

Write down the distribution factors, write down the carry-over factors plus half and minus 1. Write down the fixed end moments in the first column. Those are those numbers which you calculated; minus 20, plus 20, minus 35, and minus five in this problem. Then, what do you do? Where do you distribute? Only at B. Only where, you have the unknown rotation. So that is easy to do. Actually, in your calculators, you have, if you press (()) into twice, you can do any number of distributions. You have to work out ways of using your calculator and do this fast.

So, you distribute it. Make sure when you add it up everything turns out to be 0 and you do a carry over. So, that plus 10 carries over to AB as plus 5 and to CB, plus 5 carries over as minus 5. That is it. Put that arrow to help you to understand.

Add it all up. It is all over. It is balanced one shot. I mean, this problem if you want to do by the force method would take you a long time. This is a very powerful. You have reduced the complexity of the problem. Then, once you have done this, draw a free body diagram and bending moment diagram.

You can do the same problem by slope deflection method. By this method. (Refer Slide Time: 21:46). How do you do that?

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Example 2b: Solution by Slope Deflection Method

Advantage of symmetry:

Fixed End Moments: (with $\theta_b = 0$)

$$M_{AB}^f = -\frac{qL^2}{12} = -\frac{(15)(4^2)}{12} = -20.0 \text{ kNm}$$

$$M_{BA}^f = +\frac{qL^2}{12} = +\frac{(15)(4^2)}{12} = +20.0 \text{ kNm}$$

$$M_{BC}^f = -\frac{(80)(0.5)(3.5)}{4^2}(3.5+0.5) = -35.0 \text{ kNm}$$

$$M_{CB}^f = -[(80 \times 0.5) - 35.0] = -5.0 \text{ kNm}$$

Slope-deflection Equations:

$$M_{AB} = M_{AB}^f + \frac{4EI}{4}\theta_A + \frac{2EI}{4}\theta_B - \frac{6EI}{4}\phi_{AB} = \left(-20 + \frac{EI}{2}\theta_B\right) \text{ kNm}$$

$$M_{BA} = M_{BA}^f + \frac{2EI}{4}\theta_A + \frac{4EI}{4}\theta_B - \frac{6EI}{4}\phi_{AB} = \left(20 + EI\theta_B\right) \text{ kNm}$$

$$M_{BC} = M_{BC}^f + \frac{EI}{2}\theta_B = \left(-35.0 + \frac{EI}{2}\theta_B\right) \text{ kNm}$$

$$M_{CB} = M_{CB}^f - \frac{EI}{2}\theta_B = \left(-5.0 - \frac{EI}{2}\theta_B\right) \text{ kNm}$$

Equilibrium Equation:

$$M_{BA} + M_{BC} = M_B = 0$$

$$\Rightarrow (20 + EI\theta_B) + (-35 + 0.5EI\theta_B) = 0$$

$$\Rightarrow EI\theta_B = +10 \text{ kNm}^2$$

Final End Moments (kNm):

$$\begin{Bmatrix} M_{AB} \\ M_{BA} \\ M_{BC} \\ M_{CB} \end{Bmatrix} = \begin{Bmatrix} -20.0 \\ +20.0 \\ -35.0 \\ -5.0 \end{Bmatrix} + \begin{Bmatrix} +1/2 \\ +1 \\ +1/2 \\ -1/2 \end{Bmatrix} \{EI\theta_B = +10\} = \begin{Bmatrix} -15.0 \\ +30.0 \\ -30.0 \\ -10.0 \end{Bmatrix}$$

First, you take advantage of symmetry, find out the fixed end moments as we get in the last (()) So, this step is common, but do not find the distribution factor instead, write down the slope deflection equations. How will they look like? Therefore, AB and BA is straight forward. It is same as these equations except that you also have minus 6 EI by L into phi wherever it is relevant. Is it clear? For AB, it is clear.

How will you write it for BC? You have to use these modified expressions. So, it is MF BC plus EI by L into theta and minus EI by L in the second equation because of the cantilever action. That minus is taking care of that carry over (()). If you have doubts with this, you can go back to the earlier class and see. We have worked on the modified slope deflection method.

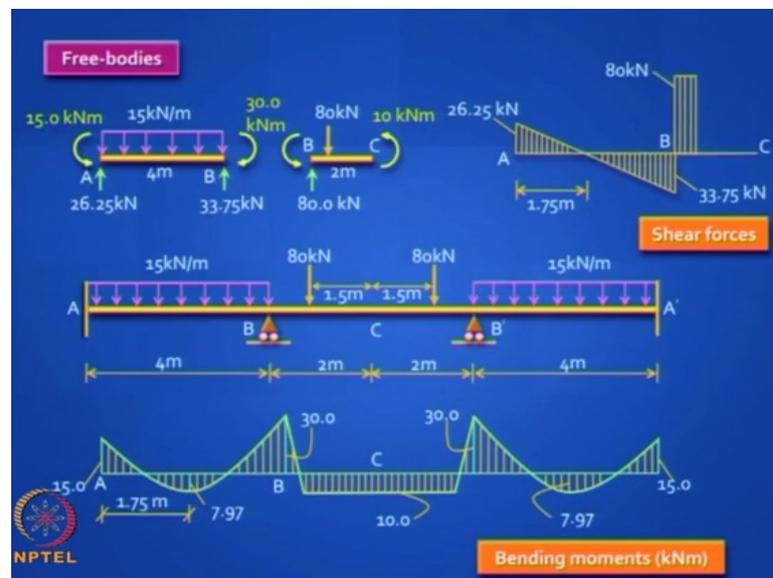
Once you have written this, what should you do? What is the next step? Equilibrium. What is an equilibrium equation?

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At B, M_{BA} plus M_{BC} is equal to 0. So, pull out the second equation and third equation and add it up to 0. You can solve for $EI \theta_B$ straight away. After you found this what will you do? Put them back, put that value back into those equations and get the answer. Easy?

So, you have a choice and I leave it to you. You can do either by slope deflection method or moment distribution method, or the proper displacement method where you use a physical approach. You can still solve this problem. It is not difficult.

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Draw the free bodies, draw the shear force diagram, and draw the complete bending moment diagram. I am going fast because these steps are easy for you to understand. Shall we move ahead?

Let us take up little more difficult problem. Apparently difficult problem, but really not. You have done this problem by force method.

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Example 3a: Slope deflection method – symmetric portal frame: Intermediate Loads

Unknown joint displacements: θ_B, θ_D

Advantage of symmetry $\theta_D = -\theta_B$

Fixed End Moments: $M_{AB}^f = M_{BA}^f = 0 \text{ kNm}$

$M_{BC}^f = -\frac{(50)(6^2)}{12} - \frac{200 \cdot 6}{8} = -300.0 \text{ kNm}$

$M_{CB}^f = -\left[\frac{(50)(6^2)}{8} + \frac{200 \cdot 6}{4} - 300.0 \right] = -225.0 \text{ kNm (sagging)}$

Single unknown rotation: θ_B ($n_c = 1$)

(Δ_c can be ignored)

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Now, what is the degree of indeterminacy in this problem? Kinematic indeterminacy.

Actually 3 because θ_B can deflect. Actually more than 3 if you assume actual deformations also. So, θ_B can go up or down, move left or right, and two translations. So, every joint is two translations and one rotation except the fixed ends at A and E. You have two joints which can move like that, B and C. So, actually it is six. If you ignore actual deformations, it reduces to 3: θ_B , θ_D and the sway of BD, but there is symmetry here. So that sway is 0. So, you have only θ_B and θ_D . Even that you can reduce.

Both are equal and opposite. Both are equal and opposite. So, you can do all that, but it is much easier to cut the beam at C. So, this is a deflected shape. Take advantage of this. Just view this. Now, it cannot move. Only θ_B can move. So, you should do this. It is the easiest thing. You can do single unknown. How long will it take you to solve this?

By the way, before we solve this, tell me - is the moment at A related to the moment at B in the column AB?

[Not audible] (Refer Slide Time: 25:28)

Yes, it will be just half. Even that you know in advance.

It will be half because there is no lateral load on AB. This is a carry-over... So, single unknown. Now, slope deflection method, find the fixed end moments. AB has no fixed end moments. BC has fixed end moments. How will you calculate them? By the way, when you cut that beam at C, that 200 kilonewton concentrated load in the middle will also get cut equally. One half will go to the left span and other half will go the right. So, this is easy to get.

Can you tell me the formula for M_{BC} ? Minus?

M_{BC}

M_{BC} or M_{BD} or whatever you want to call it. Is it not due to UDL? You should take the full span now. Minus 50 into the full span 6 square divided by 12 minus 200 into 6 by 8. These are standard formulas.

Can you tell me what is M_{CB} ? You can do it yourself. Tell me. You have to write that also. M_{BC} is clean. M_{CB} is what you get in the middle. Due to UDL, what will it be? $W L$ square by?

[Noise – not audible] (Refer Slide Time: 26:50)

$W L$ square by 8 is what you get when you have simply supported span.

$W L$ square by 12.

When you have a fixed beam, the end moments are $W L$ square by 24 and the middle moment is $W L$ square by?

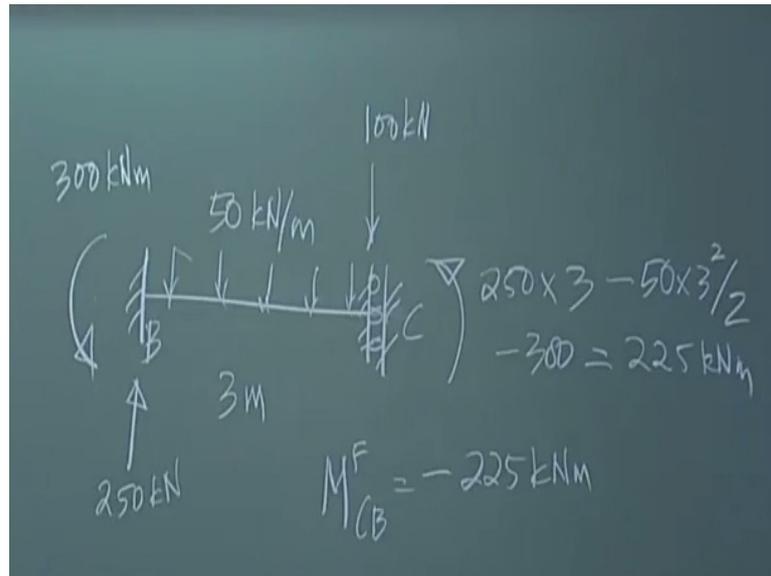
[Noise – not audible] (Refer Slide Time: 27:09)

This is 12. Middle will be?

8

1 by 8 minus 1 by 12. This is 1 by 24.

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Now, it is best that you do not worry too much about formulas. You go straight to the free body. You will never make a mistake. For example, in this problem what we are trying to do is, we are trying to find out what happens due to this loading. This is 100 kilonewton, this is 50 kilonewton per meter, and this span is 3 meter. This is B and this is C. Now, we have already found this out. It is a hogging moment and the value is 300 kilonewton meter.

You also know that there is going to be no reaction here because it is free to slide down. So, the entire reaction goes here. How much is that? 100 plus 150 is 250 kilonewton. There is no reaction. There is only...

[Noise – not audible] (Refer Slide Time: 28:40)

We have only plus half; also concentrated load is there.

Yes. I will show you that calculation on the screen. After calculating this, it is easy to calculate this. How much will this be? Just take the free body and do because there was no load here. So, this will be 250 into 3 minus? 50 into 3 is 150. So, fifty into 3 square by 2. Do you agree? This will have no contribution, no lever arm. Minus 300. How much does it work out to?

This is the easiest way to calculate that moment. How much is it? It will turn out to be 225, but please note it is anti clockwise. So, if you want to write it as M^F_{CB} , you should

write minus 225. So, here, my suggestion is, do not use formulas, use your brain. Please draw the free body diagram. After you got this, write down slope-deflection equations.

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Slope-Deflection Equations :

$$M_{AB} = M_{AB}^f + \frac{2(EI)}{L} \theta_B = (0 + 0.5EI\theta_B) \text{ kNm}$$

$$M_{BA} = M_{BA}^f + \frac{4(EI)}{L} \theta_B = (0 + EI\theta_B) \text{ kNm}$$

$$M_{BC} = M_{BC}^f + \frac{2EI}{3} \theta_B = (-300.0 + 0.6667EI\theta_B) \text{ kNm}$$

$$M_{CB} = M_{CB}^f - \frac{2EI}{3} \theta_B = (-225.0 - 0.6667EI\theta_B) \text{ kNm}$$

$$\begin{Bmatrix} M_{AB} \\ M_{BA} \\ M_{BC} \\ M_{CB} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ -300.0 \\ -225.0 \end{Bmatrix} + \begin{Bmatrix} +1/2 \\ +1 \\ +2/3 \\ -2/3 \end{Bmatrix} \{EI\theta_B\} = \begin{Bmatrix} +90.0 \\ +180.0 \\ -180.0 \\ -345.0 \end{Bmatrix} \text{ kNm}$$

Equilibrium Equation :

$$\text{NPTE } M_{BA} + M_{BC} = M_B = 0 \Rightarrow -300.0 + 1.6667(EI\theta_B) = 0 \Rightarrow EI\theta_B = +180.0 \text{ kNm}^2$$

M_{AB} is M_{AB}^f plus $2EI$ by L into θ_B and M_{BA} is M_{BA}^f plus $4EI$ by L into θ_B . θ_B is your unknown. You can club it along with EI .

M_{BC} is M_{BC}^f plus EI by L . Now, here the beam has a second moment of area which is $2EI$. You should put that; do not lose that. It is EI by L into θ_B plus and minus. Is it clear? So, with some practice, you can master this; write it nicely in a matrix form. What is the next step? Up to this stage if you do correctly, you would not go wrong. What is the next step?

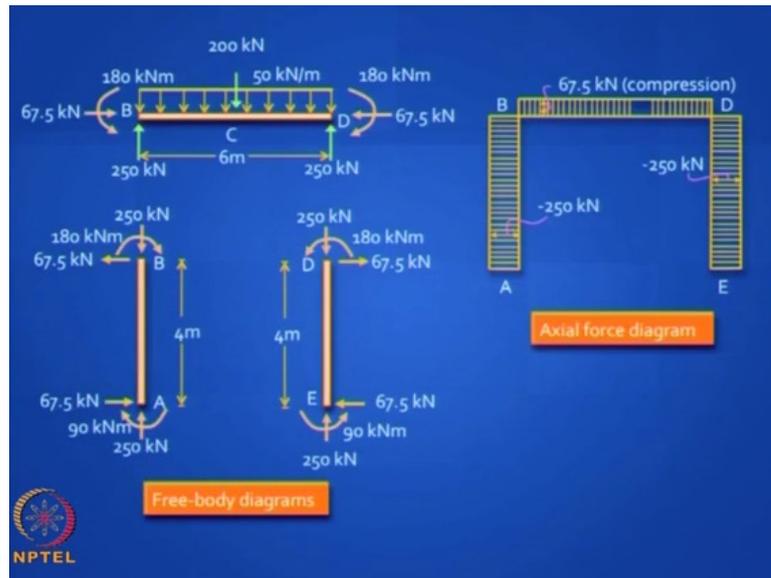
[Not audible] (Refer Slide Time: 30:49)

Equilibrium; what is the equilibrium?

M at B is...

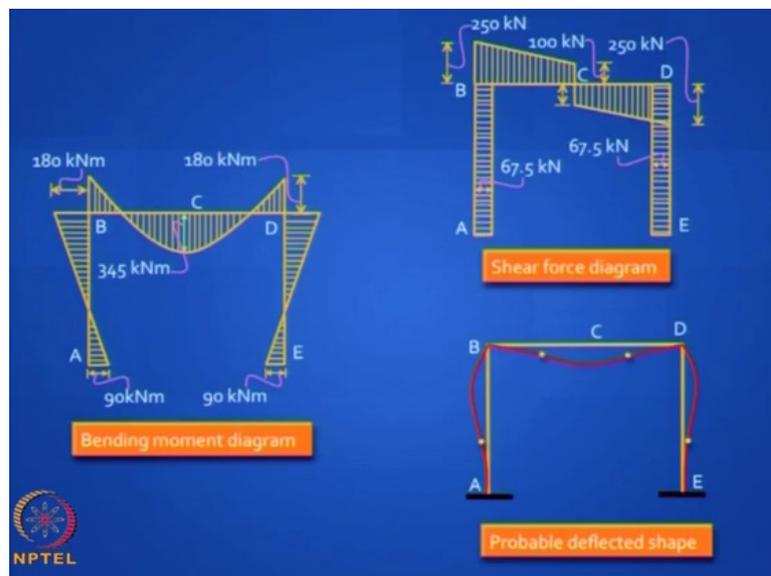
M at B is 0 , which is M_{BA} plus M_{BC} is 0 ; there is no net moment acting there. Substitute; solve the equation; you get $EI\theta_B$. Then, plug it nicely into those equations; you got the answer. Then, draw the free body diagrams.

(Refer Slide Time: 31:10)



You know all these. Draw the... We have done the same problem; Shear force diagram and bending moment diagram.

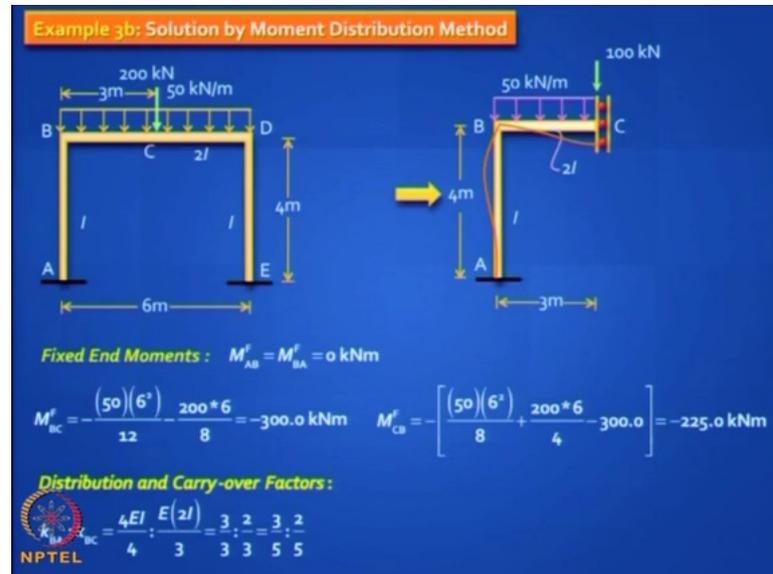
(Refer Slide Time: 31:14)



What is very interesting is that moment at A is exactly half that you get at B in the column, is it clear? Very easy. You see, compared to the force method, this is much easier and much faster. You can draw the deflected shape. Mark the points of contra

flexure and see how nicely they match with the bending moment diagram. you know which side is tension. You have a clear understanding of this kind of reactions.

(Refer Slide Time: 31:46)



If you want do this by moment distribution method, you are welcome. Similar exercise; find the fixed end moments, which you have already done. Now, do these calculations; how do you get the distribution factors. Tell me the factors at B.

k_{BA} is to? Give me the answer. I am not going to give you the answer; give me the answer.

It will be k_{BA} is to k_{BC} is $4EI$ by 4 is to E into $2I$ by 3 ; straight forward, which is 3 is to 5 is to 2 is to 5 . Not difficult. What about carry-over factors?

[Noise – not audible] (Refer Slide Time: 32:52)

To A, it is half; to C, it is?

Minus 1

Minus 1 plus half and minus 1.

Now, at B we do not have any roller support kind of thing; it may...

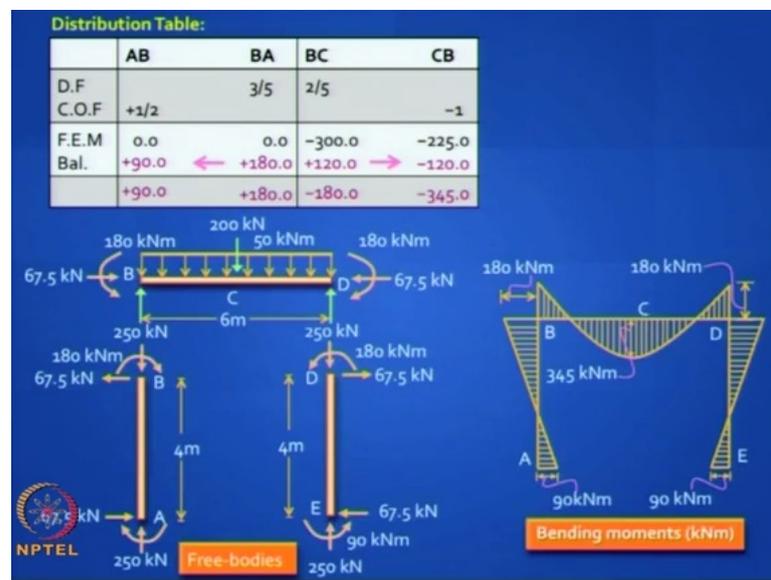
Let me ask you - when you had a continuous beam, you had no problem. This is a continuous. In the continuous beam, you had a reaction at B; you had a vertical line. Here also you have that. It is AB which is taking it. So, AB is referred to as a beam column. As beginners, you find it a little difficult to relate the continuous beam to the portal frame.

Does B deflect or not?

Does not deflect; B does not deflect because AB is inextensible. Actual deformations are negligible. Here also, B does not deflect. (Refer Slide Time: 34:02) Here, AB was horizontal; there AB was vertical. Do you have problem with that? It is the same philosophy. Is it clear? You had a little doubt because you had turned it around to 90 degrees, but what is common to the portal frame and this is; In both, the joint is rigid at B. Here, the included angle between BA and BC was 180 degrees; there it is 90 degrees, that is all. However, θ_B is same for both BA and BC.

Good question.

(Refer Slide Time: 34:43)



Next, draw the distribution table. Now, the fixed end moments are only in BC and CB; minus 300 and minus 225. Distribute 300; what will you get? 180 and 120; do the carry over; add it to all up. How simple and elegant; This called one cycle distribution and you get the answer.

(Refer Slide Time: 35:10)

Types of problems (beams/frames)

- Single unknown rotation – **nodal (joint) moment**
- Single unknown rotation – **arbitrary (non-nodal) loading**
- Multiple unknown rotations – **'non-sway' type problems**
- Unknown rotations and translations (slopes and deflections) – **unavoidable 'sway'**

Increasing difficulty in solving

Note: In some cases involving 'sway', it is possible to ignore the translational degrees of freedom (reduce n_e) by using modified element stiffnesses and modified fixed end moments.

NPTEL

If you look carefully, if you got the first type of problem, single unknown with a single joint moment, you are very lucky; very easy do. If you have intermediate loads, but still single unknown rotation, it is not too bad. It is just one cycle distribution.

If you have multiple unknown rotations you have to do simultaneously distribution; many joints; it is little tricky. We will look at that shortly, but if you have to sway; if you have support settlements, which are not known in advance, then it becomes more complicated. However, the important thing is – always when you do things manually, do minimum work; take advantage of reduce in determinacy. In some cases involving sway, it is possible to ignore the translational degrees of freedom by using modified element stiffnesses and modified fixed end moments.

(Refer Slide Time: 36:09)

Example 4a: Ignoring sway in frame - Slope deflection method

We can reduce the kinematic indeterminacy (ignore the sway degree of freedom) by considering the cantilever behaviour of AB (with modified stiffness).

Fixed End Moments (with $\theta_B=0$)

$$M_{AB}^f = M_{BA}^f = M_{BC}^f = 0 \text{ kNm}$$

Slope-Deflection Equations :

$$M_{AB} = M_{AB}^f - \frac{(EI)}{4} \theta_B = -0.25EI\theta_B$$

$$M_{BA} = M_{BA}^f + \frac{(EI)}{4} \theta_B = +0.25EI\theta_B$$

$$M_{BC} = M_{BC}^f + \frac{3(2EI)}{3} (\theta_B) = +2EI\theta_B$$

NPTEL

Let me demonstrate that. Take this problem. In this problem, which is basically a half of a portal frame ABC, you have a concentrated moment acting at B. What is the degree of kinematic indeterminacy, actually assuming axial deformations to be negligible? What are the two?

θ_B

θ_B and δ_{BC} . What is a deflected shape going to be? Will it sway to the right or left?

[Not audible] (Refer Slide Time: 36:44)

Obviously, right. Supposing you did not have the beam, you had just a vertical cantilever, it will go to the right. So, that is a deflected shape.

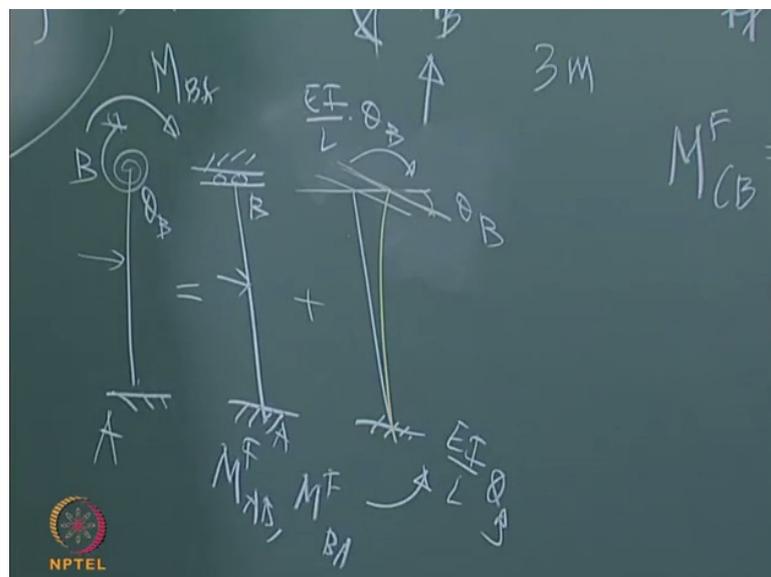
Now, we are trying to take advantage of the fact that we know the cantilever behavior. Why do we not take advantage of that and say? We know the modified stiffness of BC. We say – we do not arrest δ_{BC} in our primary structure; if you do that, you get a tremendous benefit.

Now, your only unknown is θ_B . Let us see how to do this problem. Your fixed end moments are not there; M_{AB} , M_{BA} , M_{BC} , is 0 because there is no intermediate loading; that concentrated moment is being shared; it is going to the node; it is not going

to any beam elements so, it will be shared by both the elements. Your slope deflection equations for the vertical element; M_{AB} is $M F_{AB}$ minus... See, it is a cantilever behavior you are allowing the top to move. If the top did not move, then it is $4 EI$ by L .

Now, the top is moving and you are arresting θ_{AB} ; so, you get $M F_{AB}$. Do not you agree that you get minus EI by L into θ for M_{AB} and plus EI by L into θ for M_{BA} . Your carry-over side will always be minus. Are you comfortable with this or not? You want me to explain this. Good. So, be bold and ask.

(Refer Slide Time: 38:27)



See this. What we are saying is – let us take this element AB. This is fixed here; it is a funny thing. It can move, but there is a rotational spring here. This is a real problem; the real model is this. This can move to the right.

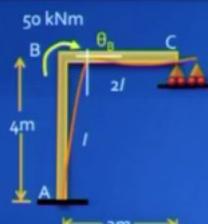
Now, if I want to find an expression for M_{BA} ; A and B; what I do is; this has a θ_{AB} ; I first arrest θ_{AB} . When I arrest θ_{AB} , (Refer Slide Time: 39:07) I am saying this and this; because I am not arresting the translates, I am arresting only the rotation. Let us say there was a load acting here, then, I should... This will give me my $M F_{AB}$; From this, I can get $M F_{AB}$ and $M F_{BA}$. Supposing there was a load; in this case, there is no load. Then, I should release this fixity; take that beam.

Now, I am just applying... I am letting it translate; I am applying so that... Or if you wish to look at this way I am now letting it rotate by θ_{AB} , but allowing it to translate.

So, the deflected shape will be? Are you getting it? (Refer Slide Time: 40:11) This is θ_B . Now, if something like this happens, what do you think the moment will be? It is clockwise; it will be EI by L into θ_B . What do you get here? Equal and opposite; EI by L . So, little thinking and you can get it. That is what those equations mean. Is it clear? It is little more tricky than the previous case, but it saves you a lot of effort. If you really do not want to take advantage, treat as a problem which we have not covered till now. You can do it later, but if you take advantage this is what it looks. Is it clear? Did this help?

(Refer Slide Time: 41:04)

Example 4a: Ignoring sway in frame - Slope deflection method



We can reduce the kinematic indeterminacy (ignore the sway degree of freedom) by considering the cantilever behaviour of AB (with modified stiffness).

Fixed End Moments (with $\theta_B=0$)

$$M_{AB}^f = M_{BA}^f = M_{BC}^f = 0 \text{ kNm}$$

Slope-Deflection Equations :

$$M_{AB} = M_{AB}^f - \frac{(EI)}{4} \theta_B = -0.25EI\theta_B$$

$$M_{BA} = M_{BA}^f + \frac{(EI)}{4} \theta_B = +0.25EI\theta_B$$

$$M_{BC} = M_{BC}^f + \frac{3(2EI)}{3} (\theta_B) = +2EI\theta_B$$

Equilibrium Equation :

$$M_{BA} + M_{BC} = M_B = +50 \text{ kNm}$$

$$\Rightarrow 2.25EI\theta_B = +50$$

$$\Rightarrow EI\theta_B = +22.222$$

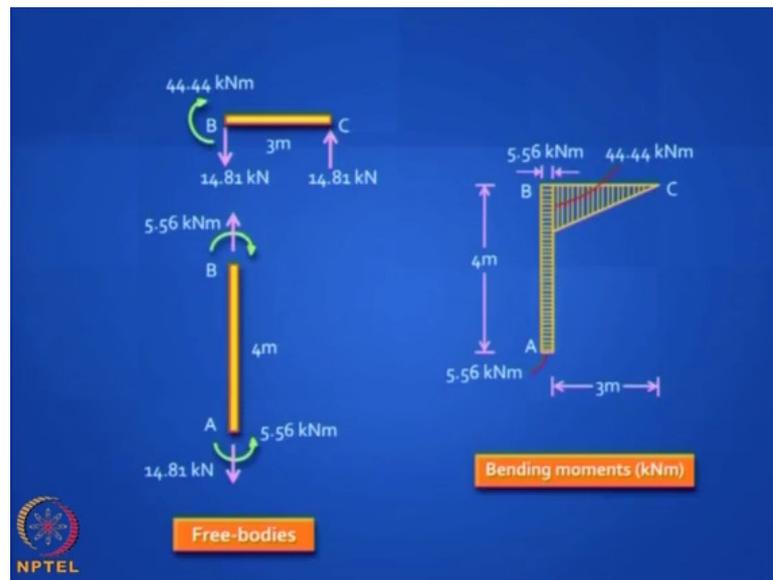
Final Moment Values:

$$\Rightarrow \begin{Bmatrix} M_{AB} \\ M_{BA} \\ M_{BC} \end{Bmatrix} = \begin{Bmatrix} -0.25 \\ +0.25 \\ +2.0 \end{Bmatrix} EI\theta_B = \begin{Bmatrix} -5.556 \\ +5.556 \\ +44.444 \end{Bmatrix} \text{ kNm}$$

NPTEL

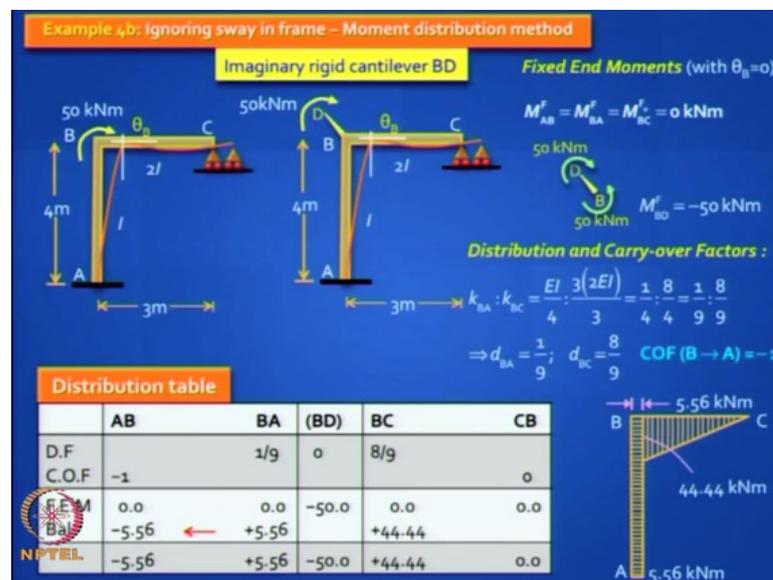
Equilibrium equation; solve it, plug it back and write the equations.

(Refer Slide Time: 41:09)



You get the answers. You get that moment of 50 kilonewton meter is shared by the beam and the column. The beam takes 44.44 kilonewton meter and the column takes 5.56 kilonewton meter.

(Refer Slide Time: 41:29)



Let us say, you want to do it by moment distribution method; it is even easier. What will you do? Same fixed end moment calculations. In moment distribution method, mind you – you are in trouble, if you do not have fixed end moment to balance.

You have to create fixed end moment. So, this is a clever method that I am showing you here; you imagine that fifty kilonewton meter was transmitted to the joint beam by means of some rigid link. I have shown a cantilever there; that yellow bit sticking out is an imaginary rigid link. If you put a moment at the end there, that will reach the joint B, but now, I have got a member with the fixed end moment. It is a clever trick that I am doing which is very useful to do. I have an element DB and I have a fixed end moment in that; $M_{F_{BD}}$ equal to minus 15. Do you get it?

That is a cantilever. I am doing it just to enable me to do the moment distribution. This is how you handle concentrated moments in moment distribution method. You do not need to do this for slope deflection method. Work out the distribution factors for the vertical elements which is a cantilever. It is EI by L for the horizontal element with the far end hinge is $3EI$ by L ; is it not? $3EI$ by L . If you work out the distribution factor, it works out to $1/9$ is to $8/9$. Your carry-over factor will be minus 1 to the end A. Draw your table do a single distribution; that element BD is only a device; so, put it in parenthesis; you do not have to worry about it from the design point of view.

Do a single distribution. $8/9$ of that 50 kilonewton meter goes to BC and $1/9$ goes to BA and gets carried over to end AB. That is it; straight away. It is a beautiful, powerful method of solving these problem. We will now... Yes?

[Noise – not audible] (Refer Slide Time: 43:38)

We are allowing horizontal moment.

[Noise – not audible] (Refer Slide Time: 43:45)

For the element BC, let us say, I have a simply supported beam with rollers. Let us say both are roller supports; I apply loads on it. Will the behavior change if it moves horizontally? That is what is happening there; that $3EI$ by L is a relationship of the moment at B in that simply supported beam in that BC. So, let it sway like a rigid body; it does not make a difference. Do you get it? In fact, this question should have come in the slope deflection method also; there also we did $3EI$ by L . Is it clear? Does it clarify?

[No]

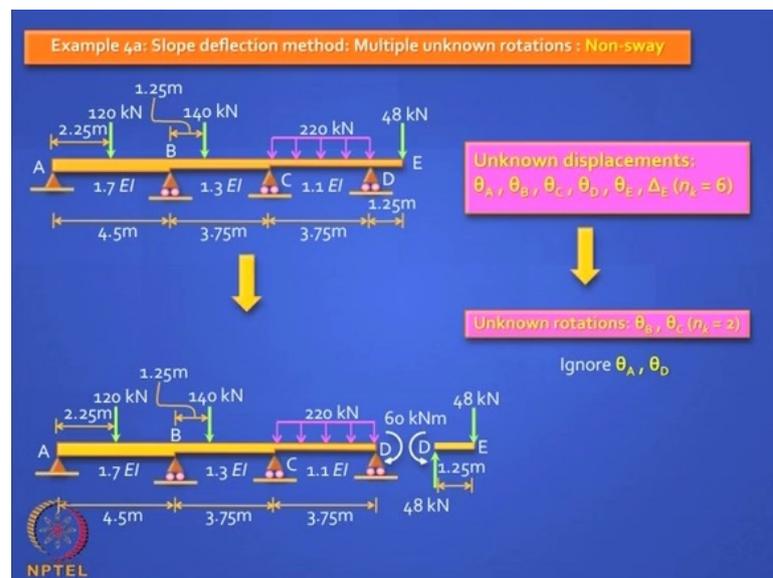
EI by L for AB. (Refer Slide Time: 44:38) See, both are same; you see this and you see this (Refer Slide Time: 44:44) I forgot to explain this. This is 3 EI by L. Both methods are identical. You do not put... The horizontal beam BC does not behave like a cantilever; it behaves like a propped cantilever when you arrest θ_B . Do you understand?

(Refer Slide Time: 45:05)

So you have to use there...

1, 3, or 4; Either EI by L or 3 EI by L or 4 EI by L; use the right one at the right place. We will now look at multiple unknown rotations.

(Refer Slide Time: 45:21)



Take this problem similar to the problem I have shown you here. So, I can go faster since I have explained this. What is the degree of indeterminacy?

[Noise – not audible] (Refer Slide Time: 45:29)

Which are the three?

[Noise – not audible] (Refer Slide Time: 45:33)

Actually, it is 5: $\theta_A, \theta_B, \theta_C, \theta_D, \theta_E$, and Δ_E ; it is 6. This problem has been solved in other books where they have reduced it to 4. That is a good thing to

do because you see that overhang part is statically determinate; you can throw it off. θ_A , θ_B and it is actually 5.

[Not audible] (Refer Slide Time: 46:06)

Yes, θ_A . So, it is 6.

Now, what many people do is, **that is a clever thing to do is;** throw away that cantilever; throw away that overhang; that is anyway statically determinate. Replace it with a concentrated moment 60, which is like in this problem. So, you got rid of two indeterminacy there. You still have D; so, what is the indeterminacy here? It is 4. It can be?

[Noise – not audible] (Refer Slide Time: 46:41)

It can be 2; A is a simple support; D is a simple support; you know the moment at A is 0. You know the moment at D is 60. Is it clear? That is a clever thing to do. We make it 2. So, from 6, we went to 4 and we went to 2; just θ_B and θ_C . How do we do this? First, you arrest those 2 and find the fixed end moments.

(Refer Slide Time: 47:11)

Unknown rotations: θ_B, θ_C ($n_i = 2$)

Fixed End Moments: (with $\theta_B = \theta_C = 0$)

$$M_{BA}^f = +\frac{WL}{8} \times \frac{3}{2} + \frac{(120)(4.5)}{8} \times \frac{3}{2} = +101.25 \text{ kNm}$$

$$M_{BC}^f = -\frac{Wab^2}{L^2} = -\frac{(140)(1.25)(2.5^2)}{3.75^2} = -77.778 \text{ kNm}$$

$$M_{CB}^f = +\frac{Wa^2b}{L^2} = +\frac{(140)(1.25^2)(2.5)}{3.75^2} = +38.889 \text{ kNm}$$

$$M_{CD}^f = -\frac{WL}{12} \times \frac{3}{2} + \frac{M_o}{2} = -\frac{(220)(3.75)}{12} \times \frac{3}{2} + \frac{60}{2} = -73.125 \text{ kNm}$$

NPTEL

What are those fixed end moments? They are very easy to calculate. You can draw them long hand. First, you need to find M_{BA}^f . What is that value? Tell me. You do a propped cantilever. What is M_{BA}^f ?

[Noise – not] (Refer Slide Time: 47:30)

Let us say it is a fixed beam; what is it if you have a load at the middle?

WL by 8.

WL by 8; plus or minus?

[Noise – not audible] (Refer Slide Time: 47:40)

Right side will be plus.

If it is propped cantilever, it will get enhanced by a factor? One and half times, is it not? In a propped cantilever it gets enhanced by one and half times. That is it. So, you got the first answer; you got $M_{F_0 BA}$. Why is it one and half time? You go back to the earlier discussion; Can you find $M_{F BC}$ and $M_{F CB}$? That is easy. You can use those formulas and get it. Wab^2 square by L^2 square and Wa^2 square by L^2 square. How will you get $M_{F_0 CD}$? You should take shortcuts only if you know how to get these answers. Due to that 220 kilonewton, how much will it be?

[Not audible] (Refer Slide Time: 48:32)

Here, it is capital W; 220 is the total load. WL by? WL by 12 if both were fixed, but you have to multiply by 1 and 3 by 2; so, it is minus that. Then, that 60 kilonewton meter also has an effect in that fixed end moment. How much will get spilled over to the fixed end when you apply 60 kilonewton meter there? Half; that is your carry-over factor. It is brilliant if you understand every step that you do. It is half. Does it make sense?

You are right; WL by 12 into one and half plus that fixed...; the moment that you get at the free end D; half of it in the same direction. So, plus half; you got it? Any questions on this? Everything depends on this. Your shortcuts depend on your ability to think through everything. Is it clear?

Once you have done this, write down the slope deflection equations.

(Refer Slide Time: 49:38)

Slope-deflection Equations:

Beam AB ($1.7EI$; $L=4.5m$; $M_{AB}=0$; θ_A can be ignored; $\theta_B=?$; $\phi_{AB}=0$)

$$M_{BA} = M_{BA}^F + \frac{3(1.7EI)}{4.5} \theta_B = (+101.25 + 1.1333EI\theta_B) \text{ kNm}$$

Beam BC ($1.3EI$; $L=3.75m$; $\theta_B=?$; $\theta_C=?$; $\phi_{BC}=0$)

$$M_{BC} = M_{BC}^F + \frac{4(1.3EI)}{3.75} \theta_B + \frac{2(1.3EI)}{3.75} \theta_C = (-77.778 + 1.3867EI\theta_B + 0.6933EI\theta_C) \text{ kNm}$$

$$M_{CB} = M_{CB}^F + \frac{2(1.3EI)}{3.75} \theta_B + \frac{4(1.3EI)}{3.75} \theta_C = (+38.889 + 0.6933EI\theta_B + 1.3867EI\theta_C) \text{ kNm}$$

Beam CD ($1.1EI$; $L=3.75m$; $M_{DC}=+60\text{kNm}$; θ_D can be ignored; $\theta_C=?$; $\phi_{CD}=0$)

$$M_{CD} = M_{CD}^F + \frac{3(1.1EI)}{3.75} \theta_C = (-73.125 + 0.8800EI\theta_C) \text{ kNm}$$

NPTEL

They are straight forward. You will use 3 EI by L for AB, 3 EI by L for CD, but not for the middle span.

(Refer Slide Time: 49:48)

$$\begin{Bmatrix} M_{BA} \\ M_{BC} \\ M_{CB} \\ M_{CD} \end{Bmatrix} = \begin{Bmatrix} -101.25 \\ -77.778 \\ +38.889 \\ -73.125 \end{Bmatrix} + \begin{bmatrix} 1.1333 & 0 \\ 1.3867 & 0.6933 \\ 0.6933 & 1.3867 \\ 0 & 0.8800 \end{bmatrix} \begin{Bmatrix} EI\theta_B \\ EI\theta_C \end{Bmatrix} = \begin{Bmatrix} +84.582 \\ -84.582 \\ +55.875 \\ -55.875 \end{Bmatrix} \text{ kNm}$$

Equilibrium Equations:

$$M_{BA} + M_{BC} = M_B = 0$$

$$M_{CB} + M_{CD} = M_C = 0$$

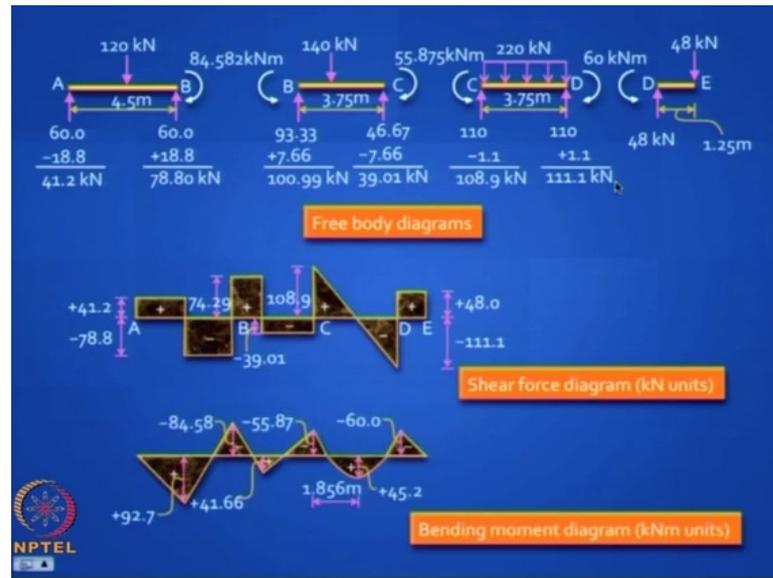
$$\Rightarrow \begin{Bmatrix} +23.472 \\ -34.236 \end{Bmatrix} + \begin{bmatrix} 2.5200 & 0.6933 \\ 0.6933 & 2.2667 \end{bmatrix} \begin{Bmatrix} EI\theta_B \\ EI\theta_C \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$\Rightarrow \begin{Bmatrix} EI\theta_B \\ EI\theta_C \end{Bmatrix} = \frac{1}{5.23142} \begin{bmatrix} 2.2667 & -0.6933 \\ -0.6933 & 2.5200 \end{bmatrix} \begin{Bmatrix} -23.472 \\ +34.236 \end{Bmatrix} = \begin{Bmatrix} -14.7072 \\ +19.6023 \end{Bmatrix}$$

NPTEL

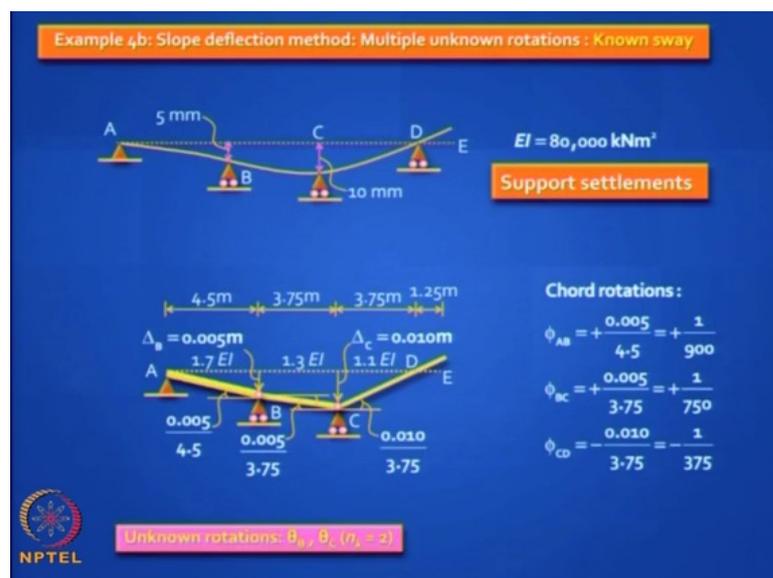
Write them down in terms of these unknowns and solve them. The equilibrium equations are M_{BA} plus M_{BC} is 0, M_{CB} plus M_{CD} is equal to 0. Solve them.

(Refer Slide Time: 50:01)



Draw your free body diagram. After you get the end moments, you can get the reactions. You know there are two parts. Look at this. After I get these end moments with plus and minus sign, this 120 is shared by these two equally - 60 and 60. This is called direct shear due to the loading. There is a moment shear; this divided by 4.5 will add up here to the right support and it will get subtracted here. So, you can do that for all the spans. So, you got the complete free body. You can draw the shear force diagram; you can draw the bending moment diagram.

(Refer Slide Time: 50:41)



I will conclude this with showing you what to do when you have sway type problem. When you have a sway type problem, we have to find the chord rotations. If δ_B is 5mm and δ_C is 10mm, your AB rotates clockwise, your BC rotates clockwise, and your CD rotates anti clockwise. You should first work out these known chord rotations. Then, the rest is simple.

(Refer Slide Time: 51:09)

Slope-deflection Equations:

Beam AB $(1.7EI; L=4.5m; M_{AB}=0; M_{BA}^f=0; \theta_A$ can be ignored; $\theta_B=?; \phi_{AB}=\frac{1}{900}$)

$$M_{BA} = M_{BA}^f + \frac{3(1.7EI)}{4.5}(\theta_B - \phi_{AB}) = 0 + 1.1333 \left(EI\theta_B - \frac{8 \times 10^4}{900} \right) = (-100.738 + 1.1333EI\theta_B) \text{ kNm}$$

Beam BC $(1.3EI; L=3.75m; \theta_B=?; \theta_C=?; \phi_{BC}=\frac{1}{750})$

$$M_{BC} = (M_{BC}^f=0) + \frac{4(1.3EI)}{3.75}\theta_B + \frac{2(1.3EI)}{3.75}\theta_C - \left(\frac{6(1.3)(8 \times 10^4)}{3.75} \right) \left(\phi_{BC} = \frac{1}{750} \right) \text{ kNm}$$

$$\Rightarrow M_{BC} = (-221.867 + 1.3867EI\theta_B + 0.6933EI\theta_C) \text{ kNm}$$

$$M_{CB} = (M_{CB}^f=0) + \frac{2(1.3EI)}{3.75}\theta_B + \frac{4(1.3EI)}{3.75}\theta_C - \left(\frac{6(1.3)(8 \times 10^4)}{3.75} \right) \left(\phi_{BC} = \frac{1}{750} \right) \text{ kNm}$$

$$\Rightarrow M_{CB} = (-221.867 + 0.6933EI\theta_B + 1.3867EI\theta_C) \text{ kNm}$$

Beam CD $(1.1EI; L=3.75m; M_{DC}=+60\text{kNm}; \theta_D$ can be ignored; $\theta_C=?; \phi_{CD}=-\frac{1}{375}$)

$$M_{CD} = M_{CD}^f + \frac{3(1.1EI)}{3.75}(\theta_C - \phi_{CD}) = 0 + 0.8800 \left(EI\theta_C + \frac{8 \times 10^4}{375} \right) = (+187.733 + 0.8800EI\theta_C) \text{ kNm}$$

You plug in back into those equations; you now have the chord rotations, but you do not have any fixed end moments.

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Equilibrium Equations:

$$M_{BA} + M_{BC} = M_B = 0$$

$$M_{CB} + M_{CD} = M_C = 0$$

$$\Rightarrow \begin{bmatrix} -322.605 \\ -34.134 \end{bmatrix} + \begin{bmatrix} 2.5200 & 0.6933 \\ 0.6933 & 2.2667 \end{bmatrix} \begin{bmatrix} EI\theta_B \\ EI\theta_C \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

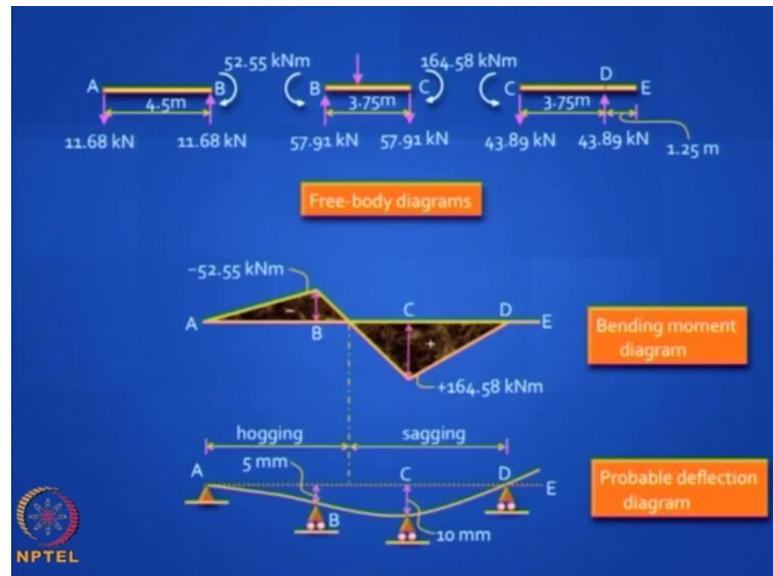
$$\Rightarrow \begin{bmatrix} EI\theta_B \\ EI\theta_C \end{bmatrix} = \frac{1}{5.23142} \begin{bmatrix} 2.2667 & -0.6933 \\ -0.6933 & 2.5200 \end{bmatrix} \begin{bmatrix} +322.605 \\ +34.134 \end{bmatrix} = \begin{bmatrix} +135.2565 \\ -26.3111 \end{bmatrix}$$

Final end moments:

$$\begin{bmatrix} M_{BA} \\ M_{BC} \\ M_{CB} \\ M_{CD} \end{bmatrix} = \begin{bmatrix} -100.738 \\ -221.867 \\ -221.867 \\ +187.733 \end{bmatrix} + \begin{bmatrix} 1.1333 & 0 \\ 1.3867 & 0.6933 \\ 0.6933 & 1.3867 \\ 0 & 0.8800 \end{bmatrix} \begin{bmatrix} EI\theta_B \\ EI\theta_C \end{bmatrix} = \begin{bmatrix} +52.548 \\ -52.548 \\ -164.579 \\ +164.579 \end{bmatrix} \text{ kNm}$$

You can just check this out yourself and find the equilibrium equations.

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Draw the free body diagrams.

We will stop at this stage. You have an exam coming up. Wish you all the best. Thank you.