Free Surface Flow

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Lecture 7

Welcome, students, to yet another module, module number two. The topic for this particular module is critical flow, and in the first lecture, we are going to start by discussing specific energy. All right. Specific energy is a common concept that you have also come across in your fluid mechanics and hydraulics course. Just to refresh your memory, I have drawn a diagram here. You see, this is a channel, right? This is the bed. This is the datum already written here.

This is one section, and this is the other section. We have drawn energy lines. This is the velocity head. This is the datum head. Different types of potential heads that are there. This is just an indicative diagram to explain the concept of specific energy again. So, if you remember, our energy is given by $z + y\cos\theta + \alpha\frac{V^2}{2g}$, given this theta. θ is the bed slope. Bed slope.

So, normally this θ is very small. So, as I said, theta is very small, cos theta will go to 1, and therefore, we can write our equation as $H = z + y + \alpha\frac{V^2}{2g}$. Do you remember what alpha is? Kinetic energy correction factor. Further, in uniform flow, if we have a uniform velocity, the value of alpha is equal to 1. Alpha is greater than 1 for non-uniform flows.

The higher the value of alpha, the higher the non-uniformity. So, if we shift the datum to coincide with the channel bottom, then what happens is So, if we shift our datum to coincide with the channel bottom, then z is equal to 0. And in that case, our energy can be simply expressed as $E = y + \frac{V^2}{2g}$. So, our datum coincides with the channel bottom.

And this is nothing but the specific energy that we started with. *E* is specific energy. Hence, specific energy is defined as the total energy head at the channel section measured with respect to the channel bottom. This statement means that our datum has been moved to the channel bottom. If you clearly see, it is nothing but an energy equation.

The concept of specific energy is very useful in defining critical depth and in the analysis of flow problems. So, this particular module is dedicated to critical flow, and therefore, the introduction of specific energy becomes very important from now on. Actually, it is very important even for further chapters, but this is one of the core and important concepts of free surface flow. You can also find that while the total energy in real fluid flow always decreases in the downstream direction, and what is the reason for the decrease in the downstream direction?

Because when there is a flow, it encounters friction, and energy is lost at least due to the bed friction. There could be other phenomena like turbulence that lead to the loss of energy. But at least due to the friction, there is a loss of energy. However, an important thing to note is that the specific energy is constant for a uniform flow. And it can either decrease or increase in a varied flow.

So, if you have varied flow, then this specific energy can either increase or decrease, and the reason is the elevation of the bed of the channel relative to the elevation of the total energy line determines the specific energy. This is quite important to note. So, one important thing, as I told you before as well, is the frictional resistance, right? So, in cases like, for example, if there is a short stretch or for any other reason, if we are able to ignore the frictional resistance, right? Then the total energy in non-uniform flow will be constant at all sections, also for non-uniform flow.

While the specific energy for such flows will be constant only for a horizontal bed channel, and in all other cases, specific energy will vary. This is important to note, and you can see that from the equation as well. So, that is not that big a deal. In the case of frictional resistance, total energy in non-uniform flow will also be constant. However, specific energy for such flows will be constant only for a horizontal bed channel.

Now, beginning with what critical depth is. So, let us see some terms like, you know, constant discharge. This is a definition sketch of specific energy. So, energy specific energy, we have seen that it is $y + \frac{V^2}{2g}$. So, we also know that Q is nothing but $V*A$, and if we write V as Q/A and substitute it, we find $\frac{V^2}{2g}$ can be written as $\frac{Q^2}{2gA^2}$.

So, if we have a channel of known geometry, E is a function of y and Q, where y is depth and Q is discharge. Keeping Q equal to constant, Q_1, the variation of E with y is represented by a cubical parabola. So, if you look at this particular figure here, this is a very specific energy versus depth diagram, right? And if you plot this y, so E against, you know, y for one particular discharge, so Q is constant, you will get a parabolic equation.

Cubic parabola, which has three roots. Okay. So now, It can be seen that there are two positive roots for the equation of e, indicating that any particular discharge Q_1 can be passed in a given channel at two depths and still maintain the same specific energy. What is important to note here is that the cubic equation has three roots, one of which is a negative root. However, for one particular discharge, for two y, y_1 and y_2, it will have the same specific energy, and these y_1 and y_2 are called the alternating depths.

So, again, in this figure, this ordinate PP', this ordinate PP', represents the condition of a specific energy of E_1. E_1 is this, E_1. The depth of the flow can be either PR, which can be one of the depths, or it can be PP, so PR'. So, one is PR and the other is PR' here, and these two possible depths have the same specific energy.

So, these two, y_1 and y_1, will have the same specific energy, called alternate depths. So, you see in this particular figure, line OS. So, this particular line is drawn such that E is equal to y. So, we draw a line that is OS such that E is equal to y. We draw it at an angle of 45 degrees, and this is an asymptote of the upper limb of the specific energy curve. You see, there is something called P. Yeah, let me just first erase it, and then you will be able to understand, right? So, you see P'R' here and P'R. This one represents the velocity head for both cases, okay?

Of the two alternate depths, one is smaller and has a larger velocity head, while the other, which means y_1', has a larger depth and consequently a lower velocity head. These are some of the significant findings from the definition sketch of the specific energy. Again.

So, if we have one particular Q_1, as the specific energy is increased, the difference between the two alternating depths increases. For example, if you keep on increasing, let us say you see if you have a specific energy here, the difference is this much. But let us say your specific energy is here; you will have this difference in these two alternating depths.

And this is what this line means. For a given Q_1, as the specific energy is increased, the difference between the two alternating depths increases. On the other hand, if E is decreased, you see, we talked about increasing E. But what happens if we decrease E? Right, the difference y_1' minus y_1 will decrease at a certain value, so this difference will keep on increasing, keep on, keep on decreasing, keep on decreasing, and a point will come where these two alternating depths will match each other, all right, and they merge with each other, and this particular point is nothing but the critical depth point.

So, there will be no value of y that can be obtained. So, theoretically, we cannot go beyond this. So, for no value, the minimum possible value of specific energy for a constant Q is this, called E_c. So, this denotes that the flow under the given condition is not possible in this region. So, the condition of minimum specific energy, so the condition of this specific minimum energy is called the critical flow condition, and the corresponding depth is y_c.

So, this depth, let me rub this one down, this y_c. Or denoted here as y_c, is known as the critical depth. Critical flow condition E_c and the corresponding depth is y_c. So, y_c is equal to the critical depth. So, what happens at the critical depth?

There are some important things to note. Specific energy E_c is minimum. And if we, you see this equation 1 that we have here, this one. This one, this specific energy equation 1, if we differentiate this, yeah, we differentiate this and we put $\frac{dE}{dy} = 1 - \frac{Q^2}{gA^3}\frac{dA}{dy}$ is equal to 0 in order to obtain maxima and minima. And we substitute instead of $\frac{dA}{dy}$ as T, that is the top width of the channel.

And if we designate the critical flow condition by suffix c, we get a condition. So, how are we going to get the condition? $\frac{Q^2}{gA^3}T$ is equal to 1 from this one here, and this is for critical condition T as top critical width gA_c^3 is equal to 1, and this is the same thing. Or alternatively, we can just write in terms of $\frac{Q^2}{g}$. How do we do that? We take this, you see $\frac{Q^2}{g}\frac{Tc}{Ac^3}$ is equal to 1, we take this one here, this one here, and it comes to be T_c exactly like this.

Or we can also say Q^2, you see $\frac{Q^2T}{gA^3}=1$, the same step. So, we take T this side $\frac{Q^2T}{gA^3}=1$, or we also know that $\frac{V^2}{2g}$ from here will be D/2, that is the hydraulic depth or $\frac{V}{\sqrt{gD}}$ equal to 1, and what is $\frac{V}{\sqrt{gD}}$? It is the Froude number, Fr. So, the Froude number will be equal to 1 for critical depth or critical flow conditions.

So, the minimum specific energy at a given discharge denotes critical flow states, which is one of the important findings. So, if any alpha value other than unity is to be used, for example, you know the energy equation we said that alpha was 1, but if we have a different alpha, then that particular equation for critical will not be just $\frac{Q^2T_c}{gA_c^3}$, this is for uniform conditions, right, but if we have any other alpha, then we substitute alpha here. So, now the equation 2 or 3. So, equation 2 or 3 is the basic equation, this one or the previous one is the basic equation that governs the critical flow condition in a channel.

You should note that the critical flow condition is governed solely by the channel geometry and discharge. So, it only depends on the channel geometry and discharge. Of course, the value of alpha is also important. Other channel properties such as the bed slope and roughness do not influence. The bed slope and roughness do not influence the critical flow condition for any given Q. So, Q and channel geometry and alpha and other things do not.

So, if we have the Froude number of the flow as $\frac{V}{\sqrt{g\, A/T}}$, for example, this is, you know, the Froude number is nothing but $\frac{V}{\sqrt{gD}}$. So, instead of D, you can write A/T, area by top width. It is easy to see that by using F in equation 2 at the critical flow y_c, F is equal to the Froude number 1 because, for critical conditions, the Froude number is 1. We get an important result that the critical flow condition corresponds to the minimum specific energy, and at this condition, the Froude number of the flow is unity. So, the idea is to find the specific energy E_c; we will see how much it is. For a channel, let us say, which has a large longitudinal slope theta and has a flow with an energy correction factor alpha, the Froude number will be defined simply.

Instead of $\sqrt{gD}$, we have this particular equation that can be used. Fr is nothing but $\frac{V}{\sqrt{\frac{1}{\alpha} g \frac{A}{T} \cos\theta}}$. So, now coming back to this figure again, let us consider any other specific energy, leaving this E_c. So, again, we are considering this PP'. The Froude number of the flow corresponding to both the alternate depths will be different from unity.

That is correct. The Froude number will be $\frac{V}{\sqrt{gD}}$. Depths are different. Different Froude numbers. And, of course, it will not be 1 because the Froude number is equal to 1 for critical conditions.

At the lower limb, you see, at the lower limb, the idea is this has a lower depth. This has a higher depth, all right. This CR and CR', okay. Depth here is This, let us say, y_1 is less than y_c, and therefore, V_1 is greater than V_c, where V_c is the critical velocity. Because of this, the Froude number at this particular point will be greater than 1 because the Froude number here is equal to 1.

I hope that is clear. You see, if you move from C to R, this has a higher y_c is greater than y_1. Therefore, the velocity here will be lower than that here. And therefore, the Froude number will be greater than 1. And this region is called the supercritical flow region.

In the upper limb of CR', you know, Similarly, if we go from the upper limb, so let me take this one down because this figure is complete in itself. So, if you go to this CR', here you see y_1' is greater than y_c, which implies V_1' is smaller than V_c, right? And therefore, this means, sorry, this means the Froude number corresponding to Fr_1' is less than Frc, and what is Frc? 1.

So, the Froude number at this location will be less than 1. So, this corresponds to the subcritical flow condition. Now, we see discharge as a variable in itself. So, in the above section, the critical flow condition was derived by keeping the discharge constant. This specific energy diagram can be plotted for different discharges: Q_1 is equal to, you know, different. We plot here, we have one Q_1, this is Q_2, this is Q_3, this is Q_4, and all greater than the previous one. In this figure, I mean, I have already told you we assume that Q_4 is greater than Q_3, which is greater than Q_2, which is greater than Q_1, and these discharges are constant along the respective. So, the discharge here, for example, this line is all constant, this line is all constant, this line is all constant, and this line is also all constant. These Q_1, Q_2, Q_3, Q_4 are different from each other. Now, let us consider a section PP' here in this plot, and we see in this that the ordinate PP', E is equal to E_1 is equal to this.

Now, after assuming this ordinate, we can see our specific energy is constant, all right. So, specific energy here is constant. Now, different Q curves, if we see, they are cutting this ordinate; it is cutting different Q curves at different points, and they have different intercepts. The difference between alternate depths decreases as the Q value increases. You see, as Q is increasing, Q. So, increasing Q means the difference between alternate depths decreases. So, now it is possible to imagine a value of Q which is equal to Q_m at a point C at which the corresponding specific energy curve would be just tangential to the ordinate PP'. So, we will have; so what I mean to say is, this particular Q is equal to Q_m', we draw Q, which means this ordinate is just tangential. The dotted line in this particular figure, which I have shown, indicates Q is equal to Q_m, representing the maximum value of the discharge that can be passed in the channel.

This is important to note. This dotted line represents the maximum value of discharge that can be passed in the channel while maintaining the specific energy. at a constant value E_1. This is very important to note. So, any specific energy curve of a higher Q value, that is Q greater than Q_m, will have no intercept.

So, if there is a specific energy of a higher Q value, we will have no intercept with the ordinate. It is very clear. So, if we have a Q that is higher than Q_m, it will never cut this PP'. Right, and hence there will be no depth at which such a discharge can be passed in the channel with the given specific energy. So, since $E = y + \frac{Q^2}{2gA^2}$, we are using this equation, and Q can be written as $A\sqrt{2g(E-y)}$.

The condition for maximum discharge can now be obtained by differentiating equation 5 with respect to y, equating it to 0, and keeping E constant. This $\frac{dQ}{dy}$ is $\sqrt{2g(E-y)}\frac{dA}{dy} - \frac{gA}{\sqrt{2g(E-y)}}$. We have just differentiated Q with respect to y. We are also differentiating A because A is a function of y. If we substitute these values, which are very common, $\frac{dA}{dy}$ is equal to T, and the previous equation Q/A is nothing but $\sqrt{2g(E-y)}$, this will give us $\frac{Q^2 T}{gA^3} = 1$. This is now the same as equation 2 and hence represents the critical flow conditions. Hence, the critical flow condition also corresponds to the condition for maximum discharge in a channel for a fixed specific energy.

It is important to note that, first, let me remove this. Not only the previous finding, but what else does the critical flow condition denote? It also denotes the condition for maximum discharge in a channel for a fixed specific energy. This is quite an important inference that we can draw. So, since we know that $A/T = y$ from the previous equation, the Froude number for a rectangular channel will be nothing but $\frac{V}{\sqrt{gy}}$. Very simple.

If we plot Q versus y at a given E, this is how we are going to obtain the curve. But most importantly, the previous finding is very important. So, I think with this, I will end this particular lecture, and I will see you in the next lecture, where we will start with the specific force. Thank you so much.