Free Surface Flow
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Lecture 60

Welcome back to this lecture, actually more of a problem-solving session on mobile bed channels. Today, we are going to solve some problems about the concepts that we have covered in this module, which is hydraulics of mobile bed channels. So, we will start with some example problems. So, the question is a wide rectangular channel in alluvium of 3-millimeter median size and relative density equal to 2.65 has a longitudinal slope of 0.0003. The question is to estimate the depth of flow in this channel which will cause incipient motion. So, this is one of the most basic problems that we are going to solve. So, let us write down what are the things that are given: dmm is given as 3.0.

So, we are going to use the formula by Swamy, which says $\tau_c = 0.155 + (0.409\ (dmm^2)\ /\ (1\ +\ 0.177\ (dmm^2)^{0.5}$. So, we have been given the values. So, we are going to calculate τ_c by substituting in the values: $0.155 + (0.409\,(3^2)\ /\ (1 + 0.177\ (3^2)^{0.5}$ and we get 2.44 Pascal.

So, for flow in a wide rectangular channel at depth *D*, τ_0 is given by $\gamma D S_0$ and at incipient motion is $\tau_0\ =\ \tau_c.\gamma D S_0$ is also given as 0.0003. So, as this is S_0 and this is relative density, this is given as equal to 2.44. Therefore, depth *D* will come out to be 0.831 meter, and this is what we needed to find. Another problem Estimate the minimum size of the gravel that will not move in the bed of trapezoidal channels. We have been given the channel shape trapezoidal, with a base width equal to 3 meters. Side slope, which is essential for trapezoidal channels, we also need to be given. Side slope is 1.5 horizontal to 1 vertical, and the bed slope, that is the longitudinal S_0, is given to be 0.004, with a depth of flow of 1.3 meters. So, we have already been given the depth.

So, how to solve this problem? So, we first find *R* which is equal to hydraulic radius. So, hydraulic radius will be for this. $((3 + (1.5 \times 1.3)) \times 1.30)/(\ 3 + 2 \times 1.3\ \sqrt{(\ 1.5 + 1)}$ So, *R* is equal to hydraulic radius is equal to 0.837 meter.

So, we also know that the dc that is the critical depth is 11 RS_0 if you remember from our lecture. So, 11 × 0.837 × 0.004 and this will come out to be 0.0368 meter. So, the d_c is, this is the minimum size of the gravel. that will not move.

So, any grain diameter that is this much or higher will not move, the ones that are smaller will be moving and this we have covered in the lecture slides as well in the theory to find the value of d_c. So, another problem a bigger a detailed problem. So, an unlined irrigation channel in an alluvium of medium size 0.30 millimeter is of trapezoidal section with bed width is equal to 3 meter and side slope 3.0 meter and side slope is equal to 1.5 horizontal into 1 vertical.

So, we have been given the shape, the cross-sectional shape, which is trapezoidal with base width and the side slope, as well as the bed slope. And the longitudinal slope, that is S_0, is equal to 0.00035. This we have been given. So, it says if this channel carries a discharge of 1.5 meters cubed per second at a depth of 0.8 meters, estimate. What are the things that need to be estimated?

This is the problem on the nature of the bed form. Of the bed form. Second is shear stress due to the grain roughness, and the third is shear stress due to the bed forms. So, we start with the things that are known to us.

So, the cross-section is given as trapezoidal, right? So, we first start with finding the area, the wetted perimeter, hydraulic radius, and other parameters. Here, we are going to use, because it is a bed form, we have to find $S*$ and R/d, and therefore, we need to do some prior calculations before. So, let us say area A first is trapezoidal, right? $(3 + 1.5 \times 0.8) \times 0.8$. This will give us 3.36 m^2. This is how a trapezoidal section will look like.

Wetted parameter $P = 3 + 2 \times 0.8 \times \sqrt{(1.5^2 + 1)}$, and this will come out to be 5.884 meters. Therefore, the hydraulic radius is wetted area divided by wetted parameter, which is equal to 3.36/5.884 = 0.571 meters. So, we have found out the area, the parameter, and the hydraulic radius R. Now, we need to calculate the value of R/d. R is 0.571 divided by, this is given as 0.00, which is 3.3 millimeters, but we need to convert it to meters, and this will come out to be 1903. And we are actually going to use the formulas. So, $0.05\ (R/d)^{-1} = 0.05.\ (1903)^{-1} = 2.63 * 10^{-5}$. We will also calculate the value of $0.014\ (R/d^{-0.46}).= 0.014\ (1903^{-0.46})$. This comes out to be $4.34 * 10^{-4}$. We will also calculate $S*$ as $S/(\gamma_s - y)\gamma$.

And this will come to be So/1.65 or $0.00035\ /\ 1.65, 2.12 * 10^{-4}$. Since $S*$ lies between $0.05\ (R/d)^{-1}$to $0.014\ (R/d)^{-0.46}$. Using the equations, the bed form is of the ripples and dunes category.

First answer, this is the first part. Second part, we have to first find the Manning's coefficient due to grains. So, ns is the cyclical equation we are going to use $(0.0003)^{1/6}/21.1$, this comes out to 0.0122, this is n_s. By Manning's formula, $Q = (1/n) * (A) * (R)^{2/3} * (So)^{0.5}$. This is given as 1 by n, 3.36 area is known, hydraulic radius is known, and this is also known $(0.00035)^{0.5}$.

To the power half, and this comes out to be $0.0433/n$, and n will come out to be 0.0288, that is Manning's coefficient for the whole channel. Now, the second part is shear stress due to grains. $\tau_0' = (n_s/s)^{3/2} \times \gamma R S_0 = 0.539$ Pascal, the second part, okay. And we also need to find out τ_0 is equal to average bed shear stress due to flow $= \gamma R S_0 = 9790 \times 0.571 \times 0.00035$, that comes to be 1.957 Pascal.

Therefore, shear stress due to bed form will be the difference of this, that is $1.957 - 0.539 = 1.418\, Pascal$. This is the last part. Now, a problem on suspended load. So, in a wide alluvial stream, a suspended load sample taken at a height of 0.3 meters above the bed. Indicated concentration of 1000 ppm by weight. The stream is 5 meters deep and So is given as 1 in 4000. Fall velocity is given for bud material as 2 centimeters per second. It says estimate the concentration of sediment at mid-depth. So, we have seen this equation, the Rouse equation $C/Ca = [((D-y)/y) \times (a/(D-a))]^Z$, let me just check the Z. Here $a = 0.3$ meter, $D = 5$ meters. So, let me just, so this was not, it was Z here. So, here also it is Z, ok. $D = 5$ meters, $y = D/2 = 2.5$ meters, Ca is given as 1000, settling velocity is 0.02 m/s. So, first we need to calculate $Z = \omega / u_* K$, ok. Now, we know since the channel is wide, $R = D$ and $u* = (gDS_0)^{0.5}$. And $Z = \omega / K(gDS_0)^{0.5}$, and assuming kappa is 0.4 as the von Kármán constant, we first find out the value of Z, the omega. $0.02/0.4 \times (9.81 \times 5 \times (1 / 4000))^{0.5}$. This will come to be 0.4515, and now using the above equation.

And using this value of Z, $C/1000 = [(5 - 2.5)/2.5 \times (0.3/(5 - 0.3)^{0.4515}$ to the power 0.4515, and this will come to be 0.2887, right? So, C will come out to be 288.7 ppm. This is one of the applications of the suspended Rouse equation. So, the question is a wide alluvial channel.

It has a bed material of median size of 0.8 millimetre. So, S_0 is given as 5×10^{-4} for the channel. The depth of the channel is given as 1.6 meters, and the velocity of flow V is given as 0.9 meters per second. We have to estimate the bed load.

Simple problem. So, we say since the channel is wide, $R = y_0 = 1.6$ meters, hydraulic radius using Manning's equation $V = (1/n) \times (y_0)^{2/3} \times (S_0)^{0.5}$. We substitute the value $0.90 = (1/n) \times (1.6)^{2/3} \times (5 * 10^{-4})^{0.5}$.

This will give us n is equal to 0.034. Now, we will find the ns using Strickler's equation. $n_S = (d)^{1/6}/21.1 = (0.0008)^{1/6}/21.1$ that is 0.0144. So, now we need to find shear stress due to the grain. shear stress due to the grain $\tau_0' = (n_S/n)^{3/2} \times \gamma y_0 S_0$ and that comes to be 0.0144/0.0340 this we have found out both $\gamma y_0 S_0$ or 0.2756 $\gamma y_0 S_0$. And $\tau_0' = \tau_0/(\gamma_S - \gamma) \times d$. So, this is to $0.2756\, \gamma y_0 S_0$. This is actually τ_0 divided by $(\gamma_S - \gamma) \times d$ or if we substitute in all the values 0.2756 into $1.6 \times 5 \times 10^{-4}$. divided by 1.65×0.0008, we get 0.167. This is non-dimensional. For bed load transport, we need to calculate first by φB, that is equal to $qB/(\gamma s\,(gd^3)^{0.5}\,((\gamma_S - \gamma)/\gamma_S)^{0.5}$ or $\varphi B = qB / 2.65 \times 9790 \times (9.81 \times (0.0008)^3)^{0.5} \times (1/1.65)^{0.5}$ and this φB can be written as $0.4234\, qB$. So, now we will apply the Peter Mayer formula. That is, φB is equal to $8\,(\tau'_* - 0.047)^{3/2}$. So, φB has come out to be $0.4234\, qB$ is equal to 8 times this 0.167 from this τ'_*. $(0.167 - 0.047)^{3/2}$.

And in this way, if you solve this, q_B will come out to be 0.785 Newton per second per meter. So in today's class, what we have seen is we have seen problems on all the aspects of mobile bed channels, whether it was bed load transport, suspended load transport, bed forms, or the computation of the critical incipient motion. I think that will be enough for today's class. Thank you so much.