Free Surface Flow
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Lecture 6

Welcome students to the last lecture of the first module. We are going to continue our problem sessions maybe we will solve 3 to 4 or maybe 2 to 3 more problems on the introduction concept. So, we finished our last lecture also on the problem where we calculated you know the discharge for the for cases when there was no energy loss and when there was an energy loss that was 10% of the initial head I mean the energy head. So, we start with yet another problem. most common structure that is used in hydraulics is sluice gate.

So, the question is a sluice gate in a 2.0 meter wide horizontal rectangular channel is discharging freely as shown in figure. So, first thing I am going to do I will draw a figure. So, I will draw a sluice gate right this is. So, this is a sluice gate, this is the free surface level and this is the so, let me draw this one a little yes.

So, this is y_2, this is y_1, this is the standard sluice gate all right. figure. Now, the question starts. If the depth a small distance upstream that is y_1 is y_1 and downstream and at downstream y_2.

You see I have shown y_1 and y_2. If these depths are 2.5 meter and 0.20 meter respectively. So, we are given y_1 and y_2. We have to estimate the discharge in the channel for some conditions. What are these conditions?

First condition is neglecting energy losses at the gate and for another case which is by assuming the energy at the gate to be 10 percent of the upstream depth y_1. Almost the conditions are same, the problem sets are different.

In the previous problem we saw a contraction, here we have a sluice gate. So, we start with solutions. So, you see if you look at this figure, y_1 is we have already been given is 2.5 meter and y_2 is 0.20 meter. So, first thing is equation of continuity. Equation of continuity is A_1V_1 is equal to A_2V_2.

See this is a horizontal rectangular channel therefore, we are writing b. So, no need for B_1 actually it is B. By_2 V_2, B gets cancelled and we can write V_2 as $V_2 = \frac{y_1}{y_2} V1$ or 12.5V_1. So, this is one important. value that V_2 can be written in terms of V_1 as 12.5V_1. So, now, we start with the first no energy loss.

For no energy loss simply Bernoulli's equation or energy equation $z_1 + y_1 + \frac{V_1^2}{2g} = z_2 + y_2 + \frac{V_2^2}{2g}$ right. And since the channel is horizontal See, you have to take q from the questions itself, it says horizontal, horizontal means z_1 is equal to z_2. So, datum gets cancelled and our equation this particular equation can be written as $\frac{V_2^2}{2g} - \frac{V_1^2}{2g} = y_1 - y_2$. or we can write we can use our previous equation $\frac{V_1^2}{2g}\left(12.5^2 - 1\right) = 2.5 - 0.2 = 2.30$.

And, if we solve $\frac{V_1^2}{2g} = \frac{2.30}{1.55.25} = 0.01481$. or V_1 is equal to 0.539 meters per second. And therefore, discharge, this is what we need to find out By_1V_1 just substitute these values B is 2, y_1 is 2.5 and this is 0.539. So, our Q will come out to be 2.696 m^3/s.

This is our the first part of the problem. So, now going to the second part and the second part is when there is energy loss. Then what is the first thing? What is the energy loss?

It is 10 percent. 0.10y_1 that is head loss is 0.25 meter that is the head loss that we have seen. So, now the Bernoulli or the energy conservation equation will be since it is you know z is cancelled out it is a horizontal bottom $y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g} + H_L$. This head loss is this. So, again $\frac{V_2^2}{2g} - \frac{V_1^2}{2g} = y_1 - y_2 - H_L$.

This H_L term is new compared to the first part. And similarly, we substitute the value $\frac{V_1^2}{2g}\left(12.5^2 - 1\right) = 2.5 - 0.2 - 0.25 = 2.05$. So, remember this we got 2.30 in our last question, last part and here we are getting 2.05. And similarly, if you are going to solve $\frac{V_1^2}{2g} = \frac{2.05}{15.25} \Rightarrow V_1 = 0.509\text{m/s}$. See, you can always apply decimal rule you can write up to two significant figures 0.51 for 0.509. Now discharge Q that is what we have to find is $Q = By_1V_1$ or you can also write By_2V_2, but I am preferring y_1 is 2.0 y_2 is 2.5 into $2.0 \times 2.5 \times 0.509 = 2.545$ This comes out to be 2.545 m^3/s.

So, this is the answer 2.545 m^3/s. So, we will see yet another problem. a little bit involving more of you know differentiation and integration. We have not used that much till now only in the first problem. So, another problem the question it says the velocity distribution in a wide channel. So, we are being given the velocity profile is given by $\frac{U}{u_{\max}} = \left(\frac{z}{h}\right)^{1/n}$. Here, u_{max} is the maximum velocity at a flow depth h. Now, the question find the depth averaged velocity First part we have to find beta that is the momentum correction factor. It is very similar to problem number 2 where we estimated the profile using the diagram.

Here we are given u by u_{max}. This is not h, this is u by u_{max} and also alpha that is the kinetic energy correction factor. So, its application of formula mostly. So, the depth average velocity u is obtained as $U = \frac{1}{h}\int_0^h u dz$. This is the formula for the depth average velocity. Now, very simple we will substitute these values 0 to h. What is u? $U = \frac{1}{h}\int_0^h u_{\max}\left(\frac{z}{h}\right)^{1/n} dz$.

And if you integrate this with regards to you know z, you are going to get $\frac{n}{(1+n)}$.

$\frac{n}{(1+n)} u_{\max}$. So, now therefore, the velocity distribution can be expressed in terms of depth averaged, how velocity as $U = u_{\max}\left(\frac{z}{h}\right)^{1/n}$ or you see we what we capital U we have calculated.

So, we can write u_{max} as $\frac{(1+n)}{n}$ U$\left(\frac{z}{h}\right)^{1/n}$, U is just simple substitution. So, the first part is finished here, the a part that is find the depth average velocity. Easier to write in the u in terms of you know depth average velocity. Now, we are representing a small u in terms of depth average velocity this u. So, now we will continue. for a wide channel, first we will find the momentum correction factor.

The formula is very simple. $=\frac{1}{h}\int_0^h \frac{u^2}{U^2}dz=\frac{1}{h}\int_0^h \frac{1}{U^2}\left(\frac{1+n}{n}\right)^2 U^2\left(\frac{z}{h}\right)^{2/n} dz$. Or if you simplify C capital U, this U gets cancelled, it will be terms comprising of h and n and finally, we get if you simplify $\frac{(1+n)^2}{(2+n)\times n}$. This is the momentum correction vector that is desired. Now, the last part is for a wide channel, the energy coefficient alpha for the C this is important that we realize that this is for wide channel. So, $=\frac{1}{h}\int_0^h \frac{u^3}{U^3}dz=\frac{1}{h}\int_0^h \frac{1}{U^3}\left(\frac{1+n}{n}\right)^3 U^3\left(\frac{z}{h}\right)^{3/n} dz$.

And if you integrate this and solve this, you will get $\frac{(1+n)^3}{(3+n)\times n^2}$. So, there is very famous law, the seventh power law. In general n is 7, but I mean this is not related to this particular question, but just letting you know there is one seventh power law where u/umax is z/h raised to the power 1/7. So, a little one last problem that we will solve.

Very similar problem but it is always good to do some practice. The velocity distribution in a channel may be approximated by again we have an equation $V=V_o\left(\frac{y}{y_o}\right)^n$ in which V is the flow velocity at depth y. And what is V_o? V_o is the flow velocity at depth y_o and n is a constant. Again, derive alpha and beta. Alpha is the kinetic energy and beta is momentum coefficient correction coefficient or momentum correction coefficient is kinetic energy correction coefficient.

So, let us consider a unit width of the channel. So, B is equal to 1. In this case, then we can replace area A in the equation for the energy and momentum factor by which term? By flow depth.

it is easier to solve that way why. So, what we have assumed is we have assumed a unit width. So, simple V_m is nothing $=\frac{\int VdA}{\int dA}$ and for a unit width. this it becomes V_m is equal to $=\frac{\int Vdy}{\int dy}$. Now, continuation let us substitute the expression of V in this equation in this equation that is $V_m=\frac{\int_0^{y_o} V_o\left(\frac{y}{y_o}\right)^n dy}{\int_0^{y_o} dy}=\frac{V_o}{{y_o}^n}\frac{y^{n+1}}{n+1}\int_0^{y_o}\frac{1}{y_o}dy$ or $V_m=\frac{V_o}{1+n}$. This is V_m in terms of V. V is equal to $V=V_o\left(\frac{y}{y_o}\right)^n$, $V_m=\frac{V_o}{1+n}$ and dA = dy. alpha we can simply get $\alpha=\frac{\int_0^{y_o} {V_o}^3\left(\frac{y}{y_o}\right)^{3n} dy}{\left(\frac{V_o}{1+n}\right)^3\int_0^{y_o} dy}$. And if you simplify this, you will get $=\frac{(1+n)^3}{(3n+1)}$. And similarly, we find beta that is the momentum correction factor same, but just the powers are different $\beta=\frac{\int_0^{y_o} {V_o}^2\left(\frac{y}{y_o}\right)^{2n} dy}{\left(\frac{V_o}{1+n}\right)^2\int_0^{y_o} dy}$ or just writing intermediate step as well, or beta is equal to $\beta=\frac{(1+n)^2}{(2n+1)}$ and this is the answer that we required. So, there could be so many other problems but I think the variety and the type of questions in the introduction concept I think we have covered in detail and with this we are finishing our module 1 which comprises of 6 different lectures and I will see you again in the beginning of module 2. Thank you so much.