Free Surface Flow

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Lecture 59

Welcome back, students. In the previous lecture, we studied the bed form resistance and also the modes of sediment transport. So, this particular lecture will be dedicated to seeing the formulations required for the calculation of bed load and suspended load transport. With that being said, let us get started. First, we are going to see bed load. So, the transport rate of sediments in the bed load q_B is usually, so bed load q_B is usually referred to in units of weight per second per unit width, that is, Newton per second per meter. So, a very large number of empirical and semi-analytical expressions are available to estimate the bed load q_B and these are calculated in terms of sediment, fluid, and flow parameters. So, two scientists called Boyce in 1879 were among the first to propose an expression for q_B as a function of the excess of τ_0.

Over the critical shear stress τ_c, and the relationship which he gave us is $q_B = \alpha(\tau_0 - \tau_c)$. And since then, a very large number of empirical parameters for excess shear stress, that is $(\tau_0 - \tau_c)$, have been proposed by various scientists and investigators. One of the most widely used empirical equations for q_B is by Meyer Peter and Muller. relates q_B in a dimensionless manner as. So, the defined $\varphi_B = 8(\tau^{*\prime} - 0.047)^{3/2}$This equation where φ_B is the bed load function and $\varphi_B = (q_B/\gamma_s\sqrt{gd^3}) \times 1/(\gamma_s/\gamma - 1)^{3/2}$

Alright. And what is $\tau^{*\prime}$ = dimensionless grain shear stress and $\tau^{*\prime} = \gamma RS_0/(\gamma_s - \gamma)d = ((n_s/n)^{3/2}) \times \gamma RS_0/(\gamma_s - \gamma)d$ So, the different meanings of different terms: q_B is equal to bed load in Newton per second per meter, d is the mean size of sediment, R is equal to hydraulic radius of the channel, γ = unit weight of water = ρg, γ_s is unit weight of sediment particles, and η = Manning's coefficient for the whole channel. n_s= Manning's coefficient of particle roughness. S_0 = longitudinal slope of the channel. And so, here it was R'.

It was a mistake before. And this R' is hydraulic radius corresponding to the grain roughness. So, in this equation here, in equation *, the term 0.047 corresponds to the asymptotic value in the Shields diagram. This corresponds to the asymptotic value in the Shields diagram. So, this is one of the equations for calculating the bed load transport. Now, we will see suspended load. So for this, consider a steady channel flow of depth D carrying Sediment in suspension. I will draw the figure soon in suspension, so the sediment

Particles which are lifted up from the bed are kept in suspension due to turbulence, which we have already discussed before, while the particles settle down due to their weight. The particles are lifted up due to the turbulence, and while, because of the weight, they try to settle down, and this results in the inner concentration profile $C = f_n(y)$, with sediment concentration *C* being distributed in a vertical manner to achieve equilibrium of the forces acting on the particles. And this is the figure that we are talking about. This is the depth *D*; this is the velocity profile; this is the concentration profile.

Now, we are going to see in a steady flow the upward diffusion of the sediment is balanced by the settling of the sediment particles. So, the upward diffusion is balanced by the settling of the sediment particles, and the basic differential equation governing this is given: $C\omega + \epsilon_s(dC/dy) = 0$

This is the sediment continuity equation. Here, *C* = concentration of sediment by weight, ω = fall velocity of the sediment particles, ε_s = mass diffusion coefficient (diffusivity). This is generally a function of y. Also, at any height y above the bed, the shear stress $\tau_y = \tau_0(D - y/D)$. And for the turbulence part, we use Prandtl's mixing theory; τ_y can also be written as $\tau_y = \rho\, \epsilon_m(du/dy)$. Here, ϵ_m = diffusion is the diffusion coefficient for momentum.

So, by considering the form of velocity distribution in the channel, $du/dy = u^*/ky$. Here, *k* = Kármán's constant, and its value is approximately equal to 0.4. $\epsilon_s = \epsilon_m$, we can write $\epsilon_s = ku^*\ (y/D)(D - y)$. And if we use this equation, substitute this value into the main equation: $C\omega + \epsilon_s\, dC/dy = 0$. The main equation, we get $C\omega + ku^*(y/D)\ (D - y)dC/dy = 0$ or $dC/C = (-\omega D/ku^* y(D - y))dy$ Now, we will integrate this equation assuming $\omega/ku^* = z = constant$ and integrating this equation between $y = a$ and y yields integration $\int_a^y \mathrm{dC/C} = \int_a^y \mathrm{ZD/(y(D-y))\ dy}$ will get $C/C_a = [((D - y)/y)(a/(D - a))]^2$ This is whole square. And this is the Important equation. What is we will define where C_a, what is C_a? C_a= concentration at any height a above the bed. The equation which you derived before; this gives the ratio of concentration *C* of the suspended material at any height y above the bed. the bed to the concentration C_a at any reference level A. So, this is called Rouse equation.

This equation is called the Rouse equation. So, knowing the concentration profile, and the velocity profile in the vertical, the suspended sediment load q_s. Per unit width of the channel in the vertical can be estimated as $q_s = \int_a^D Cudy$ a_1, and what is a_1?

a_1 = level corresponding to the edge of the bed load layer. Usually taken as 2 times the diameter. So, from 0 to 2D, it is considered bed load, and above that, the regime for the suspended load. So, this method requires estimation of C_{a1} by other means and methods.

Several methods exist in So, today, what we have learned is the details about the bed load and the suspended load. How do we calculate that? Equations of the bed load: Peter Mayer and Muller, and also this Rouse equation. And in the next class, we will try to solve some problems relating to the concepts that we have covered in the last four lectures.

I think I will close the class for today and see you in the next class. Thank you.