Free Surface Flow
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Lecture 57

Welcome back, students. In the last class, we started with the introduction to the hydraulics of bed channels, where we began with the introduction, the incipient motion, and then looked at the Shields curve. So now, we are going to continue our journey with the Shields curve. So far, we have seen that to use the Shields curve to estimate the critical shear stress for a given particle size d greater than 6 millimeters, we can use the equation $\tau_c = 0.905d$mm. Very simple, right?

But to use the Shields curve to estimate the critical shear stress for a given particle size less than 6 millimeters, one has to adopt a trial-and-error procedure. This is because τ_c occurs in both non-dimensional parameters of the curve. Swami and Mittal have expressed the Shields curve results in an explicit relationship between τ_c and d using an empirical non-dimensional formula. So, for a specific case of water at 20°C and sediment with a relative density of 2.65, I am going to write the empirical relationship. So, we are going to write an empirical relationship by Swami and Mittal. They say that $\tau_c = 0.155 + [0.409(dmm^2)] / \sqrt{[1 + 0.177(dmm^2)]}$. And this holds true for water at 20°C where $\upsilon = 1 \times 10^{-6}\ m^2/s$ and sediment relative density of 2.65.

So, this is one equation that can be used to obtain the critical shear stress. Critical shear stress is very important because, from this, if you see, we can go—you see, there are many other formulas, but this particular formula here can be used to obtain the bed shear stress, right? If this bed shear stress is more than the critical bed shear stress, we will find if the sediment has moved or not. And d_{mm} is particle size in millimeter, and τ_c is the unit of τ_c is N/m^2. So, now this equation this particular equation is based on the limiting value of the Shields curve as 0.06 and is very convenient in calculating to an accuracy of about 5 percent error. The value of τ_c for particle sizes up to 5.5 millimeters. So, you can also use it just below 6; it can measure. No, I mean, actually, the calculation is accurate up to 5.5 mm, but for larger particle sizes, it is much better to use this equation.

So, we said that the limiting value was 6 millimeters in this case, right? To be able to apply for less than 6, we have to use trial and error, but Swami's equation has shown Swami and Mittal that it could be used for up to 5.5 millimeters. Okay, okay. So now consider an alluvial channel with Re_{c^*} greater than 400, that is having particle size greater than 6 mm. See, particle size greater than 6 mm these Shields curve and other things are very well applied.

So, we can write $\tau_c/(\gamma_s - \gamma) = 0.056$ So, if dc is the size of the particle that will just remain at rest in channel of bed shear stress τ_0 , then $d_c = 0.056/(\gamma_s - \gamma)$. But for uniform flow of hydraulic radius and bed slope So. We have also seen that τo is nothing but γRSo. Thus, $d_c = \tau_0/0.056(\gamma_s - \gamma)$. Now, we will take $\gamma_s/\gamma = 2.65$ Then $d_c = 10.82RS_0$ or approximately $11\ RS_0$ where r is hydraulic radius S_0 is the bed slope and this is equation is valid. For d_{mm} greater than 6 mm and this provides a quick estimation for the size of the sediment particle that will not be removed. This equation provides a quick method for estimating the of sediment particle that will not be removed from the bed of a channel.

So, this much or greater than that will not be removed, alright. So, this was the basic concept of the Shields curve. Now, we are going to look at another concept called bed forms. So, when the shear stress on the bed of an alluvial channel due to the flow of water is larger than the critical shear stress τ_c , as we have seen, the bed will become dynamic and will have a strong interaction with the flow. So, depending on the flow, sediment, and fluid characteristics so depending on the flow characteristics, sediment, and fluid characteristics the bed will undergo different levels of deformation and motion.

So, the bed will deform, right? As a result of careful observations, the following characteristic bed features are recognized. So, due to sediment transport, some important bed features are bed form types. One is a plain bed with no sediment motion. Second is ripples and dunes. Third is transitions. It is a plane bed with sediment motion and standing waves, and the fourth is anti-dunes.

So, these bed features are called bed forms or bed irregularities. So, these are called bed forms. Okay, so now let me show you how they look. You see these are plane beds with sediment motion. These are ripples, you see? Okay, this is a dune. This is washed-out dunes, this is plane wave, this is anti-dune, this is breaking wave, and standing waves. This is what it looks like.

We are not going to go into much detail. This type of complete detail will be covered in another course. I will, of course, tell you some basic things about it in the next slide. So, first is plane bed with no sediment motion.

So, this corresponds to a situation where $\tau_0 < \tau_c$. So, nothing happens; there will be no motion, and the bed will remain plane. So, the friction offered here to the flow is due to the resistance of the grain only. Second is ripples and dunes. So, among these, first is ripples.

So, if τ, that is shear stress in the channel, is increased, this can be done in many ways, right? By increasing either discharge or the slope such that τ_0 if τ_0 is such that τ is moderately $> \tau_c$. This is important: is τ_0 moderately $> \tau_c$. The grains will begin to move. They will move, and the bed will be covered by a sawtooth type of ripple pattern, that is, ripple as in this one here. Now, dunes.

So, if the shear stress that is, shear stress is further increased or gradually increased. or gradually increased, the ripples gradually grow into larger sizes. In this case, then, a different form known as dunes appears. These are dunes, and they appear with ripples riding over them, okay. The third one is the transition among that A, which is a plane bed with sediment motion. So, further increase of the shear stress whether it could be due to another reason after the dune bed pattern phase will lead to a transition phase where the bed undulations get washed away and progressively achieve ultimately an essentially plane bed surface. The sediment transport rate would be considerably larger in the dune phase. The flow, however, will be in the subcritical range, with the Froude number of the flow being near to unity.

So, let me just write this down. So, further increase of τ_0 , that is shear stress after the dune bed pattern phase, will lead to a transition phase where the bed undulations get washed away to achieve a plane bed.

And this is what a plane bed with sediment motion means. Here, sediment transport rate would be considerably larger than in the dune phase. Just note, the flow will be in the subcritical range. Now, the next standing waves, okay. So, now we are, I mean, slowly and gradually increasing the bed shear stress. So, further increase if we further increase τ_0, that is beyond the plane bed stage brings the Froude number nearer unity and beyond. It means we see that the plane bed phase is where the flow was still subcritical but near unity, but further increase would bring the Froude number nearer unity or even higher than that.

This would lead to the formation of symmetrical sand waves with associated water surface standing waves. This means the water surface the water surface undulations will be in phase with the sand waves.

So, just see, these are the standing waves. So, it is important to note that the above two bed features the standing waves and the plane bed with sediment motion are clubbed into one class called transition. Right. So, both of these the transition phase of the bed form are very unstable.

This is important. So, both the standing waves and the plane bed with sediment motion are highly unstable. The fourth type is the antidunes. So, if you further increase the τ_0 shear stress, if the shear stress in the channel is further increased beyond the transition phase, the symmetrical sediment wave.

And so, the symmetry is achieved in the standing wave. The symmetrical sediment wave and the associated standing wave slowly start moving upstream. Okay, so if we increase the bed shear stress even higher than that of the standing wave phase, the channel the symmetrical sediment wave and the associated standing wave will slowly start moving upstream. Now, what's going to happen? The waves gradually grow steeper and then break. The bed form at this stage is called the anti dune. So, how does it look?

This is the breaking wave. So, that means it has broken. So, the next topic that we are going to cover in the subsequent lectures will be the bed form and resistance. We will see about the bed load and the suspended load sediment transport and other aspects associated with it. We will also try to solve some problems.

But for today, I think that is it, and I will see you in the next class.