Free Surface Flow
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Lecture 53

Welcome back, students, to this lecture on rapidly varied unsteady flow. In the last two lectures, we have extensively covered the four different types of rapidly varied unsteady flow transients. That is surges: positive surge moving downstream, positive surge moving upstream, negative surge moving downstream, and negative surge moving upstream. In all those cases, what we have done is numerical treatment, found out the celerity, and performed some numerical analysis of these four cases. Now, moving ahead, we will examine a dam break problem.

What happens when a dam breaks? So, this is a dam break problem, and this is the sketch of the dam break. So, a particular case we studied earlier was Type 4, which was a negative surge. We studied a typical case where the situation involves $V_1 = 0$. This situation models the propagation of a negative wave upstream due to the instantaneous complete lifting of a control gate at a reservoir.

This ideal sudden release of flow from a reservoir simulates the sudden breaking of a dam holding back a reservoir, and as such, this problem is known as the dam break problem. So, Type 4 was a negative surge moving upstream, and its special case with $V_1 = 0$ simulates a dam break problem. So, let me talk about this particular figure here.

In this particular figure, it shows the flow situation due to the sudden release of water from an impounding structure. This is, as I mentioned, a special case of type 4 wave with $V_1 = 0$. The coordinate system used here is $x = 0$ and $y = 0$ at the bottom of the gate. x is positive in the downstream direction from the gate and negative in the upstream direction of the gate. y is positive vertically upward.

So, if we try to use the previous equations, that is, the equation we solved for type we had this V_w as $3(\sqrt{gy}) - 2(\sqrt{gy_1})$. And the velocity at any section can be given as $2(\sqrt{gy_1}) - 2(\sqrt{gy})$. The water surface profile of the negative wave is minus x, as we have seen previously as well. The profile is a concave upward parabola.

The conditions at the gate are interesting. At the gate, $x = 0$, and using the suffix 0 to indicate the values at the gate, we get $(\sqrt{y_0}) = 2/3\ (\sqrt{y_1})$. This is the initial condition, meaning at the gate, $y_0 = 4/9\, y_1$. Note that y_0 is independent of time and as such is constant. The salient features of the wave profile are as follows.

So, we are talking about the salient features of the dam break problem. The dam break problem is actually a type 4 problem. With $V_1 = 0$, and after putting V_1 equal to these, these are the things that we obtained. So, at $y = 0, x = 2t\,(\sqrt{gy_1})$. At $y = y_1, x = -t(\sqrt{g\, y_1})$, and at $x = 0, y_0 = 4/9\, y_1$.

So, the velocity at the gate Vo, by equation 1, is $2/3\,(\sqrt{g\, y_1})$, $V_0 = 2/3\,(\sqrt{g\, y_1})$. And the discharge intensity will be $q = V_0\, yo = (2/3\,(\sqrt{gy_1})) \times (4/9\, y_1)$ or $8/27\,(\sqrt{g\,(y_1^3)})$, which is also independent of the time t. If we note that the flow is being analyzed in a horizontal frictionless channel, and as such, the depth y_1 with $V_1 = 0$ represents the specific energy E. At the gate axis, we can write $y_0 = 2/3\, y_1 = 2/3\, E$, which is the critical depth. Also, at $x = 0$, the Froude number can be calculated as $V_0 / (\sqrt{gy_0})$, and we can substitute the values of V_0 and y_0, just substituting in the value of $V_0 = 2/3\,(\sqrt{g\, y_1})$ and $y_0 = 4/9\, y_1$, which is equal to 1. Thus, we know that the flow at the gate axis is critical and the discharge is maximum.

Further, it is easy to see that upstream of the gate, the flow is subcritical, and downstream of the gate, it is supercritical. If we analyze the Froude number upstream and downstream. And this is the simple ideal analysis of a sudden release from an impounding structure, found to give satisfactory results for the major part of the profile. However, in real situations, the downstream end is found to have a rounded positive wave instead of a parabolic profile with its vertex on the x-axis. In an actual dam break, the tapered leading edge of the ideal profile is modified due to the action of ground friction, causing a positive surge to move downstream.

See, this type of simple analysis will give a good approximation. This is analytical. This is one of the very easy solutions. Correct analysis for the most part, but sometimes it might not because the assumptions that we are taking might not always hold true. But for you, more importantly, this dam break problem is a type 4 problem.

What is the type 4 problem? Negative surge moving upstream with $V_1 = 0$. If you have understood this, you will be able to solve the dam break problem analytically. Okay, so now another partial lifting of the downstream gate.

If we lift, you see there are gates at downstream sections of the river. If we lift it partially, what happens? Something like this, partial lifting of the downstream. So this is a variation of the dam break problem in case What is the variation?

That is the partial instantaneous lifting of the downstream gate from the initially closed position. A simple case of a sluice gate in a rectangular channel is we will analyze. So in dam break suddenly everything was opened but here we have lifted it partially. So, if we consider the sluice gate to be suddenly raised by an amount a from an initially closed position and if a is greater than $4/9\ y_1$ then it amounts to. So, if you remember this was from dam break problem, right? So, if this a, if it is this a, the lifting is greater than this value $4/9 y_1$, then it amounts to full rest position as indicated in the previous section. And we have to do analysis of the dam break. However, if this a is less than $4/9\ y_1$, then it is a partial closure and analysis for such case, we are going to show below. So before the operation of the gate water upstream of the gate is at rest at depth y_1 because this is this partial lifting also we have to assume $V_1 = 0$.

The only difference is that the a is less than $4/9\ y_1$. The gate is lifted instantaneously and partially so that h_0 is equal to draw down at the gate. A negative wave will be produced by this action and it travels upstream with a wave velocity V_w and a forward flow velocity V is created. Since $V_1 = 0$, V_w will be $3(\sqrt{gy}) - 2(gy_1)$ and v will be $-2(\sqrt{gy}) + 2(\sqrt{gy_1})$. that is negative wave at $x = 0, y_0 = y_1 - h_0$ and velocity $V = V_0$.

Thus,V_0 can be written as $2\left(\sqrt{gy_1}\right) - 2\sqrt{g(y_1 - h_0)}$.The discharge in a rectangular channel $Q_0 = B\ y_0\ V_0$, which is constant as V_0 and y_0 do not change with time. Q_0 can be expressed in terms of H_0 and y_1 by substituting for y_0 and V_0. And Q_0 will be $B(y_1 - h_0)(2(\sqrt{gy_1} - 2(\sqrt{g(y_1 - h_0)})$. We just substitute y_0 in different terms.

On simplification, an expression for the discharge can be obtained in non-dimensional form as this is an important equation, though complicated: qy under root $Q_0/By_1(\sqrt{gy_1}) = 2 \times (1 - h_0/y_1) \times (1 - (\sqrt{1 - h_0/y_1}))$. Wave profile: the profile of the negative wave at any time t is given by, if you remember, $-x$ was $V_w \times t$, right, like this. Where y is the depth of the flow at any x, t. So, this concludes not the entire topic, but this concludes the theoretical part. So, we have seen. So, let me say before starting the problems, let me just summarize what we have done till now.

What we have done is we have analyzed different types of surges for different cases. Type 1, type 2, type 3, type 4. Type 4 found the most application because of the dam closure and partial lifting of the gate. And we saw the numerical treatment, we saw the continuity equation, and we saw the momentum equation. Now, we will see some problems which we are going to solve for the rapidly varied unsteady flow.

It's easier to solve. I mean, it's easier to understand and solve the questions. The question is about a 3-meter-wide rectangular channel. It has a flow of 3.6 m^3/s with a velocity of 0.8 m/s. If a sudden release of additional flow at the upstream end of the channel causes the depth to rise by 50 percent, determine the absolute velocity of the resulting surge and the new So, the question is to find the absolute velocity of the resulting surge, that is V_w. So, the question is: there is a wide rectangular channel that is 3 meters wide, and the flow is also given with the velocity. If there is a sudden release of additional flow at the upstream end of the channel, and that means it causes a positive surge, okay? The water level rises by 50 percent, so it's best to first try to draw it. So, this is a rectangular channel, right? And there is a surge here. This is V_w here. So, this is y_2, this is y_1, this is V_2, this is V_1, right? This is V_w. So, this is our sketch, okay. So, if we try to, you know, there is another sketch with.

So, is there will be another sketch? make this by superimposing it with minus V_w. But let us find some important parameters. We know $V_1 = 0.8\ \mathrm{m/s}$ and y_1 is 3.6, discharge is given 0.8 *3.0 that is 1.5 meter. So, y_1 is given 1.5 meter. And it is also given $y_2 / y_1 =$ 50 percent that is 1.5. That means $V_w = y_1 \times 1.5$, 1.5×1.5. So, y_2 is 2.25 meter, that is 2.25 meter and this V_1 was given 0.8 m/s. V_2 is also 0.25 meter positive. So, for a positive surge moving downstream in a rectangular channel.

Either you can take the equations directly from the slides or you can start from basic by applying continuity equation and momentum equation. I am taking the equations directly $(V_w - V_1)^2/gy_1$ is equal to $(1/2) \times (y_2/y_1) \times ((y_2/y_1) + 1)$ If you do not remember, this is type 1. Look it in, so, this is the equation. Now we are going to substitute different values. So, $(V_w - 0.8)^2/(9.81 \times 1.5) = (1/2) \times 1.5 \times (1.5 + 1)$ right or we can write. $(V_w - 0.8)^2 = 27.591$. This will give two roots, select positive root Vw will come out to be 6.053 m/s. This is the, what is this called? You see this absolute velocity of the resulting surge. Now, we also need to find out the flow rate. So, for flow rate by continuity equation,that is $A_1 V_1' = A_2 V_2'$ or A_1 is $B y_1 \times (V_w - V_1) = B y_2 \times (V_w - V_2)$ So, this B B gets cancelled, we can write $y_1 \times (V_w - V_1) = y_2 (V_w - V_2)$. And V_2 can be written using this equation $y_1/y_2 \times V_1 + (1 - (y_1/y_2)) * V_w$ and $V_2 = ((1.5 / 2.25) \times 0.8 + (1 - (1.5/2.25)) \times V_w$ we found out in the last slide 6.053.

It comes out to be, if we solve this, it comes out to be, let me write it down once again. So, new V_2 we have found out, new flow rate will be $Q_2 = By_2V_2$ or B was 3, y_2 came out to be 2.25 and this came out to be 2.551. Q will be 17.22 m^3/s, Q new flow rate 17.22 m^3/s So, see this particular question was about a type I problem where a positive surge was moving downstream. We were given the discharge, we were given the velocity, so if this was given, we could have we were easily we were able to calculate y_1 was 1.5. And therefore, y_2 was also given as it was rose by 50%. So, we had calculated y_2. And now using the our equation for momentum, we found out the value of V_w, V_w, was the absolute velocity of the resulting surge. And then finally, using the continuity equation, we find out the velocity at 2.

So, here in section 2, the velocity we were able to find out. And then, if we were, then we were also able to find the new flow rate, that is by 2×. And that came out to be 17.22 m^3/s. So, this is one of the typical problems. Now, in the next classes, we are going to see some more problems on rapidly varied unsteady flow.

And that is all for this lecture. Thank you so much.