Free Surface Flow

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Lecture 52

Welcome back, students. Today, we are going to continue again what we started last lecture. Last lecture, we started rapidly varied steady flows. In the first lecture, we gave the introductions and started about surges: positive surges, negative surges, surges moving upstream, and surges moving downstream. We also brought out the similarity between two types of surges: positive surges moving downstream and positive surges moving upstream, relating to moving hydraulic jumps and trying to bring out the analogy between steady-state hydraulic jumps and moving hydraulic jumps.

We use the same equations for the moving hydraulic jump as we do for the steady-state hydraulic jump, except we have to account for the relative velocity, Vr_1. So, Vr_1 will differ when there is a surge moving downstream versus when there is a surge moving upstream. That was covered in the last lecture. Today, we will continue with the celerity of the surge. The velocity of the surge relative to the initial flow velocity in the channel is known as the celerity of the surge, *Cs*.

So, what is celerity? The velocity of the surge relative to the initial flow velocity in the channel is known as the celerity of the surge, denoted by Celerity is denoted by *C* or *Cs*. *C* stands for celerity; *S* stands for surge. Thus, for a surge moving downstream, *Cs* is simply $V_w - V_1$.

This we have already seen. For the surge moving upstream, it will be $V_w + V_1$. This is also quite simple. Now, if we have a rectangular channel, we can write, so we are using the previous equations for the moving hydraulic jump and or even the steady state hydraulic jump, we find out the value of *Cs* as given as here. Let me just write it down, half of $g\, y_2 / y_1 * (y_1 + y_2)$ for the wave of very small height. So, if the surge that is happening is very small, then that means y_2 and y_1 are approximately approaching each other and *Cs* becomes under root gy this is the so if there is a very less difference between upstream and downstream. So, this is the same diagram as for the positive surge moving downstream. If we consider a surge like this moving downstream, if the surge is considered to be made of large number of elementary surges of very small height, pile one over the other, then for each of these V_w will be $V_1 + (\sqrt{gy})$. So, here the assumption is we have, if we consider the bigger surge to be made up of large number of elementary surges of very small height piled up one over the other, then for each of these V_w will be $V_1 + (\sqrt{gy1})$.

And if we consider the top of the surge, that is point *M* here, this point moves faster than the bottom of the surge. That is correct. That is, it moves faster than this. And this causes the top to overtake the lower portions. And in this process, the flow tumbles down and to the wave front to form a roller of a stable shape.

Thus, the profile of the positive surge is stable, and its shape is preserved. And this. Yeah. However, in a negative surge, if by a similar argument, the point *M* on the top of the surge moves faster than a point on the lower water surface. This results in the stretching of the wave profile; the wave profile will be stretched.

The shape of the negative surge at various time intervals will be different, and as such, the analysis used in connection with positive surges will not be applicable. This is the reason for the instability; this is not stable. For channels of very small lengths, the simple analysis of a horizontal frictionless channel gives reasonably good results. However, when the channel length and slope are large, friction and slope effects have to be properly accounted for. We do not consider friction, but if it is too large a channel, then we might have to.

Further changes in geometries, such as the cross-sectional shape, break in grade, and junctions along the channel, influence the propagation of the surge. These are the factors that will affect the propagation of the surges. Now, talking about elementary negative waves. This is a figure. So, the shape of the negative surge varies with time, as we discussed, due to the stretching of the profile by varying the values of V_w along its height.

And this is the elementary negative wave control volume. This is typical for Negative. Surges. Thus, for the purpose of the analysis, the negative surge is considered to be composed of a series of elementary negative wavelets of celerity $\sqrt{gy}$ superimposed on the existing flow.

So, the same analogy applies: we will consider that the large negative surge is composed of a series of elementary negative wavelets of celerity $\sqrt{gy}$. So, if you consider one such elementary negative wave height of wave of height Δy, this The motion is converted again the same process we have to convert the motion to an equivalent steady flow state to be able to apply Newton's law. And how do we do that? We superimpose a velocity minus V_w on the system.

So, this becomes $V_w - V_2$, and this becomes $V_w - V_1$. Now, consider the rectangular channel of constant width; the continuity equation will be written as $a_1\, v_1 \;=\; a_2 v_2$ or $y_2\,(V_w - V_2) \;=\; y_1\,(V_w - V_1)$. This is section 2; this is section 1. So, if we use the continuity equation, we say V_1= v, y_1 = y, V_2= v - δv, and y_2 = y - δy, because we are considering a small change—we just use a different notation. If we get $(y \,-\, \delta y)\,(V_w \,-\, v \,+\, \delta v) \;=\; y\,(\,V_w -\, v)$. Or $(y \,-\, \delta y)\,(V_w \,-\, v \,+\, \delta v) \;=\; y\,(V_w \,-\, v)$. Or we try to simplify again: $y\,V_w \,-\, yv$ and then $y\,\delta v$. So, this $y\,\delta v$ here minus $V_w y$, so minus $\delta y\,V_w$ here. Plus $v\,\delta y$. So, this becomes minus minus becomes plus, minus $\delta y\,\delta v$ is equal to $y\,V_w$minus yv, and this gets cancelled, and this also gets cancelled, and if we neglect this, Because this is too small—this is a product of the differentials—we can get $y\,\delta v \,-\, V_w\,\delta y \,+\, v\,\delta y \;=\; 0$. Or $y\,\delta v \;=\; V_w\,\delta y \,-\, v\,\delta y$, or $y\,\delta v$. This δy becomes common: $V_w -\, v$, or $\delta v\; by\; \delta y \;=\; (V_w \,-\, v)/y$.

Now, similarly, we can write momentum equations similarly: $(y_1^2)/2 \,-(y_2^2)/2 \;=\; 1/g\;y_1\,(V_w \,-\, V_1)[(V_w \,-\, V_2) \,-\, (V_w \,-\, V_1)]$. Or $y_1^2/2 \,-\, y_2^2/2 \;=\; y_1/g\,(V_w \,-\, V_1)\;[(V_1 \,-\, V_2)]$, or if we further simplify—instead of, you see— *y2* can be written as *y2* can be written as this one, and *V2* was *V1*, you see. *V2* was $V \,-\, \Delta V$. So, if we bring in the same transformation here, This will give us $\delta y \;=\; \delta v/g\,(V_w \,-\, v)$, or $\delta v/\delta y \;=\; g/(V_w \,-\, v)$. This is the second equation. We see the first equation using the momentum continuity equation and the second one using the momentum equation.

So, if we equate 1 and 2, we get $(V_w \,-\, v)/y \;=\; g/(V_w \,-\, v)$, or $(V_w \,-\, v)^2 \;=\; gy$, or *v* this is nothing but celerity. So, $c^2 \;=\; gy$, as celerity is equal to plus or minus the root gy, and this is the celerity of the negative wave. If we use this equation, In this, we can get $\delta v/\delta y \;=\; g/(V_w \,-\, v)$, or $\delta v/\delta y \;=\; g/\sqrt{gy}$, or this equation: $\frac{\delta v}{\delta y} = plus\ or\ minus\ under\ root$ root g square divided by gy or $\delta v\;/\;\delta y \;=\; \pm\,(\sqrt{g\;/\;y})$. So, we know as *δy* goes to 0, we get $dv\;/dy \;=\; \pm(\sqrt{g/y})$. This is simple integration, differentiation and integration that is going on. Now, this is the basic differential equation governing a simple negative wave which on integration with proper boundary conditions enables the determination of characteristic of negative wave. So, two important equations And second one was *c* is equal to $(\sqrt{gy})$.

Now, this was the basic negative wave. We are going to talk about negative wave moving downstream. So, if we consider a sluice gate in a wide rectangular channel passing a flow with a velocity of *V1*. and a normal depth of flow of *y1* in a channel downstream of the gate. So, there is a sluice gate, channel is wide rectangular, velocity *V1* and a normal depth of flow *y1* and the channel, so the flow is towards the downstream of the gate.

Now, if we consider a sluice gate to partially close instantaneously, so what will, it will negative surge. Let the new velocity and depth of the flow at the gate be V_0 and y_0 respectively. The closure action of the gate would cause a negative wave to form on the downstream. This is what you have written, generate a negative surge.

So, this closure action of the gate would cause negative surge to form on the downstream of the channel. and this wave would move in the downstream direction. So, which direction? Downstream. So, negatives are moving downstream, an example, something like this.

So, V_2, y_2, V_y, V_1, y_1 and this is So, the velocity *v* and depth *y* at any position *x* from the gate is obtained by integrating the basic differential equation of a simple negative wave. So, if we integrate this, but you remember this equation. in previous slides when we were discussing about elementary negative wave. elementary negative wave.

For the negative wave moving downstream positive sign in the basic differential equation is adopted and the resulting basic differential equation is so we adopt positive sign because it is moving in downstream direction since it is moving in positive sign why because it is moving downstream. and x positive in the downstream. So, we write $dv/dy = (\sqrt{g/y})$ and on if we integrate this we get $v = 2(\sqrt{gy}) + constant$. Now, we use the boundary condition, $v = v1$ at $y = y1$. So, if we do that, we write the value of constant that the $v1 = 2(\sqrt{gy1}) + constant$ therefore, constant is equal to this.

And then now we substitute the value of constant into the equation and we get $v = v1 + 2(\sqrt{gy}) - 2(\sqrt{gy_1})$. Since wave travels downstream, celerity c is $V_w - v$ that is $(\sqrt{gy})$. Hence, we can write vw is nothing but $v + (\sqrt{gy})$. So, this $vw = v + (\sqrt{gy_1})$. And now if we use this vw can be written as $v1 + 2(\sqrt{gy}) - 2(\sqrt{gy_1}) + (\sqrt{gy})$.

And therefore, this will become $2gy$ and this gy will become $3(\sqrt{gy})$. So, if the gate movement is instantaneous that is at $t = V_0$ at $t = 0$ V_w is in the direction of positive *x* and hence the profile of the negative wave surface is given by V_w is dx/dt. So, V_w can also be written as dx/dt that is $x = V_w * t$. So, *x* can be written as $(v1 + 3(\sqrt{gy}) - 2(\sqrt{gy_1})) * t$. And this is the expression for the profile of the negative wave in terms of *x*, *y* and *t*. This equation is valid for the values of *y* between y_0 and y_1. What is y_0?

So, from 1 we can write previous equation $(\sqrt{gy}) = 1/2\,(v - V_1 + 2(\sqrt{gy_1})$ that is from the previous equation, this equation. And if we use this in this equation, we get $x = [V_1 + 3/2\,(v - V_1 + 2(\sqrt{gy_1}) - 2(\sqrt{gy1})] * t$. or *x* = you see there will be simplification because $v1 - 3/2\,v1$ this will become $-V_1/2 + 3/2\,v + (\sqrt{gy_1})$or $x = 2/3 * x/t + V_1/3 - 2/3\,(\sqrt{gy_1})$. See this is initial initial initial means at the beginning initial condition initial condition right and this is downstream

So, we can determine any velocity v if we know the things at the beginning of the this surge. Similarly, I mean like we have this positive surge moving downstream, sorry we have a negative surge moving downstream, we have now negative surge moving upstream. So till now we have dealt three cases, positive surge moving from the beginning of this module, positive surge moving downstream, positive surge moving upstream. Third one was negative surge moving downstream. Now the fourth one that is negative surge moving upstream.

We remember if we talked about it, we said that there are four cases and this is the fourth case. So in this case, so this shows a negative surge produced by, how it is produced? Instantaneous raising of a sluice gate located at downstream end of the horizontal frictionless channel. So important thing is that the sluice gate is located at downstream end, right? and this is instantaneously raised, generate negative search.

Now, the negative wave which starts at the gate is shown moving upstream. So, this is downstream and this surge here. Surge moving upstream. Now, the basic differential equation is given by, so you remember we had this $dv/dy = plus\ minus\ (\sqrt{g/y})$. So, for negative surge moving downstream, we had positive sign So, therefore, we will have negative sign for negative negative surge moving upstream

On integrating, we get v is equal to minus $2\sqrt{gy}$ plus constant. We will adopt the same process as we did for the negative surge moving downstream. Now, if we use suffixes 1 and 2 to denote conditions before and after the passage of the wave, And using the boundary conditions $v = v1$ at $y = y1$, similar treatment, we get $V_i = V_1 + 2(\sqrt{gy_1} - 2(\sqrt{gy}$, as we were analyzing previously. Now, wave solidity is given by the same $V_w + v = (\sqrt{gy})$ or $V_w = (\sqrt{gy}) - v$, and if we put this in one, we get $V_w = (\sqrt{gy}) - V_1 - 2(\sqrt{gy_1}) + 2(\sqrt{gy})$. Or $V_w = 3(\sqrt{gy}) - 2(\sqrt{gy_1}) - V_1$. This is the equation. Now, if we consider the same thing, vw can, instead of $+dx/dt$, we write -dx/dt, the profile of the negative wave is $-x = V_w t$. And therefore, we can transform that equation as $-x_i = 3(\sqrt{gy})$, so this is nothing but vw. And this vw comes from before, you see this one.

So, we can also write x can be $V_1 + 2(\sqrt{gy_1}) - 3(\sqrt{gy}) \times t$. This is the initial condition. Any depth y and this is at any time t. So, we dealt with four cases: the first one was positive surge moving downstream. The second case was positive surge moving upstream. These two cases were also called moving hydraulic jump. The third case was a negative surge moving downstream, and the fourth case was a negative surge moving upstream. And these were the four types of surges that are rapidly varied unsteady flow. So, I think this is enough for today's lecture. In the next lecture, we will start with a dam break problem and then continue with some problems in the next class.

Thank you so much.