Free Surface Flow

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Lecture 51

Hello students, today we are going to start a new module, which is rapidly varied unsteady flows. Rapidly varied unsteady flows are the second category of unsteady flows. The first one was gradually varied unsteady flow, which we studied last week. There, we covered the theory, solved some of the derivations, and also saw the progressive wave, the concept of which will be utilized further for the solution of rapidly varied unsteady flows. With that being said, we will start the topic right away.

So, the rapidly varied transient phenomenon you remember we said that transients, or rapidly varied unsteady flows, are also called transients. The rapidly varied transient phenomenon in an open channel is called a surge. This term we saw in the last class as well. It occurs whenever there is a sudden change in the discharge, depth, or both. This is very different from gradually varied unsteady flows because here everything changes suddenly or the process happens over a shorter distance.

These types of situations occur, for example, during the sudden closure of a gate. If you close the gate suddenly, this type of situation might occur. So, a surge that produces an increase in depth is called a positive surge, and the one that causes a decrease in depth is called a negative surge. So, what we can understand is that the rapidly varied unsteady phenomenon in an open channel is called a surge. This surge is positive when there is an increase in water depth, and this surge is also can cause decrease in water level and then it will be called as negative surge. So, this is about the surge causing either increase and or decrease in depth. However, the surge can travel, right. It can travel to the right, to the left, to the forward or to the backward or in morely in terms of commonly said in terms of rivers and streams, it can travel either upstream or downstream direction and thus giving rise to four basic types, right. So, it can travel upstream or downstream.

Now, there is a positive surge, there is a negative surge, there is a surge travelling upstream, there is a surge travelling downstream. These 2×2 combinations will give us 4 different types of surges. That is 2×2. positive, negative, upstream, downstream. So, this is a diagram of a positive surge moving downstream.

So surge is positive because it increased, it caused an increase in water depth. And you see the direction of the surge is towards downstream. So, this is increasing the surge is called increase in the water depth and it is moving towards right that is towards the downstream and therefore, it is called positive surge moving downstream. So, this is type 1.

Similarly, the positive surge is moving upstream. So, this surge here is downstream. Upstream. So, this is a positive surge because it has raised the water level, but it is moving upstream. So, it is a positive surge moving upstream, and this is type 2, type 1, type 2.

Thirdly, there is a negative surge that is moving downstream. So, you see the movement. So, how do we know the movement? The movement is known by this. Do you see it is moving? The movement is downstream, right? And you see, compared to the normal water level, there is a decrease in water depth.

So, this is a negative surge because it caused a decrease in water depth, and it is moving downstream. So, a negative surge moving downstream this is type 3. The last one, again, you see this surge is moving. This is the surge. Now, this is moving upstream.

And this has caused a decrease in water depth, and therefore it is a negative surge moving in the upstream direction. So this is type 4. So positive waves generally have steep fronts and are stable. So positive surges, you can also say positive surges generally have steep fronts and are stable.

This is important to note. Consequently, they can also be considered uniformly progressive waves. You remember we studied uniformly progressive waves in our last lecture. We read about uniformly We read about uniformly progressive waves.

Negative surges, on the other hand, are unstable. So positive surges are stable, negative surges are unstable, and therefore their form changes with the advancement of the surge. In the case of positive surges, their shape is stable; they do not change. Now, being a rapidly varied flow phenomenon, friction is usually neglected, as we have discussed in our previous module as well, that friction is usually neglected in this simple analysis of surges. The reason is, this phenomenon occurs over a short distance of time.

And you know, it is $\tau_0 Pdx$. So, this is very, very small. So, therefore, friction is neglected here. So, the important message from this particular slide is that positive surges are stable, whereas negative surges are unstable, and their form continuously changes with the advancement of the surge. Positive surges can be considered to behave like uniformly progressive waves in the analysis of a rapidly varied flow.

Friction is neglected because this process occurs over a short distance of time, as in any rapidly varied flow, whether it is rapidly varied steady flow or rapidly varied unsteady flow. Now, we will analyze one by one. Positive surge moving downstream first. So, if we consider a sluice gate in a horizontal, frictionless channel horizontal means there is no slope, and frictionless, of course, means no friction is present the channel is suddenly raised to cause a quick change in depth and hence a positive surge traveling down the channel. So, there is a sluice gate resting on a horizontal, frictionless channel, and it is suddenly raised up, which will cause a quick change in the depth.

The water will flow down, flow out of it, and the depth will increase. Decrease, and hence, a positive surge traveling down the channel will occur, something like this: you see, this is a sluice gate; this is section 2; section 1 is horizontal. And frictionless as well. See, this is a positive surge, and it's moving in this direction with V_w. All right.

Now, suffixes 1 and 2, as I have referred to the conditions before and after the passage of the surge, respectively. So, the overall phenomenon of surge is happening here. Let me just say it in this way. So, the absolute velocity V_w of the surge can be assumed to be constant. So, V_w will be assumed to be impractical.

In practice, it might not be true. But what we will assume is that V_w is constant. The unsteady flow situation is brought toward I mean, we remember in our previous module we brought the system to rest by applying the opposite V_w to the system or the same magnitude of velocity V_w but in the opposite direction. So, the unsteady flow situation is brought to a relatively steady state by applying a velocity, -V_w to all the sections, and this is done so that we are able to apply Newton's law of motion. Something like this, you see.

So, after we have applied this velocity, at section 2 becomes $V_w - V_2$, the velocity at section 1 becomes $V_w - V_1$. This is depth y_1, this is depth y_2, this is actually section 2, this is section 1, and this is the control volume marked by the dotted line. And this is the equivalent steady flow diagram for this particular situation. So, in view of the possible loss of energy between section 2 and 1 in the equivalent steady motion, the linear momentum equation is applied to the control volume enclosing the surge to obtain the equation of motion. We made the unsteady flow into a steady flow by applying minus V_w to the system, and therefore, we are able to apply the linear momentum equation to the control volume. This is the control volume marked by the dashed line.

So, by continuity equation we know that $A_1V_1 = A_2V_2$, you see $A_1\ V_1 = A_2\ V_2$. So, we will write $A_2(V_w - V_2) = A_1(V_w - V_1)$ simple. Now, the momentum equation through the assumption of hydrostatic, see at section 1 and 2, since the situation has become steady, because of application of this velocity $V_{w.}$ we can apply and have this assumption of hydrostatic pressure distribution at both the sections 1 and 2. And if we apply that, we will get $\gamma A_1 \overline{y_1} \quad \gamma A_1 \overline{y_1}$.

These are the hydrostatic forces $F_1 - F_2$ = Momentum Flux So, $\gamma A_1 \overline{y_1} - \gamma A_2 \overline{y_2} = \gamma / g\, A_1 (V_w - V_1)[(V_w - V_2) - (V_w - V_1)]$ And, from this particular equation, first equation, we can write so just simply utilizing and writing in terms of V_2 in terms of V_1 and V_w. So, we what we do is we write V_2 in terms of V_1 and V_w from this equation and the result is this particular equation $V_2 = A_1/A_2\ (V_1) + (1 - A_1/A_2)\ V_w$. Now, if we substitute this relation in equation 2, that is this particular. So, wherever there is V_2 here, we substitute this equation. And if we do that, we will write $(V_w - V_1)^2 = g\ (A_1/A_2) \times (1\ /\ (A_2 - A_1))(A_2 \overline{y_2} - A_1 \overline{y_1})$.

This will give us equation number 3 or $V_w = V_1 + \sqrt{g\ (A_1/A_2)(A_2 \overline{y_2} - A_1 \overline{y_1})/(A_2 - A_1)}$. Since, surge is moving downstream V_w. So, if the surge is moving downstream $V_w - V_1$ is positive sign of the square root is considered integral. So, we are going to because this will be both right this is a quadratic equation right. So, why we are considering positive root because the surge is moving downstream and $V_w - V_1$is a positive ok. So, for a rectangular channel, if we consider our channel as rectangular and considering unit width of the channel, the continuity equation can be written as $y_1(V_w - V_1) = y_2(V_w - V_2)$.

So, here is = y_2 because B gets cancelled out. in case of rectangular channel. And momentum that is what that was equation number 2 can be simplified as this equation, this momentum equation is this one. $1/2\gamma\ y_1^2 - 1/2\gamma y_2^2\ =\ \gamma/g\ y_1(V_w - V_1)(V_1\ -\ V_2)$. and from this particular equation, we can write V_2 as same process.

Actually, if we instead of A_1 and $A_{2,}$ if we put A_2 as By_2 and A_1 as By_1, we are going to get what we are rectangular channel use A_2=By_2 and A_1=By_1,. This is what exactly we are doing it from the initial basic. So, $V_2 = (\ y_1/y_2)\ V_1\ +\ (1 -\ y_1/y_2)\ V_w$ and this we will substitute in this equation. Substituting for equation 4 and on simplifying we get $(V_w - V_1)^2/\ g\ y_1\ =\ 1/2\ \times\ (\ y_1/y_2)\ (\ (y_1/y_2)\ + 1)$.

Now, the equation sets 1, 3, 4, and 6 contain five variables: y_1, y_2, V_1, V_2, and V_w. So, if three of them are known for example, there are five: y_1, y_2, V_1, V_2, and V_w. If three of them are known, the other two can be evaluated. In most cases, trial-and-error methods have to be adopted. So, this was the general equation, but the general equation is this one.

And this was for a positive surge a positive surge moving downstream. Now, we will see a different case. We will see You will see a positive surge moving upstream. This is case 2. So, this kind of surge occurs upstream of a sluice gate when the gate is closed suddenly. The first one happens when we open the gate suddenly, but the positive surge moving upstream will occur when the gate is closed suddenly, and in the phenomenon, It also happens in the phenomenon of tidal bores. Tidal bores what exactly they are is out of the scope of this course. But sluice gates you have studied a lot about them in hydraulics and other hydraulics courses. So something like this. You see this is moving upstream. surge moving upstream and this has caused. So, similarly, this is section 2. This is section 1.

So sub-suffixes 1 and 2 refer to the conditions at sections of the channel before and after the passage of the surge respectively. So, the unsteady flow is converted into the same process that we did for the positive surge moving downstream. Here also the unsteady flow is converted into equivalent steady flow by the superposition of the velocity V_w directed downstream same process and it will look something like this. So, this will become $(V_1 + V_w)$ and this one will become $(V_2 + V_w)$ because the surge was moving in this direction and negative So, surge direction is this.

Therefore, superimposition direction will be this. And therefore, it will be added in both the cases both with V_1 and V_2. So, if you consider the unit width of a horizontal frictionless rectangular channel, let us consider from the beginning we have a rectangular channel, the continuity equation will be $y_1(V_1 + V_w)= y_2\ (V_2 + V_w)$. So, basically $A_1 V_1 \ = \ A_2\ V_2$ right and $By_1(V_1 + V_w)= By_2\ (V_2 + V_w)$ And this B and B will get cancelled out and therefore, we will get equation number 1 that is $y_1(V_1 + V_w)= y_2\ (V_2 + V_w)$.

Now, it is seen that the equivalent flow is similar to a hydraulic jump with an initial velocity of $(V_1 + V_w)$ and an initial depth of y_1 . The final velocity is $(V_2 + V_w)$, and the depth after the surge is y_2. So, this is equivalent to a hydraulic jump. This is an important observation. Observation. So, this is a hydraulic jump. Equation also. But we apply the momentum equation and we get $1/2\gamma\ y_1^2 \ - 1/2\gamma\ y_2^2 \ = \ \gamma/g\ y_1(V_w + V_1)[(V_w + V_2) - (V_w + V_1)]$. So, basically, this is momentum flux, and these are the pressure forces. And this will reduce to $\gamma/g\ y_1(V_w + V_1)(V_2 \ - \ V_1)$. If we use the previous equation, V_2, if you use the previous equation, this one, we can write V_2 in terms V_1 and V_{w1}.

This is what we have done here. And if we put this equation in this equation and substitute this relation, we can simplify equation 2 as $(V_w + V_1)^2/gy_1 \ = \ 1/2 \ \times \ (\ y_2/y_1)\ (\ (y_2/y_1)\ + 1)$. Now, from this equation, this equation number 3. Two of the five variables y_1,

y_2, V_1, V_2, and V_w can be determined, the same thing if three other variables are given. It is to be remembered that in real flow, V_w is directed upstream.

The velocity V_2, however, may be directed upstream or downstream depending on the nature of the Bohr phenomenon or the search phenomenon. But the maths will take care of it. So, we discussed two cases till now. The first case was a positive surge moving downstream, and we used the continuity equation and momentum equation. In the second case, we analyzed a positive surge moving upstream. And, in both cases, what we observed was we have three variables: y_1, y_2, V_1, V_2, and V_w. We have only two equations. So, if we know three of these variables in our system, we can determine.

There is something called a moving hydraulic jump. So, this is how a moving hydraulic jump will look. I will come. So, the type 1 and type 2 surges that is, positive surge moving downstream and moving upstream, respectively are often termed as moving hydraulic jumps. So, the two previous analyses that we have just done are referred to as moving hydraulic jumps.

And this is done in view of their similarity to a steady-state hydraulic jump in horizontal channels. So, this will be clear from the study of the simulated steady-flow situations of the above two types of flows if we do simulations. So, this similarity could be used advantageously to develop shortcuts to predict some flow parameters of the positive surge phenomenon. Basically, sometimes it is also defined as surges are also defined as moving hydraulic jumps. But there are four types of surges, but the first two type 1 and type 2 are called moving hydraulic jumps.

So if the velocity is relative to the wave velocity V_w, this is the surge velocity V_w, which is the flow situation as it would appear to an observer moving along the surge with velocity V_w. The relative velocity at a section V_{r1} can be represented as follows. We are going to show that. So, we have written $V_{r1} = (V_w - V_1)$. We are writing V_{r1}, V_{r1} is equal to, so for type 1 surges, surge moving downstream, V_{r1} can be written as $(V_w - V_1)$, which we have already seen before. And for type 2 surge, V_{r1}, type 2 means moving upstream, V_{r1} will become $(V_w + V_1)$, which we have also seen before.

So with this notation, both figures can be represented by a single figure like this. This is the moving hydraulic jump, which is essentially the same as that of a steady flow hydraulic jump. This is something, you see, this is the process of is the process of hydraulic jump. Further, the equations obtained by the application of the momentum equation in type 1 and type 2 cases can be expressed by a single equation, which we will show below by considering the two surges as a moving hydraulic jump.

So, we can always write this particular equation $(V_{r1})^2/\ g\ y_1 = (F_{r1})^2\ =\ 1/2\ \times$ $(\ y_2/y_1)\ (\ (y_2/y_1)\ +1)$. Similar to the hydraulic jump equation, which is the same form as the steady hydraulic jump, using this similarity of form, we can also find the ratio of (y_2/y_1) of the moving hydraulic jump. So, what we have done until now is we have tried to find out the similarity between these surges and the hydraulic jump, and we will use the equations derived for the hydraulic jump for the moving hydraulic jump as well. Now, which is equivalent to the sequent depth ratio of a steady state, which is given by, so y_2, this is the normal one, normal, just that this F_{r1} this F_{r1} in the case will be found using this equation. The energy loss also in the moving hydraulic jump would be similar, will be similar to the steady state hydraulic jump.

And the formula will be given by $E_1\ =\ (y_2 - y_1)^3/4y_1y_2$. We have just brought out the similarities. Which is independent of F_{r1}. Note that equations 1 and 2 are applicable to both type 1 and type 2 surges when the relative surges when the relative velocity v. So, what we have to do is adopt a proper V_{r1} for $V_{r1} = (V_w - V_1)$ for positive surge moving downstream and for type 2, $(V_w + V_1)$. So, I think this is a good point to stop the lecture here.

In the next lecture, we will start with the celerity of the surge. Thank you so much. See you in the next class.