Free Surface Flow
Dr. Mohammad Saud Afzal
Department of Civil Engineering
Indian Institute of Technology Kharagpur

Lecture 50

Welcome back students to the last lecture of our gradually varied unsteady flows where we are going to discuss a very specific numerical technique and we will solve a problem about that as well. In the last class, if you remember I introduced about the numerical methods for Saint Venant's equation and we said there are two, one is approximate method, one is complete numerical method. And in complete numerical method, there are direct methods, there is a method of characteristic, there is finite element method and then we further divided and subdivided into implicit and explicit scheme. So, today we will be concentrating more on the direct numerical method with implicit technique. But we will talk about little bit talk about direct method also and what the explicit technique does and what does implicit technique does.

So, just something about direct numerical method. So, a wide variety of finite difference schemes exist for solving Saint-Venant equations. a few of these are very common. in this finite difference approximations to the partial derivatives in the x_t plane is done. The substitution of these approximations to partial derivatives in the Saint-Venant equations result in algebraic equations for the unknowns. In these schemes, Δt and Δx values are fixed to have a rectangular grid in the So, this is, in general, how the direct numerical methods work. So, direct numerical methods are of two types: one is explicit, and one is implicit. So, the first is the explicit method in the explicit finite difference scheme, the Saint-Venant equations. Saint-Venant equations are converted into a set of algebraic equations in such a way that the unknown terms what are the unknown terms? *v* and *y* at the end of the time step are expressed by the known terms at the beginning of the time step. So, this is, in general, about the explicit method. So, in this, the same equations are converted into a set of algebraic equations, so that the unknown terms at the end of the time step are expressed by the known terms. So, on the left-hand side will be the unknown terms, and on the right-hand side will be the known terms. Just a brief idea.

Now, the implicit method. In implicit finite difference schemes, the partial derivatives and the coefficients are replaced in terms of values of the variables at known and unknown time levels of the nodes. of an elemental cell of size Δt and Δx. Secondly, the unknown variables

therefore appear implicitly. Implicitly means appearing on both the left-hand side and right-hand side of the equation. I will write equations. The set of algebraic equations for the entire grid system will have to be solved simultaneously; you have to solve it simultaneously in these methods.

Because of the large number of time steps required with a large number of time steps required by an explicit method to route a flood in a channel, implicit methods can use large time steps without any stability problems. So, explicit methods may overcome may have this without any stability problems. So, several implicit, several implicit finite difference schemes have been proposed, okay.

Important ones. Preissmann, this is the one that we are going to study: Amein, Strelkoff, Abbot. So, Abbot and Lonesq beam and Warming. So, the most famous one is the Preissmann scheme, and that is one of our targets to study. Preissmann scheme. It uses a 4-point weighted method at point P. What is point P? So, let us say this is So, this is let us make a grid. So, let us say this is point P. This point, this point, this point, this point. So, let us say this is i, j. This is $i+1, j$. This is $i+1, j+1$.

This is $i, j+1$. So, and this whole is Δx, this whole is Δt, this is time, this is distance x time t. So, for a given variable M, M can be such as depth let us say, for example, depth y, stage h, or it can be anything discharge Q. A weighting coefficient is used coefficient is used to approximate the derivatives, and the coefficients are as follows. the time derivatives $\delta m/\delta t$ are is not equal to but approximation $(M_i^{j+1} + M_i^{j+1} - (M_i^j + M_{i+1}^j) / 2\,\Delta t$. The space derivatives are $\alpha\,(M_{i+1}^{j+1} + M_i^{j+1})/\Delta x + (1 - \alpha)(M_{i+1}^j - M_i^j)/\Delta x$. the value of M as a coefficient is certainly equal to $\frac{\alpha}{2}(M_{i+1}^{j+1} + M_i^{j+1} + \frac{(1-\alpha)}{2}.(M_{i+1}^j - M_i^j)$ where i and j are an x and y respectively.

The value of α is equal to $\Delta t' /\Delta t$ locates the point P along the time axis in the finite difference grid. the Preissmann scheme is unconditionally stable for if α lies between 0.50 and 1. for typical applications a value of α in the range 0.55 to 0.70 is recommended in order to avoid higher order numerical oscillations.

Now, consider the Saint Venant equation in the form with the discharge. So, now we consider a Saint-Venant equation with discharge Q function of x , t and stage means and stage h as function of x and t as the dependent variable. dependent variable. So, we use $S_f = (Q\,|Q|)/ K^2$.

We can write continuity as $\delta h /\delta T + (1 /T) \delta Q/ \delta x = 0$ and momentum as $\delta Q/ \delta T + \delta/\delta x (Q^2/A) + g A \delta H/\delta x + g \times A (Q |Q|)/ K^2 = 0$ and the application of the Preissmann schemes, the derivatives and coefficient results in $\delta H/\delta T = (h_i^{j+1} + h_{i+1}^{j+1}) - (h_i^j + h_{i+1}^j)/2\Delta T$ and $\delta Q/\delta t$ approximately results in $(Q_i^{j+1} + Q_{i+1}^{j+1}) - (Q_i^j + Q_{i+1}^j)/2\Delta T$ and $\delta Q /\delta x = \alpha (Q_{i+1}^{j+1} - Q_i^{j+1})/ \Delta x + (1 - \alpha) (Q_{i+1}^j - Q_i^j) / \Delta x$. So, what we have seen is that we have applied continuity and momentum equation and use the implicit Priessmann scheme and we have obtained these values. So, what we are going to do very briefly, we are going to see one problem. So, question is determine the time derivative and space derivative of the flow rate Q by using Priessmann scheme. alpha is given 0.65 and discharge is given as below. x is equal to 1000 meter and x is equal to 1500-meter time at 3 hours at 4 hours. Q is given as 125 m³/s, here Q is given as 140 m³/s, here Q is given as 115 m³/s, here it is given as 120 m³/s. These are the distances upstream and downstream.

So, what we do here is, we say Δt is 1.5 hours or 5400 seconds, and Δx is the difference of 500 meters. So, Q_i^j is 125 m³/s, Q_{i+1}^j is 115V, Q_i^{j+1} is 140 m³/s, and Q_{i+1}^{j+1} is 120 m^3/s. $\delta Q/\delta t$ is calculated using these formulas here: this one, this one, and this one. We will use these formulas and simply start writing down the values: $(140 + 120) - (115 + 125) /(2 \times 5400)$, and that comes to 0.01852 m³/s/m.

Similarly , $\Delta Q/\Delta X$ α was $0.65 \times (120 - 140)/500 - (1 - 0.65)/500 \times (115 + 125)$. 0.019 m³/s/m. So, these are the answers a simple problem, a direct application problem. The formulas used were given by the Preissmann scheme, which we applied for the continuity and momentum equations. And therefore, this resulted. So, we saw this Preissmann scheme, which is implicit, and we worked through a problem. I think that will be the end of today's lecture and this particular module.

I will see you again in the next module, which is about rapidly varied and steady flows. Thank you so much.