Free Surface Flow
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Lecture 5

Welcome students to yet another session. This is the fifth lecture of our module one that is introduction to basic concept. In next 30 minutes or maybe even an hour, we will try to solve some of the basic problems that are relating to the introduction of the basic concepts. Basic concepts are mostly from our course hydraulics and fluid mechanics.

So, all right. So, we will go to start the first problem right away. So, the first problem is the question is the velocity distribution in a rectangular channel of width *B*. So, the rectangular channel has a width *B* and depth of flow yo was approximated as *v* is a function of depth and here *k1* is a constant.

We need to calculate the average velocity. So, this is something that is very very important when we come to fluid flow. We will calculate the average velocity for the cross section and also correction coefficient alpha and beta.

So, this is our problem very simple and the way we approach this problem will be you know I will start writing you know. So, we are given the width is *B* and depth is flow is depth of the flow is yo So, first thing area of cross section option A will be very simple, it is a rectangular channel. So, it will be By_o and from we know that how the formula for average velocity in general

dv is $\frac{1}{By_o}\int_0^{y_o} v\left(Bdy\right)$. and which is so $1/y_0$, *B* and *B* gets cancelled, 0 to y_0, K1 we write v as $\frac{1}{y_o}\int_0^{y_o} k_1\sqrt{y}dy$ that becomes *ydy*. and if we integrate this we get $\frac{2}{3}K_1\sqrt{y_0}$. This is the average velocity, very simple to calculate and the key step here is this formula. Now, we also need to find the kinetic energy correction factor alpha and momentum correction factor that is beta.

So, we will go to the next slide. kinetic energy correlation factor. So, alpha is, it is a formula, $= \frac{\int_0^{y_o} V^3 (Bdy)}{V^3 B y_o}$.

And then we will substitute the value of velocity and velocity V is $K_1 \sqrt{y_o}$. So, this will become $= \frac{\int_0^{y_o} K_1^3 y^{3/2} Bdy}{\left(\frac{2}{3} K_1 \sqrt{y_o}\right)^3 B y_o}$, this is velocity. cube, this is capital V, which we found out in the previous slide, By_o. And if you solve this, so y^3 when you integrate this and you try to solve this, you are going to get a value of 1.35. If you remember, in the lectures, we told that this value will always be greater than 1.

The higher the non-uniformity, the higher the value of alpha and beta. Similarly, we will try to some correction factor which is beta The formula of beta is $= \frac{\int_0^{y_o} V^2 (Bdy)}{V^2 B y_o}$ note the difference in kinetic energy it is cubic. So, I think this got yeah.

So, and in the momentum it is V^2 and this capital V is nothing but the one that we found out in the previous slide. $= \frac{\int_0^{y_o} K_1^2 y Bdy}{\left(\frac{2}{3} K_1 \sqrt{y_o}\right)^2 B y_o}$. And if you solve this, you are going to get a value of 1.125. And it is also again important to note that this value is greater than 1.

So, in for the ideal cases uniform flow these correction factor and with 0 slope these correction factors are exactly equal to 1. So, we solve another problem. So, first for this particular problem, we will see there is a diagram first. So, this is the sea bed, this is the river bed or the bed. And this is this distance is given as capital D. And then we have you know the.

So, this is u u u and so let us say this is y and this distance is this is *aD*. So, this is called the velocity distribution. This velocity distribution is given to us. Now, the question is this is the for this velocity this particular velocity distribution we have to determine the kinetic energy kinetic correction factor alpha and momentum correction factor beta for this profile ok.

So, this is the question all right. So, the way that we do is, so I will start doing the solution from here. So, from this figure, we can see that velocity, this velocity into depth is $u \times (1 - a \times D)$. So, this is the key step. to write it down in this particular way. And then again it is just simple this is application of the formula and see now you see the this starting point is no longer 0.

It is at $a \times D$ and it goes up to depth D and $\frac{\int_{aD}^{D} u^3 dy}{V^3 D}$ or if we substitute this u. So, if using this particular formula we substitute u into the alpha then we get $\frac{u^3(D - aD)}{u^3(1-a)^3 D} = \frac{1}{(1-a)^2}$. This is kinetic energy correction factor. Again, for beta same thing, same formula starting from aD to D, $\frac{\int_{aD}^{D} u^2 dy}{V^2 D}$. And if you substitute the value of V in terms of a small u and integrate you get $\frac{u^2(D - aD)}{u^2(1-a)^2 D} = \frac{1}{(1-a)}$. So, this is momentum correction. Now, you see this a whatever small value will it have both the denominator will be less than 1. Therefore, this particular quantity will be greater than 1 and same is for alpha as well. So, next problem.

The width of a horizontal rectangular channel. It is important to note as see there are many steps or many things observations then you will understand while reading the question. So, first clue here what I mean whatever whatever, I have written you see it is a horizontal rectangular channel. Horizontal means there is no bed slope, rectangular means it is a rectangular cross section is determined. So, it says the width of the horizontal rectangular channel is reduced.

From 3.5 meter to 2.5 meter and the floor is raised by 0.25 meter in in elevation at a given section at the upstream section the depth of flow is 2 meter. So, basically they have given y_1 alright and the value of alpha also is given 1.15 ok. Now, it says if, the drop in water surface see when the width of the horizontal rectangular channel is reduced that means it will have a contraction right and the drop in the water surface elevation at the contraction is 0.2 meter right. It says calculate the discharge for following cases. What are the following cases?

First case is the energy loss is neglected. Second case is the energy loss is 10 percent of the upstream velocity head. So, one assumption that can be taken is that alpha can be taken as uniformly 1.15 everywhere.

So, this is the entire question. So, what should be the first step? So, the first step should be to draw the diagram of this particular question that will ease our problem. So, we will start I mean you can even I mean so what it means the first thing see there is something like this and then there is a contraction right something like this and this one is 3.51 that is B_1 and this one B_2 is 2.5.

This is the first line that we have drawn and the second thing is we are going to draw a bed. and this is the water this is the line for the water this is the entire thing is 2 meter 2 meter and we this is y_2 right and this drop is 0.2 m right this is what it says. So, this will ease our problem a lot right and this is a step hold on. right. So, this is actually y_2 and this is 0.25 m yeah and this is the energy lines right.

So, this is something that we have taken clue from the question and this is just a rough diagram you can make a better diagram right. So, if we refer to this particular figure right and even from the question some things are very clear y_1 is 2.0 meter all right and you see from this geometrical what will be y_2? y_2 is going to be 2 - 0.25 - 0.2 meter that is that equals to 1.55 meter. And in these cases, we will simply apply equation of continuity.

Continuity equation is $B_1y_1V_1$ is equal to A_1V_1 is equal to A_2V_2. So, instead of A_1V_1, I am writing B_1y_1, $B_2y_2V_2$. So, let me write down A_1V_1 is equal to A_2V_2. So, it is much more clear to you.

Therefore, by equation of continuity V_1 will be $= \frac{2.5\times1.55}{3.5\times2.0}V_2 = 0.5536V_2$. This is V1 in terms of V2. This is general thing irrespective of our conditions. Now, what we are going to do, we are going to solve it for our first case, no energy loss. So, in that case, no energy loss, so we apply energy equation that is the Bernoulli's

Between where section 1. and section 2 just after the contraction. So, general equation is $z_1 + y_1 + \alpha_1 \frac{V_1^2}{2g} = (z_1 + \Delta z) + y_2 + \alpha_2 \frac{V_2^2}{2g}$, z1 + Δz because our datum is at the bottom. here we already know α_1 is 1.15 and α_2 is I mean you can take 1.15 actually but we can I mean I am just assuming 1.0 just the values will change. So, that does not make a much of a difference all right.

So, this problem is continued. $\frac{V_2^2 - 1.15V_1^2}{2g} = y_1 - y_2 - \Delta z$ or we say we substitute V_1 in terms of V_2, $\frac{V_2^2}{2g}\left[1 - 1.15\left(0.5536^2\right)\right] = 2 - 1.55 - 0.25$. Or if you simplify this, we get V_2 is equal to 2.462 meters per second. And therefore,

discharge Q is 2.5 x 1.55 x 2.462 that is 9.54 m^3/s and this was the desired quantity that we needed to find out. This was when there was no energy loss. Now, there is an energy loss that the part B is energy loss is 10% of the velocity. So, first thing we need to calculate head loss that is 0.1 of that what is this $\alpha 1 \frac{V_1^2}{2g}$ and this is the energy loss $0.115\frac{V_1^2}{2g}$.

So, we said that the loss is 10 percent and we apply the same equation. So, let me for your convenience, let me write down $z_1 + y_1 + \alpha_1 \frac{V_1^2}{2g} = (z_1 + \Delta z) + y_2 + \alpha_2 \frac{V_2^2}{2g} + H_L$ this term is important right for the conservation. So, now this is continued.

So, what we can write down is $\alpha_2 \frac{V_2^2}{2g} - \alpha_1 \frac{V_1^2}{2g} + H_L = y_1 - y_2 - \Delta z$. And if we substitute the value of alpha 2 is equal to 1, alpha 1 we can take 1.15 as well, but does not matter whatever see the values will just change. And head loss we have already found out 0.115 because it is 10 percent of So, if we substitute this in the above equation, we are going to find out $\frac{V_2^2}{2g} - 1.15\frac{V_1^2}{2g} + 0.115\frac{V_1^2}{2g} = 2 - 1.55 - 0.25$. And, since we know since you know from our previous slide before just starting we using the continuity equation we already found out this V_1 was $0.5536V_2$, we substitute this and what we get is $\frac{V_2^2}{2g}\left(1 - 0.9 \times 1.15 \times 0.5536^2\right) = 0.2$. And if we solve this, we get V_2 is equal to 2.397 m/s. And, discharge will again be A_1V_1 is equal to A_2V_2 or just simply $2.5 \times 1.55 \times 2.397 = 9.289\text{m}^3/\text{s}$. So, this is the discharge in the resulting case.

So, if you see in the first one, we had a discharge of 9.54 m^3/s and in the second case where we have energy loss, we have 9.29 let us say m^3/s and this is answer to our second part of the problem So, I thank you all and in the next lecture we will also solve some problems before we close this module. Thank you so much.