

Free Surface Flow
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Lecture 49

Welcome back, students, to another lecture of our Module 10, which is gradually varied unsteady flows. We were discussing uniformly progressive waves. So, in the last class, we were talking about—I am just repeating the slides—we wrote the continuity equation for a monoclinal wave that has been brought to rest by applying a velocity V_w in the opposite direction. And therefore, this Q_r was $A_1(V_w - V_1)$ is equal to $A_2(V_w - V_2)$ is equal to $A(V_w - V)$. And using this, we simplified V_w as dQ/dA . And we saw that the maximum value of V_w can be obtained by these equations.

And we concluded that V_{wm} is nothing but $K_w * V_n$, where V_n is the normal velocity. So, we said K_w is 1.67 for a wide rectangular channel, 1.44 for a wide parabolic channel, 1.33 for a triangular channel, and this equation serves to be quite useful in field conditions. Now, taking these steps further. So, we are going to use the previous equations, considering the equivalent steady-state flow. $d/dt (V_w - V)$ is equal to $d/dx (V_w - V) dx/dt + d/dt (V_w - V)$ is equal to 0.

V_w is equal to dx/dt is equal to constant. On simplification, we get dV/dt is equal to $-V_w dV/dx$. Also, since V is equal to $V_w - Q_r/A$, we can write dV/dx is equal to $QT/A^2 dy/dx$. So, if we substitute in the equation of motion, it can be expressed as $S_0 - S_f$.

dy/dx is equal to $S_0 - S_f - (Q_r^2)T/gA^2$. See, it is a gradually varied sort of condition, just in place of Q , it is Q_r . Now, just writing some characteristics, this is the above equation, this is the differential equation of a monoclinal rising wave. Note the similarity, which I have already explained before, with the differential

equation of gradually varied flow (GVF). So, how do we obtain the profile? The profile of the wave is obtained by integrating this equation. Which equation?

The previous equation, this equation. This equation can be simplified by considering the denominator to be approximately equal to unity for small velocities, that is, by neglecting the effect of the velocity head as dy/dx is equal to $S_0 - S_f$ or $S_0 * (1 - S_f/S_0)$.

or $S_0 (1 - (Q^2)/(Qn^2))$ where $k(S_0)^{0.5}$ which is equal to normal discharge at any depth y and Q is actual discharge and Qn is the actual discharge at that depth. So, on rearranging, what do we get? Q/Qn is nothing but $(1 - dy/dx)/S_0$. This is another important equation.

uniformly progressive wave for any point on the wave profile, what we can do is we dy/dx is equal to $dy/dt + Vw$ dy/dx is equal to 0 and hence dy/dx is equal to $- dy/dt$ divided by Vw . And if we substitute this in our previous equation, the above equation in the previous equation will lead to Q/Qn is equal to $(1 + (dy/dt)/VwS_0)$ This equation indicates that during the rising stages in a flood flow, the actual discharge is larger than the discharge by the normal stage discharge S_0 , this equation indicates that during the rising stages in a flood flow, the actual discharge is larger than the discharge read by the normal stage discharge relationship in this particular equation. So, now the rising stage, now we have to discuss about the falling stage. conversely during the falling stages in a flood flow, the actual discharge is lower than that indicated by Discharge curve equation above which equation this equation is used in hydrometry. To correct the normal discharges read from a stage-discharge curve when The depth is changing at a rate dy/dt . In using this Formula for natural channels.

If we want to use this formula for natural channels, it To assume that Vw is 1.4 times of Vn , where Vn is nothing but Qn/A in cases where Vw Is not known. Important to note also that the energy slope S_f is used in place of S_0 , So, So, this actually sort of concludes this uniformly progressive wave part, but we will continue with some classifications and some numerical methods for the solution of these types of gradually varied unsteady flow equations.

We talked briefly about numerical methods Will do some classification today in this lecture and then proceed later also. So, we have seen that gradually, even gradually varied unsteady flow has mainly one equation. What is that equation? Saint-Venant equation, right?

So, the solution of Saint-Venant equations by analytical methods has been obtained only for simplified and restricted cases. What are those? We have graphical solutions, Graphical solutions have been in use for a long time, but are seldom preferred these days. Why? The reason is the development of modern digital computers. has led to, so computers are becoming better and better every day, with numerical techniques. So, there are a variety of

techniques, each having its own advantages, disadvantages, stability, and different accuracy; those are the things. But all these schemes or techniques can be classified into two categories. So, what are those two categories? One is approximate

And the second is complete numerical methods. We are talking about approximate methods. They are based on equations of continuity and on a curtailed or shortened equation of motion.

So, approximate methods use the continuity equation to the fullest, but the equation of motion that is based on the momentum equation is drastically curtailed or shortened, okay? The storage routing methods popularly adopted by hydrologists, such as Muskingum, method, kinematic wave, and diffusion analogy belong to this category, whereas complete numerical methods aim to solve

the basic Saint-Venant equations through numerical modeling. So here, not only continuity but the entire Saint-Venant equation, which is based on the full momentum equation, is generally solved. Several individual methods under this category are available, and they can be further classified into Subclasses. So, this particular table mainly two classifications. One is the approximate method and the complete numerical method. Several approximate methods, one is the storage routing method, Muskingum methods, third is the diffusion analogy, fourth is kinematic wave models. In complete numerical, there are several, mainly three. One is the direct method, second is the method of characteristics, third is the finite element method. I will write the MOC is equal to and FEM is the finite element method.

In the direct method, there are two, implicit and the third one, second one is explicit, implicit and explicit. In the method of characteristics, there are two, one is characteristic nodes right and the second one is a rectangular grid. Both of these have two methods, implicit and explicit here also. implicit and explicit. So, this is the complete classification for dealing with numerical methods for Saint Venant's equation.

In the next so, I will be closing soon very soon this class, but we will be not going into much details in any of those, but in the next class we will learn about one new specialist technique. which is one of the implicit methods in direct method. And we will start the next class with talking a little bit about direct method and then concluding with the one maybe one solved example with that particular scheme. So, that is all for today. See you in the next lecture.

Thank you.