Free Surface Flow
Dr. Mohammad Saud Afzal
Department of Civil Engineering
Indian Institute of Technology Kharagpur

Lecture 48

Welcome back, students. In the last class, at the end, we started with a problem where we were trying to convert the Saint-Venant equations in terms of discharge. So, what we did was, just as a revision, the momentum equation of the Saint-Venant equation can be written with discharge as primary variables like this. So, this was the question.

We started by saying that if y is the flow depth and u is the velocity, we first wrote the continuity equation, this one. And then, in the next step, we wrote the momentum equation that we know. Now, we are going to start with the further steps, and the further steps are now. So, before summarizing, let me write down the important equation first: $dQ/dx + dA/dt$ is equal to 0. Also, the momentum equation as

S_0 - S_f is equal to $dy/dx + u/g\ du/dx + 1/g\ du/dt$. And proceeding further. du/dx can be written as $d(Q/A)/dx$, or we can write du/dx is equal to $A\ dQ/dx - Q\ dA/dx$ divided by A^2. So, what did we do? We wrote $d/dx\ (Q/A)$ as this, or du/dx is equal to $1/A$.

$dQ/dx - Q/A^2\ dA/dx$ or du/dx is equal to $1/A\ dQ/dx$. So, we know that we can write A is equal to Tdy. Using this here in A, we can write $QT/A^2\ dy/dx$, OK. $u/g\ du/dx$ is equal to $u/Ag\ dQ/dx - QUT/gA^2.\ dy/dx$. So, you see in this one, du/dx was this.

So, multiplying with u/g, we get $u/Ag\ dQ/dx - QuT/gA^2\ dy/dx$. Since u is equal to Q/A, we can write $u/g\ du/dx$ is equal to Q/gA^2. So, u we are replacing in terms of Q, $Q/gA^2.dQ/dx - (Q^2)T/gA^3\ dy/dx$. This is our objective to replace all u's in terms of discharge or $u/g\ du/dx$ can also be written as Q/gA^2 into dQ/dx. What is this? This is Froude number whole square.

So, we can write $-(F^2)\ dy/dx$. This is another important equation. du/dt is equal to $d/dt\ (Q/A)$. So, with this, we start the manipulation again for du/dt. We did for du/dx; now we will do the same for du/dt. So, we can write du/dt is equal to $(AdQ/dt - Q\ dA/dt)/A^2$. u is Q/A or du/dt will be $1/A.\ dQ/dt - Q/A^2\ dA/dt$. We also know

Continuity equation: what do we know? That $dA/dt + dQ/dx$ is equal to 0, or this is another equation. So, or we can write dA/dt is equal to minus dQ/dx. So, this equation will become du/dt is equal to $1/A\ dQ/dt\ -$ or this minus becomes this $+\ Q/A^2\ dQ/dx$. So, we use this in this. So, we use this equation in this, and we get this one, or we divide it by.

So, now *1/g du/dt* can be written as $1/Ag\ dQ/dt\ +\ Q/gA^2\ dQ/dx$. This is another important equation. Using all these equations in the main equation, which is the momentum equation. we get S_0 - *Sf* is equal to this is the same $+Q/gA^2\ dQ/dx\ -\ A^2\ dy/dx\ +\ 1/Ag\ dQ/dt.\ +Q/gA^2\ dQ/dx$. Now, this will get added. So, we can write S_0 - *Sf* is equal to *1/Ag dQ/dt* this term $+2Q/(A^2)g\ dQ/dx\ +\ S0$, *dy* and *dx* term also, *dy dx* terms, this term and this term we add. So, it becomes $(1 - F^2)$ common and *dy/dx*, and this is the thing that we had to prove.

So, if you go back and look, this was the condition that we needed to prove. Some features about uniformly progressive waves. So, this is a new sort of topic, but it will help us in understanding the transition from gradually varied unsteady flow to rapidly varied unsteady flow. Progress, so you know, uniformly The waveform is assumed to move with its shape unchanged.

So, in a uniform progressive wave, the waveform is assumed to move with its shape unchanged. So, there is no change in the shape of the waveform. A particular case of this type of wave is a monoclinal wave. What is the monoclinal wave? It consists of only one limb

joining two differing uniform flow water levels where upstream and downstream of it. So, a rough sketch. So, let us say this is the bed, this slight angle theta is here. This is the free surface here also freezer.

This is depth *y2*, and this is depth *y1*, and this is the wave having a velocity *Vw*. See, this is joining two different limbs. This is the upstream length, this is the downstream upstream water level, and this is the most simple one to assume. So, some of the important features to note from here about this type of monoclinal wave are that the wavefront moves with a uniform absolute velocity of *Vw*. We have already mentioned that the velocity is *Vw*.

So, for an observer who moves along the wave, who moves along with the wave at the same velocity. Which velocity? Velocity appears to be stationary.

And therefore, an equivalent flow can be represented by superimposing a velocity equal in magnitude but opposite in direction on the entire system. So, if we are in the same reference frame as that observer who is moving along with the wave at velocity Vw, this system, this is angle theta. Monoclinal wave y2, this is one section, and this is another section, right? This is *y1*.

So, this will appear to be moving $Vw - V2$ also let us say this is section 2 yes *y2* means section 2 and this is *y1* is section 1 and this will be minus *V1*. So, this is how the entire system will look like after we superimpose it with velocity of *Vw* but in the opposite direction and now for such a system the continuity equation is thus written as how that is *Qr* is equal to $A1\,Vw - V1$ is equal to $A2\,Vw - V2$ or $A(Vw - V)$. And what is this *Qr*? This is called over RUN, *QR* is nothing but over RUN.

y_2 moving with velocity V_2, monoclinical wave moving with *Vw* and this one is y_1 moving with V_1. This can be easily transformed into This is *y2*, this is *Vw* minus *V2*, this is *y1*, this moving in *V1* minus *Vw* and stationary So, this is the summary of this particular setup. Now, if we simplify this particular $(A_1V_1 - A_2V_2)/(A_1 - A_2)$ or dQ/dA. So, from this we can it may be seen that the maximum value of Vw is obtained as *(Vw)m* is equal to *dQ/dA* and *dA/dy* is equal to *T* or *Vw* maximum is equal to $1/T\ dQ/dy$ So, for a wide rectangular channel, the normal discharge per unit width is *qn* is equal to $1/n\ y^{(5/3)}\ So^{0.5}$.

dqn/dy is equal to $5/3 * 1/n * y^{2/3} * S_0{}^{0.5}$ *or* $5/3\,Vn$, where *Vn* is *qn/y*. Normal velocity. Thus, *Vw* maximum is *Kw Vn*, where *Kw* is equal to *1.67* for a wide rectangular channel. It can be shown that Kw is equal to *1.44* and *1.33*

for wide parabolic and triangular channels, respectively. Field observations have indicated that for small rises in the flood stage, the absolute wave velocities can be roughly estimated by the above equation. Which equation?

This equation: *Vw_max* is equal to *Kw Vn*. So, we will continue this derivation further in our next class. So, that is it for today. Thank you so much.