Free Surface Flow

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Lecture 47

Welcome back. And today, we are going to start the equation of motion for gradually varied unsteady flows. It is more of a numerical treatment where we are going to use the force balance, as we have always been using. So, with that being said, we continue. of motion.

The first one was the continuity equation, and this is GVUF, or gradually varied unsteady flows. So, first, let us draw a figure. This is and this is the x-direction, and this is the bed. We are taking a general configuration, which is why we are drawing it like this.

This is the free surface. So, we have, and this is, let us say, z, and this second section we can say z plus dz. It will look counterintuitive because you might think that dz, if you have added something, it should be higher, but dz could be negative as well. So, the idea is the distance from the datum of the bed at this point and at this point are different. So, this is one section, section 1, and here we are going to draw section 2. So, the hydrostatic PA, this is pressure $PA + d$ change in and here it is PA. Here we will have frictional force F_f, and here we will have F_w, which is the force of weight. We will have a component in this direction; this angle is theta. We will write these values as well. So, I think pressure—everything is written here, yes. So now, we would also like to draw a—let us see—this is the centroid, and this is Z_c here, and this h.

So first, let us write down the value of F_w, which is the weight, that will be equal to ρ. This is $F_w \sin \theta$. What will be the value of F_f? It will be $\tau_o * Pdx$, where P is the perimeter, the weighted perimeter. So, this being and this is the perimeter that we have defined, the weighted perimeter.

So, the equation of motion for gradually varied unsteady flow (GVUF) in a prismatic channel is derived by the application of the momentum equation to a control volume encompassing two sections of the flow. So, we are going to use the momentum equation for the control volume. So, we have one, two sections here.

So, these are the our two sections, section 1, section 2. As we have already discussed before, since the flow is gradually varied hydrostatic pressure solution is. So, as in any gradually varied flow computation also for gradually varied unsteady flow. We are going to use the hydrostatic pressure distribution to our analysis.

So, what we are going to do? We are going to apply Newton's second law of motion along the *x* direction. In broadly, if it says *F* is equal to mass into acceleration, the driving force will result in acceleration which is equal to *F/m*. So, we will get *P* that is pressure into area minus pressure into area plus $d/dx\ (PA)$ that is pressure into $dx + Fw$ the weight component *sin θ* - the frictional force is equal to $m * ax$.

So, important thing to note is mass of fluid element is equal to *ρAdx*. So, what is F_f? We have already written. It is τo * *Pdx* and what is τo? It is the bed shear.

So, this one was m. What is F_w? F_w is nothing but *ρgA dx*. Now, what is a_x? It is a streamwise acceleration, also can be written as ax is equal to $du/dx + du/dt$. So, using all these thus we get. So, this *PA* and *PA*, this *PA*, this *PA* will get canceled. So, minus *d(PA)*, that is pressure into area $dx + Fw,$ can be written as $\rho gA\ dx\ sin\ \theta - \tau o\ Pdx,$ this is wetted parameter equal to $\rho Adx\ (u\ du/dx + du/dt)$ instead of a_x. And we know that for small bed slopes, *sin θ* is equal to S_0.

Therefore, we can write $d(PA)\ dx + \rho gA\ dx\ sin\ \theta - \tau 0\ Pdx$ is equal to $\rho A\ dx\ (u\ du/dx + du/dt)$. Sorry, instead of *sin θ*, we can write So. So, in the next step, what do we do? We divide both sides of the equation by the weight of the fluid, which is *ρgA dx*. If we do that, we are going to get—what are we going to get? $-1/\rho gA\ d(PA)/dx + S0 - Sf$ is equal to $U/g\ du/dx + 1/g\ du/dt$. And why S_f? Because S_f is $\tau_0 P/\rho gA$, or *A/P* is *R*. So, $\tau_0/\rho gR$, therefore, this becomes Sf, and we have acceleration terms here.

So now we are going to concentrate on the first term on the left hand side. So, the first term—which is the first term? This is the first term on the left-hand side. So, the first term on the side called LHS

All the above equations can be expressed as how? So, $1/\rho gA * d(PA)/dx$. How can it be written? So, it can be written as *1/ρgA* will remain the same. Now, d/dx of

P can be written as ρgzc depth to the centroid into area or it can also be written as *1/A d/dx ρg* and *ρg* will get cancelled from here and here $d/dx\ (ZcA)$ Z_c is the depth of the centre head or it can also be written as *1/A d/dx* of *A* integral of whole area *dh/dx* **dA* or finally, it can be written as *dh/dx*. So, if you look at this previous drawing, right and take the definitions this is what you can write. So, Z_c **A* is nothing but *d/dx* of Z_c integral of *h dA* right and then we can write we can take *dx* inside it becomes *dh/dx* * *dA dh/dx* integral of this quantity this is important value. So, finally, we get this particular equation main equation. Finally, we get Sf is equal to So $- dh/dx - U/g\ du/dx - 1/g\ du/dt$. So, this is kinematic uniform flow.

If you include these terms diffusive and if you go until this point you get quasi steady flow. or steady non-uniform flow and until the end you get dynamic unsteady non-uniform flow uniform flow. You see with different acceleration terms being included, this is *d/dt*, this gives rise to the unsteadiness, this gives rise to non-uniformity *du/dx* and this gives rise to unsteadiness. So, this is the complete final equation of motion for on steady GVUF. This equation is also called, or is called, the general dynamic equation, or we can also say the general dynamic equation for unsteady flow or GVUF. The next point is very important.

It is also known as the De Saint Venant equation. This is a very famous equation in hydraulics. This is also called the De Saint Venant equation. So, we will see a problem in the next slide. We will see not a problem, but sort of a derivation.

The question is: show that the momentum equation of the Saint Venant equations can be written with discharge $1/Ag\ dQ/dt + 2Q/(A^2)g\ dQ/dx + (1 - F^2)dy/dx$. is equal to So - Sf, where dy/dx is equal to So Froude number *F*, *F^2 is* $(Q^2)T/gA^3$, *F* is nothing but the Froude number.

So, basically in this question, we have to show that the momentum equation of the Saint-Venant equation can be written with discharge, ok. So, ideally, we have to transform the Saint-Venant equation. In this equation, instead of u, we have to put *Q*, ok? Or instead of velocity, we have to put discharge, ok. How to approach this problem? We will start and then, you know, continue.

So, we say let *y* be the flow depth and *u* be the velocity. So, the standard definition is *y* is always the flow depth and u is the velocity. First, we write the continuity equation. Then, the continuity equation is $dQ/dx + dA/dt$ is equal to 0.

So, this is the continuity equation, the same equation that we started with. The unsteady flow width. If you remember, this equation has no lateral discharge term. Now, we can say the momentum equation is, we have already derived the momentum equation before, S_f was equal to $S_0 - dy/dx - u/g$.

$du/dx - 1/g\ du/dt$, or we can write $S0 - Sf$. We can bring other terms to that side: $dy/dx + u/g\ du/dx. + 1/g\ du/dt$. So, this is the unsteady term non-uniformity. So, if you look at the problem, we are asked to transform this equation with the help of the continuity equation and, with the help of the continuity equation, to this particular equation. If you see this complete equation, the difference is Q is now put in place of u. So, we have now started this solving procedure and have just written down the basic equations that we knew before.

This is the continuity equation, and this is the momentum equation. So, I think at this point, I will close, and we will start this problem from the same point here in our next lecture. Thank you so much, and I will see you in the next class.