Free Surface Flow
Dr. Mohammad Saud Afzal
Department of Civil Engineering
Indian Institute of Technology Kharagpur

Lecture 45

Welcome back, students. Last time, we solved problems for a triangular channel in our rapidly varied flow class, where for the triangular channel, we saw how to calculate the discharge. We calculated and saw how to determine the Froude numbers upstream and downstream, both, and in the end. We were able to calculate the energy loss. Now, we will see one, maybe two to three more problems in this last lecture of rapidly varied flow before we proceed to our unsteady flows module. So, let me start by writing down the question first.

We will solve a different type of problem. So, an overflow spillway has its crest at an elevation of 138 meters and a horizontal apron at an elevation of 100 meters on the downstream side. So, the question is: estimate the tailwater elevation required to form a

hydraulic jump when the elevation of the energy line just upstream of the spillway crest is 140 meters. Now, we have been given: take the value of C_d as 0.735, and also neglect energy loss due to So, there is an overflow spillway which has its crest at the elevation of 138 meters and a horizontal apron at an elevation of 100 meters on the downstream side.

Now, we have to estimate the tailwater elevation that is required to form a hydraulic jump when the elevation of the energy line just upstream of the spillway crest is 140 meters. We have to take C_d as 0.735, and also, we have to neglect energy loss due to the flow over the spillway. So, we will start writing down the things that are given. We have been given the elevation of the energy line over the crest as equal to 140 meters. So, we have also been given the elevation of the top of the crest as equal to 138 meters.

The elevation of the top of the apron is equal to 100 meters. We have also been given C_d as equal to 0.735. So, the first thing that we need to see is what is the head of water over the and that is 140 - 138, which is equal to 2 meters. We also need to find out the discharge per unit

over the spillway, q is equal to $2/3\ C_d(\sqrt{2g})\ H_1^{3/2}$. So, the values we have already been given. So, q will be equal to 2/3 into C_d, which is 0.735, multiplied by $\sqrt{2 \times 9.81}$, and the head is 2 raised to the power, head is 2. If we q comes out to be 6.13.

Let me write it down again here. So, it is easy for you. q is equal to $2/3 \times 0.735 \times (\sqrt{2 \times 9.81}) \times (2^{3/2})$. And *q,* that means *q* is equal to 6.139 meter cube per second per meter. Now, specific energy

Apron E is equal to how much? 140 minus 100 is 40 meters. So, if we apply the energy equation Here, E_1 is equal to $y_1 + V_1^2/2g$ or$y_1 + q^2/2g(y_1^2)$, and then we can equate what t is equal to $y_1 + 6.139^2\ 2 \times 9.81 \times y_1^2$or $y_1^3 - 40y_1^2 + 1.921$ is equal to 0, okay.

This is a cubic equation in y. Which, when solved, gives y_1 as 39.9999, the other is 0.220 or -0.219. So, we also know The flow over the apron will be supercritical flow, which means the depth should be small.

This implies a depth of the flow will be equal to 0.220 meter. So, velocity V_1 will be q/y_1 or 6.139/0.220 that is 27.905 meters per second. Pre jump Froude number F_1 will be $V_1/\sqrt{gy_1}$ equal to $27.905/\sqrt{1 \times 0.220}$ that is 18.995 that is the Froude number at the beginning of the jump.

ok from the sequent depth ratio of hydraulic jump y_2/y_1 is $1/2\ [(1 + 8Fr_1^2) - 1]$. So, y_2/y_1 is equal to half $(\sqrt{(1 + 8 \times 18.995^2)}) - 1$ or y_2 is equal to $0.220/2\ [\sqrt{(1 + 8 \times 18.995^2)} - 1]$ or y_2 is if you solve 5.801 which means required tail water elevation is going to be 100 + and 205.81. So, this is a different type of problem that relates hydraulic jump. See we have used the formula for sequent depth ratio as well here and the situation was different. We had a spillway, we had an apron and we had to do some calculations that were very much required to be able to solve this question. So, now we will go and try to solve a problem of basically a weir now, ok. And then we will solve two small problems here. Actually, it will be a lot easier, ok? One using standard formula, one using trial and error, ok. A 2-meter wide rectangular channel has a discharge of 0.0350 meter cube per second. Question is, find the height of a rectangular weir spanning the full width of the channel. that can be used to pass this discharge while maintaining upstream depth of 0.850 meter, ok. So, what are we going to do? We are going to solve the height of a rectangular weir here, ok.

First, we will see how a trial and error procedure is used. So, in this first in this process, we have to assume first a value of C_d, which could be an intelligent guess. In my opinion, I am assuming the value of C_d is 0.640, and then we are going to use this standard equation of discharge: that is $2/3\, C_d\, (\sqrt{(2g)}) L\, H_1^{3/2}.H_1^{3/2}$ can be written as $Q/(2/$

$3\, C_d\, (\sqrt{(2g)})L)$, and then substituting in the values, $H_1^{3/2}$ will be $0.350/(2/3 \times 0.640\ (\sqrt{19.62})$ that is 2×9.81 into 2.0, that is L, ok.

It is a 2-meter-wide, ok. This gives us H_1 as 0.205 meter and P as 0.850 - 0.205, which is equal to 0.645 meter. Then, as you remember, I explained the procedure: H_1/P is 0.318 meter. And Cd is $0.611\ +\ 0.08\ \times\ 0.318$, which is 0.636. So, we use the second iteration.

And we calculate $H_1^{3/2}$ as 0.0926 divided by 0.636 into 0.640. This gives us 0.09318, and H_1 is 0.206, P is 0.644, and H_1/P is 0.320. And C_d is 0.637. So, we see there is not much difference in the values of H_1/P and C_d So, we accept the value of C_d.

And the final values of H_1 are equal to 0.206 meter, and P is equal to 0.644 meter, implying the height of the required weir is, therefore, P is equal to 0.644 meter. This is what was asked in the question, ok. So, a similar question, but using a formula that I have already discussed in these lectures. A 2.5-meter-wide rectangular this is a new question a rectangular channel has a rectangular

The full width of the channel. And the weir height is 0.75 meters measured from the bottom of the channel. So, what discharge is indicated? when the weir is working under the submerged mode with a depth of

flow measured above the bed channel of 1.75 meters and 1.25 meters on the upstream and downstream of the weir. So, there is a 2.5-meter rectangular channel and it has a rectangular weir, I mean, spanning the full width of the channel. A weir height is 0.75 meters. So, we have been given the weir height, and that is measured from the bottom of the channel.

The question is what discharge is indicated when the weir is working under the submerged mode with a depth of flow measured over the bed of the channel of one point. So, the depth of the flow is 1.75 meters and 1.25 meters on the upstream and downstream of the weir, respectively. So, this is relating to the last part where the weir was, you know, under a submerged mode. So, we have been given the weir height indicated by P is equal to 0.75 meters. So, H_1 will be 1.75 minus that is from here 1.75 that is 0.75 is equal to 1.00 meters, and the second compared to this one 1.25 minus 0.75 that is 0.50 meters.

So, the first case H_1/P is going to be 1 divided by 0.75, and that is equal to 1.3333. And similarly, H_2/P will be—sorry, H_2/H_1. 0.5/1.0, that is 0.50. Here, the coefficient of discharge is going to be 0.611 plus 0—we are using the formula: 0.611 plus 0.8 times this particular value H_1/P. C_d is nothing but $0.611\ +\ 0.8H_1/P$, and that comes to be 0.718—sorry, not meter, C_d is dimensionless.

So, let me just write down the values in continuation so that it is easier on this page. P is 0.75 meter, which we derived—I mean, we found out from the previous. So, H_1 came out to be 1 meter, and H_2 came out to be 0.5 meter, okay. H_1/P came out to be 1.333, and C_d came out to be 0.718. Now, we will see Q_f is $2/3\ C_d(\sqrt{(2g)})\ LH_1^{3/2}$, or Q_f is 2/3

into $0.718 \times (\sqrt{(2 \times 9.81)}) \times\ 2.5^3/2$ because H_1 is so it comes to be 5.30-meter cube per second—basically, Q_f is okay, so we are going to use the Villemonte equation. This is a very important equation for submerged weir flow. And noting the coefficient n that is used in this equation is 1.5 in the Villemonte equation.

So, Q_s is given as $Q_f\ [1 - (H_2/H_1)^{1.5}]^{0.385}$, and Q_f was 4. So, Q_f was 5.30. $[1 - (H_2/H_1)^{1.5}]^{0.385}$. So, Q_s is 4.48-meter cube per second, and this is the final answer for the discharge.

So, these two were different types of problems. One was submerged—the last one—the other was a different type of weir. Before that, we solved a problem on hydraulic jump. And in the previous lecture, we solved a problem related to hydraulic jump in a triangular channel. And I think that is good enough for this particular module.

I would like to thank you all for listening carefully and attending the classes. I will meet you again in Module 10 of our course, which mostly covers the introduction to unsteady flow. Until now, from Module 1 to 9, we have been studying steady flow. So, now we will get an introduction to unsteady flow in open channel hydraulics. So, thank you so much.

See you in the next class.