Free Surface Flow
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Lecture 44

Welcome, students. Today, we are going to continue our topic, and we will see one problem that relates to the end depth or the free overfall. Then, we will move on to some discussion about the sharp-crested weir. If some time remains, we will solve one problem. So, the problem here is: if we consider a free overfall, we have to obtain a relationship for y_e/y_c in terms of bed slope S_b and critical slope S_c for a wide rectangular channel, and we have to use Manning's equation also. So, if we have a wide rectangular channel, the hydraulic radius R is equal to y. And if we follow, I mean, if we use the continuity equation at cc and ee, as we saw in the last lecture, we say Q_c is equal to Q_e, right? That means now the whole question turns in terms of Manning's equation, and you know Manning's equation can be written as Q_c can be written as $A_c/n\,R_c^{2/3}S_c^{1/2}$, whereas Q_e can be written as $A_e/n\,R_e^{2/3}$ and $S_b^{1/2}$. So, A_c can be written as B y_c, and A_e can be written as By_e, and similarly, R_e can be written as y_c, and R_e can be written as ye. Using these four in this, we can write B y_c/n, R_c instead of R_c we write y_c, and $S_c^{1/2}$. Similarly, A_e/n becomes B ye/n, and R_e becomesy_e $S_b^{1/2}$. Then, B gets canceled, n gets canceled, and we have y_c and $y_c^{2/3}$.

So, this becomes $y_c^{5/3}$ and $S_c^{1/2}$. Here, similarly, we have ye and $y_e^{2/3}$, which becomes $y_e^{5/3}$ $S_b^{1/2}$. Or, we take y_c here and bring S_b here. So, it becomes $(y_e/y_c)^{5/3}$ is equal to$(S_c/S_b)^{1/2}$, or y_e/y_c is equal to $(S_c/S_b)^{0.3}$. We multiply it by 3 raised to the power 5.

So, half into 3 raised to the power of 5, that is 0.3 to the power of 0.3. So, now what is a sharp-crested weir? So, what is a weir? A weir is a structure built across a channel to raise the level of water, with the water flowing over it. So, in the previous lecture, we were talking about spatially varied flow; we were talking about a side weir.

Now, here, this is built across the channel. If the water surface, while passing over the weir, separates at the upstream end and the separated surface jumps clear of its thickness, the weir is called a sharp-crested weir. So, again, what is a sharp-crested weir? If the water surface, while passing over the weir, separates at the upstream end and the separated surface jumps clear of its thickness, the weir is called a sharp-crested weir. It is also known as a notch or thin-plate weir.

You might have heard about a V-notch. So, this sharp-crested weir is also called a notch or a thin-plate weir. Sharp-crested weirs are extensively used as fairly precise flow-measuring devices in laboratories, industries, and irrigation practices. So, if you remember, we use a V-notch for measurement in laboratories. Hydraulics and Water Resources Lab at IIT Kharagpur.

So, weirs come in many geometrical shapes, but the rectangular or a triangular ones are the most commonly used. What is a rectangular weir? So, this figure shows the definition sketch of flow over a sharp crested rectangular weir. So, you see This is sharp crested rectangular weir.

The water surface of the stream curves rapidly at the upstream of the weir and plunges down in a parabolic trajectory on the downstream. This surface is known as upper nappy. You see this nappy, this is sharp crest, the zoomed figure and this is clinging nappy and this is upper nappy here. So at the weir crest, the flow separates to have a free surface which initially jumps up to a level higher than the wear crest before plunging down and this surface is known as lower nappy. So, upper nappy upstream and lower nappy downstream.

If the wear extends to the full width of the channel, the lower nappy encloses a space having air initially at the atmospheric pressure. As the flow proceeds for some time, some of the air from this pocket is entrained by the moving water surface and the pressure in the air pocket falls below the atmospheric pressure. This in turn causes the nappy surfaces to be depressed. This change is a progressive phenomenon, a limiting case of air pocket completely evacuated is called a clinging nappy as shown in figure. So we go back, you see this is nappy, this is air vent which we are talking about and this is a clinging nappy.

The weir flow, as shown above, assumes a tailwater level far below the crest and is called free flow. Now, the discharge equation. It is usual to derive the discharge equation for free flow over a sharp-crested weir by considering an ideal undeflected jet and applying a coefficient of contraction to account for the deflection due to the action of Thus, for a rectangular weir of length L spanning the full width b of a rectangular channel. So, we have a rectangular weir of length L and which has a full width b of a rectangular channel.

The ideal discharge through an elemental strip of thickness dh at a depth h below the energy line is given by $dQ_t = L\,(\sqrt{2gh})\,dh$. Thus, the ideal discharge Q_t can be written as $L\,(\sqrt{2g})$ integral of $V^2/2g$ to $H_1 + V^2/2g$. So, these are the two limits h ($\sqrt{h}$)dh. And, if we solve this, the actual discharge is C_c into Q_i, where C_c is the coefficient of contraction. So, Q can be written as $(2/3)\ C_c\ L\ (\sqrt{2g})\ [((H_1\ +\ V_0^2/2g)^{3/2})\ -\ (V_0^2/2g)^{3/2}]$.

So, you see this equation is inconvenient to use. The discharge equation is usually written in terms of H_1. So, the depth of flow upstream of the weir measured above the weir crest can be written as $Q\ =\ (2/3)\ C_d$. C_d is a coefficient.

So, C_d is a coefficient, which coefficient I will tell you—the coefficient of discharge, which takes into account the velocity of approach Vo. C_d is given by this equation: $C_d = C_c\ \times\ (1 +\ V_0^2/2g)^{3/2}\ -\ (V^2/2gH_1)^{3/2}$. So, in ideal fluid flow, C_d is a function of $H\ _1$and P, H_1/P, and this variation has been studied by a scientist called Stretkoff. In real flow, C_d should, in general, be a function of the Reynolds number and Weber number, in addition to the weir height factor H_1/P. If the Reynolds number is sufficiently large and if the head H_1 is sufficiently high to make surface tension effects negligible, the coefficient of discharge can be written as a function of H_1/T. So, the variation of C_d for a rectangular sharp-crested weir is given by the well-known Rehbock formula.

So, this is the variation of C_d that can be given, and it is valid for H_1/P less than 5.0. Now, what is submergence? In free flow, it was mentioned that the tailwater level is far below the crest to affect the free plunging of the nappe. If the tailwater level is above the weir crest, the flow pattern would be much different from the free flow case, and such a flow is called submerged flow. So, the idea is that the tailwater level is above the weir crest, and that is called submerged flow. So, this is a submerged sharp-crested weir. Now, the ratio of H_2/H_1, where H_2 is the downstream water surface elevation measured above the weir crest, is called the submergence ratio. You see, this is the height of the weir crest, and this height is H_1, and this is H_2.

So, H_2/H_1 is submergence. So, estimation of Q_s can be made by using the Villemonte formula. In this case, Q_s is equal to Q_1, where Q_1 is the free flow discharge under the head H_1, n is the exponent of the head in the head-discharge relationship, and for a rectangular channel where n is equal to 1.5, this formula can be used. So, the minimum value of H_2/H_1at which the discharge under the given head H_1 deviates by 1 percent from the value determined by the free flow equation is termed as modular limit or submergence limit. So, what is submergence limit?

The minimum value of H_2/H_1at which discharge under a given head deviates by 1 percent. So, this is the theoretical part, but we will not stop here; we will continue by having some solved examples for this. So, we will start with one problem about the hydraulic jump. So, a hydraulic jump takes place in a horizontal triangular channel.

So, horizontal means So is equal to 0. having side slopes 1.5 horizontal to 1 vertical. The depths before and after the jump are 0.30 meter and 1.20 meter, respectively. The question is to estimate first the flow rate.

Second is Froude number at the beginning of the jump. at the end of the jump and finally, energy loss in the jump. So, before solving we have to see what are the integral points here. So, the integral points here are, first it is a horizontal channel. That makes our life easy that is So is equal to 0, that is slow.

However, cross section is triangular with let us say slope of m. We will solve in terms of m and later we will substitute. So, m in the end is 1.5. and if we are also given y_1 is equal to that sequent depth is 0.30 and y_2 is given as 1.20 meter. The first step is always to understand and draw how the channel is going to look like.

So, I will draw a brief. a triangular channel, it could be y. So, this is how the channel looks like. So, now, we will go and solve for So, P is the force $\gamma A y$ bar that is the hydrostatic pressure force and γA is nothing but my^2 and that is centroid is at y bar for is equal to y/3 for a triangular channel and area is equal to my^2 for a triangular channel.

And how? Just you see, A is 1/2 y into $2my$ if this is 1 in m slope. So, A is my^2. So, this is the hydrostatic pressure force $\gamma m y^2$into y/3. Also, in terms of momentum flux, we can write $\rho(Q^2)/2$, that is ρ, so $\rho(Q^2)/2$, $\rho(Q^2)/A$ or $\rho(Q^2)/2my^2$ since A is my^2.

Using force balance, we can write $P + M$ is equal to $\gamma m y^3/3 + \rho(Q^2)/2my^2$ is equal to is equal to constant. So, also we can write gmy_1^3, putting at both sections 1 and 2 by 3, plus Q taking out rho, rho, rho cancelling out $\rho(Q^2)/2my_1$ is equal to gmy_2^3. So, y_2 cube by 3 $+ (Q^2)/2my_2$.

So, this was Qy_1^2 and this one was my_2^2. Q^2/m common from this one and this one, we can write $(1/y_1^2 - 1/y_2^2)$ is equal to $gm/3$, we can take common and write $y_2^3 - y_1^3$. This we can write or we can write Q^2/g is equal to $m^{2/3}$ $(y_2^3 - y_1^3)$ $(y_2^2 - y_1^2)/(y_1 y_2)^2$.

we can write Q^2/g. We are doing nothing but just simplifying here. We can take y_1^3 common, we can write $((\eta^3) - 1)(\eta^2)(y_1^4)/((\eta^2) - 1)y_1^2$ where η we can write y_2/y_1 we are designating as η. or Q^2/g can be written as $(m^2)/3$ y_1^5 or $((\eta^3) -$

$1)(\eta^2)/(\eta^2) - 1)$. So, this is one equation that we get. Now, putting in some of the parts, we know here m is what?

In this particular problem, m is 1.5, y_1 is 0.3. is y_2/y_1 is equal to 1.2/0.3 that is 4. We know η, we know m, we know y_1. Q^2/g is equal to $1.5m^{2/3} \times (0.3^5) \times (4^3 - 1) \times 4^2$ divided by $4^2 - 1$ and this will come out to be 0.12247 or Q^2 is equal to into 9.81 or Q is coming to be 1.096 meter cube.

per second. So, far we have for a triangular channel. See, we see what was the first question that was asked. We have to calculate the flow rate.

Flow rate we have already calculated, right. And now, the Froude number. So, for a triangular channel, the Froude number is written as $Q/A\sqrt{(gA/T)}$ in general terms. Or, F^2 can be written as $(Q^2)T/gA^3$ standard, and this is $(Q^2)T$ is $2my$ for a triangular channel. And this is $g(m^3)y^6$ or $2Q^2/g(m^2)(y^5)$.

This is the general Froude number. So, if we set the Froude number to 1, the square will be at section 1, it will be 2×1.096^2 divided $1.096^2(9.81 \times 1.5 \times 0.3^5)$, and this comes to be 44.88. Therefore, the Froude number comes to be 6.693. Supercritical.

I mean, using the same formula for F_2 will be $2 \times (1.096^2)/(9.81 \times 1.5^2)$. Now, this is different for, and this comes to be 0.04375, or F_2 is 0.209. So, this is And this is supercritical. So, this was our Froude number at the beginning of the jump, and the Froude number at the end of the jump, F_1 and F_2.

Now, the last part is to calculate the energy loss. calculation of energy loss. El is nothing but $E_1 - E_2$. E_1, E_2 is the specific energy at section 1 and 2 respectively since energy is represented by specific energy for a horizontal bottom case.

H is equal to E + delta z and H_1, E_1 plus delta z_1, H_2 is E_2 + delta z_2. Since delta z_1 is equal to delta z_2 is equal to 0, H_1 is equal to E_1 and H_2 is equal to So, specific energy can be written as $y_1 + V_1^2/2g$ minus $y_2 + V_2^2/2g$ and A_1 is 1.5×0.3^2 that is 0.135 meter square and V_1 is 1.096 divided by 0.135 that is 8.119 meter. A_2 is $1.5 \times (1.2^2)$ that is 2.160-meter square or V_2 is 1.096/0.16.

2.16 right 2.16 and this comes out to be 0.507 meter. Therefore, energy loss will be $0.3 + 8.119^2/2 \times 9.81 - (1.2 + 0.507^2/2 \times 9.81)$ or energy loss is going to be $3.6 - 1.213$ or E_1 is going to be 2.447 meter. This is the energy loss. So, we have calculated the discharge, we have calculated F_1 upstream, F_2 downstream of the hydraulic jump and we have also calculated the energy loss.

The good part was that instead of dealing with the rectangular section, it was a little challenging as well, but we tried for a triangular section. So, I think with this, we will end the class here today, and we will meet in our final class on rapidly varied flow next. We will try to solve one or two more problems before we proceed to our next topic. So, thank you so much. See you in the next class.