Free Surface Flow
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Lecture 43

Welcome students to the third lecture of our rapidly varied flow. In this lecture, we will try to cover efficiency and classification of hydraulic jump, ok? So, until the last lecture, we have seen energy losses, relative energy loss, and now we are going to start with the efficiency of the jump. So, the main question that arises now is: what is the efficiency of the jump?

So, it is quite simple. It is the ratio of specific energy after the jump to the specific energy before the jump. So, jump efficiency E_2/E_1, that is So, with all the analysis that we have done until now, we shall be in a position to calculate E_2. Or if we are given E_1, E_2 can also be written as E_1 - delta E. So, if delta E is known, we have a formula. E_1 we know from before. So, E_1 - delta E divided by E_1.

So, it is going to be 1 minus delta E/E_1. So, efficiency will always be less than 1. So now, we define that for a horizontal rectangular channel. So, the formula that comes out to be in terms of Froude, I mean, when we relate efficiency of the jump to the upstream Froude number Fr_1, ok? And this is a complicated equation in Fr.

Now, after efficiency, there is something called the length and height of the jump. So, the height of the jump is given by simply $y_2 - y_1$. So, if y_1 is the upstream depth and y_2 is the downstream depth, in a hydraulic jump, it is well known that the depth increases on the downstream side. So, the height of the jump hj is given by $y_2 - y_1$. And the length of the jump is the horizontal distance between the toe of the jump to a section where the water surface levels off after reaching the maximum depth.

So, you remember in one of the problems in the gradually varied flow some lectures ago, we solved where we used the direct step method for calculation of the length of the jump as well. Now, because the water surface profile is very flat towards the end of the jump, large personal errors can be introduced in the determination of the length. So, generally, what happens is experimental results or experiments are generally done also to estimate the length of the jump. And therefore, in the observation where the water surface is going to level off or becomes exactly horizontal, it will depend from one person to another, and therefore, large personal errors.

So, now there is a scientist called Elevatorski. He has experimentally shown that the length of the jump can be written as 6.9 times the height of the jump. So, this formula. Now, talking about the classification of hydraulic jump, what are the different classifications?

You know, in the beginning, we talked about classical jump, but we will now define it more professionally. So, this classification is based on extensive studies, you know, by two scientists, Bradley and Peterka. The hydraulic jump in horizontal rectangular channels. So, not everywhere, but horizontal rectangular channels are classified into 5 different categories. So, 5 different categories, and it is defined on the basis of the upstream.

Froude number, that is Fr_1. So, Fr_1 it is important to note that it is supercritical, which means it is greater than 1; Fr is greater than 1. So, the first one is undular jump, where the Froude number lies between 1 and 1.7. So, what are the characteristics? So, the first characteristic is the water surface is undulating no undulations are there and it has a very, very small ripple on the surface, and the sequent depth ratio is very small, that is, y_2 by y_1 is small.

And relatively, energy loss is practically 0. So, there is not much larger difference in the depth; y_2 will almost be equal to y_1, and therefore, the relative energy loss is also going to be practically 0 in this type of undular jump. When we increase the Froude number, so how it looks like I mean, this is how it looks like. So, this is undergoing a phenomenon; you see the difference is very little. You cannot say the difference between the water depth.

So, this is how an undular jump looks. Now, when the Froude number increases further if the Froude number is between 1.7 and 2.5. I have one question before I proceed: why is the Froude number not even lower? So, just repeating the same thing, a hydraulic jump is possible if and only if the upstream condition is supercritical, and a supercritical condition requires Froude numbers greater than 1. So, the undular jump, therefore, occurs between 1 and 1.7. Coming back to the weak jump again, the Froude number varies between 1.7 and 2.5.

And now, what are the characteristics? Here, when this jump occurs, the surface roller makes its appearance. So, at a Froude number of 1.7, it has been observed by scientists that the surface rollers I will explain what the rollers are in the figure gradually increase in intensity toward the end of this range. So, if it goes up to 2.5, the intensity of the surface rollers will increase. Here, the energy dissipation is also very small not much energy is lost and the water surface is smooth after the jump.

So, this is how it looks you know, these are the surface rollers, and you see the water surface is smooth. The water surface is smooth after the jump. Now, if you increase the Froude number further, we will have an oscillating jump, and the range of the Froude number is between 2.5 and 4.5. So, this category of jump is characterized by an instability of the high-velocity flow in the jump, which oscillates randomly between the bed and the surface. So, these oscillations produce large surface waves that travel considerable distances downstream.

This is an important observation that in oscillating jumps, they produce large surface waves that travel considerable distances downstream. So, you see these are the rollers as indicated, and this is the oscillating jet, and these are the surface waves that have been created. Now, if you increase the Froude number further, Fr_1, if it is between 4.5 and 9, this type of hydraulic jump will be termed as a steady jump. So, in this range of Froude number, the jump—the hydraulic jump—is quite well established. The roller and the jump action are fully developed to cause appreciable energy loss.

So, this is marked by significant energy loss, and the relative energy loss ranges from 45 to 70 percent in this class of jump. I mean, we studied you remember in the previous lecture, we talked about the relative energy loss, that was delta E/E_1; delta E/E_1 was the relative energy loss. So, this is how it looks like, you see. So, it is you can see that the roller and jump action are fully developed.

The last category is a strong or choppy jump, in which the Froude number is greater than 9. In this class of jump, the water surface is very rough and choppy very, very rough. The water surface downstream of the jump is also rough and wavy. So, when the flow becomes subcritical, that also is very, very rough and wavy that water surface. And, the sequent depth ratio that is, y_2/y_1 is very large, and energy dissipation is very efficient, and the relative energy loss is greater than 70 percent.

Therefore, $\Delta E/E_1$ is greater than 0.7, and the way it looks is this one. But it is important to understand here that Fr_1, which is the upstream Froude number, determines the category of the jump for all calculation purposes. Now, a small problem I mean, sort of a derivation itself, just as a problem we have to show that in a horizontal rectangular channel to create a hydraulic jump, this is the condition that would be required, where y_1, y_2, and y_c are the depths.

So, y_1 is before the jump, y_2 is after the jump, and y_c is the critical depth, respectively. So, remember, while deriving the Bélanger equation, we obtained a term. You can actually start from the beginning as well, and this was the equation we got. We used it previously also: ${y_1y_2}^2 + {y_2y_1}^2 - 2q^2/g = 0$. Or q^2/g can be written as half of

So, if you see this, you can write $2q^2/g = {y_1y_2}^2 + {y_2y_1}^2$, or q^2/g can be written simply as half So, if you take y_1y_2 common from here, y_1y_2 will be $y_2 + y_1$, exactly like this. This is 1. But for a rectangular channel, the critical depth is also given by 1. $y_c = (q^2/g)^{1/3}$, or $y_c^3 = q^2/g$. And what needs to be done? This q^2/g has to be substituted here.

So, if you compare when $n = 2$, we can get y_c^3 is equal to $y_1\, y_2(y_2 + y_1)/2$. And this is very simple. Actually, this was mostly derived during our class, but just to show you how this will work. Now, a small problem will let us solve all the quantities using the formulas that we have studied till now. So, the question is: water flows under a sluice gate to discharge into a rectangular plain stilling basin having the same width as the gate.

So, the width of the gate and the stilling basin is the same. So, after the contraction of the jet, the flow has an average velocity of 24 meters per second. And the depth of flow is 1.8 meters. So, that means we have been given some of the quantities. Now, it is asking us to determine the sequent depth y_2, height of the jump, length of the jump, loss of energy in the jump, efficiency of the jump, type of the jump, and the ratio of Froude number 1 by Froude number 2.

So, we have been given that the velocity V_1 is 24 meters per second, and we have been given y_1 is equal to 1.8 meters. So, the first step is to check if the flow is subcritical, supercritical, or not. So, $Fr_1$is $V_1 /\sqrt{(g\, y_1)}$ V_1 is 24, g is 9.81, y_1 is 1.8, and our Froude number is 5.7. So, the Froude number is greater than 1, which means supercritical condition, and this can cause a hydraulic jump, which implies hydraulic.

Jump will take place. Now, you remember we derived a formula y_2/y_1: it was $1/2\,[-1 + \sqrt{(1 + 8{Fr_1^2})}]$. So, now we will substitute the values. And find y_2. So, we know y_1, y_1 is 1.8, and we know Fr_1, we calculated from here 5.71. We put it here, and after calculation, we get sequent depth y_2 is equal to 13.66 meters. So, second part.

So, this was the first part; this is the second part. First part was sequent depth y_2 here. So, this is actually the first part; y_2 was 13.66 meters. Second is height of the jump. Height of the jump is $y_2 - y_1$.

So, height of the jump is simply $y_2 - y_1$, that is 13.66 - 1.8. So, 11.86 meters is the second part of the problem. Now, let us see what they have asked again. So, they are asking the length of the jump. The length of the jump formula was Hj was 6.9 times of Hj. So, 6.9 times of Hj is 6.9 of 11.66.

The length of the jump is 81.83 meters. Now, the loss of energy in the jump—the loss of energy—we have a formula that is $(y_2 - y_1)/4y_1y_2$. So, y_2 is 13.66 and y_1 is 1.8. So, if we put this in, we are going to get the energy loss as 16.96 meters. Now, what is the efficiency of the jump?

The efficiency of the jump is—I mean, we can use the formula also, but we can also use E_2/E_1, that is (E_1 - delta E) divided by E_1, or if we want to use— So, Fr_1 was equal to 5.71, and put it into this formula—that is, 5.71 here, 5.71, 5 point—and this will give us E_2/E_1 as 45.5 percent. And the type of jump will depend on the upstream Froude number. So, Froude number 1 is 5.71. So, it is a steady jump because a Froude number between 4.5 and 9 is considered a steady jump.

You can just go and verify the type of jump from the table here. You see, a steady jump is between 4.5 and 9, and our Froude number is 5.7. This, therefore, means we have the type of jump as a steady jump. Now, the final part: Fr_1/Fr_2— Fr_1 came out to be 5.71, but we need to calculate Fr_2 as well. So, A_1—we can use the simple A_1V_1 is equal to A_2V_2, or By_1 —breadth was the same, if you remember, we said in the question itself.

So, y_1V_1 is equal to y_2V_2. We know y_1. We know V_1, and we know y_2. So, V_1 came out at 1.8, and this was given as 24. So, this is 13.66. So, our V_2 comes out to be 3.66.

And therefore, we will calculate Froude number 2, which is 3.16 divided by under root, and it comes out to be 2.273. And therefore, the ratio of Froude number 1 to Froude number 2 is 20.91. This is the final part of the question that we had. So, all the quantities were calculated here if you just go back. The first was your sequent depth, which was calculated. Then, the height of the jump was calculated from the formula, the length of the jump was calculated from the formula, and the loss of energy was calculated from the formula.

Then, we also calculated the efficiency of the jump using the formula. And we calculated the type of jump based on Froude number 1. Then, in the end, we calculated the ratio of Fr_1 and Fr_1. Fr_1 we calculated initially. For Froude number 2, we used the continuity equation, calculated the Froude number 2 first, and then the ratio as 20.91. Now, what is a free overfall?

So, canal fall or drop or free overfall has a gradually varied profile upstream, and the portion of the fall is rapidly varied flow. So, for any fall, the upstream section is generally gradually varied flow, and the portion of the fall is rapidly varied flow. So, something like this. So, this CC until so, this is a GVF profile, and this has a GVF profile, and this has an RVF profile—GVF profile and RVF profile, OK. So, again, the depth just over the fall, that is the section EE, is called the brink or end depth.

So, this one is the brink or end depth. Also called as ye. Just a little bit upstream of this depth, at distance xc from here, the critical depth yc occurs at section CC. So, just before some distance xc, it is critical depth, and before that, it is gradually varied flow.

So, now, if we apply the momentum equation at CC here and EE (EE is this), neglecting the weight component and frictional forces, what we are going to get is So, $P_c - P_e$, this one here, OK, is equal to $M_2 - M_1$, that is $wQ/g\ (V_e - V_c)$, V_e is here, and V_c is here, OK. That is the first equation. We can simply write down the value of P_c and P_e from the force equation—that is, hydrostatic pressure force at CC is P_c, and hydrostatic pressure force at EE is P_e. This is the unit weight of water, that is equal to ρg, and this is velocities at E minus velocities at CC, or velocities at E is V_e, and velocities at CC is V_c.

Now, for a rectangular channel with a bottom width B, we can write continuity equation that is $A_e V_e$ is equal to $A_c V_c$ or $By_e\ V_e$ is equal to $By_c V_c$ because here A_c is By_c and A_e is Bye. This is continuity equation. So, we can write$y_e V_e$ is equal to $y_c V_c$ which is also equal to q that is discharge per unit width. Thus we can get simply V_c as so just to make it $y_c V_c$ is equal to q therefore, V_c can be written as q/y_c here. Similarly, we will have V_e is equal to q/y_e .

Ve is equal to q/y_e . Now, see this is a free overfall right and if we assume Pe to be 0 as the nappe thickness small at the brink and using this and this in the equation number 1, which is equation number 1? This is the equation number 1 all right. So, we are able to write P_c is nothing but $wQ/g\ (q/y_e - q/y_c)$. Because P_e is 0 or P_c can be written as P_c is equal to $wA_c y_c$ bar, this is hydrostatic force, pressure force and y_c is what the location of the centroid below the free surface at cc which will be equal to for a rectangular channel equal to, so y_c bar will be $y_c/2$

for a rectangular channel. w and w will get cancelled here and B can come down this side So, it will be y_c into $y_c/2$ is equal to Qq/Bg and then Q/B is also equal to q but anyways we will see if we do this, this Q/B then it becomes $q^2/g\ (1/y_e - 1/y_c)$, $y_c^2/2$ and this is nothing but. So, if you see, if you remember y_c^2 is equal to q^2/g. So, instead of this, we will write y_c^3. This equation becomes $y_c^2/2$ is equal to $y_c^3 * (1/y_e - 1/y_c)$.

So, this will cancel; this will remain y_c. So, this will become y, this will In the next step, this will be $2y_c * (1/y_e - 1/y_c)$ is equal So, or the same thing, 1 is equal to $2y_c$ into $(1/y_e - 1/y_c)$. Now, this can be written as $1/y_e - 1/y_c$ can be written as $ye\ y_c$, $y_c - y_e$, and this has come here.

And then in the next step, y_c can go; y_c, this y_c and this y_c gets cancelled, or we can write ye is equal to $2y_c - 2y_e$. This basically, this y_e going this side, and then this can come later this side and It will become $3y_e$ is equal to $2y_c$, $3y_e$ is equal to $2y_c$, or y_e is equal to $2/3\ y_c$. This overfall.

So, we needed what we needed to determine here was, if you have a, if you look clearly, we needed to determine this y_e that was free; this was the brink or end depth, and that is or that is equal to the value that we have got here. See, after all the calculations that we have got here is $2/3\ y_c$. So, 66.6, 66.66 percent of y_c; this is the free overfall, all right. So, I think with this, I will end this lecture here and continue again where we start with a problem based on this free overfall, and I will see you in the next class. Thank you.