Free Surface Flow
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Lecture 42

Welcome back, students, to the continuation of our rapidly varied flow. In the last class, we derived an equation that is valid for hydraulic jump for any arbitrary cross section. So, today, what we are going to do is start with that particular equation, and after that, we will simplify it for different channel cross sections. So, in the last class, if you remember, we started with this. So, in the last class, we ended with this particular equation, which is a general equation for hydraulic jump for any arbitrary channel cross section.

So, the next step would be to write the equation. Hydraulic jump for any known cross section. In our case, we shall start with rectangular cross section. So, now, hydraulic jump in a rectangular channel is the simplest, and in the beginning of the topic, in the introduction section, we said that the jump in a horizontal rectangular channel is referred to as a classical jump.

Now, starting from the equality of specific forces, if you remember, in the middle while deriving the equation of hydraulic jump for any arbitrary cross section, we also came up with the equality of specific forces. So, in hydraulic jump, the specific energy no longer remains the same. However, the specific force remains the same. So, we will now equate the specific forces—instead of starting with the final equation—we will start with the equality of the specific forces at both sections 1-1 and 2-2.

1-1 indicates the section just before the hydraulic jump has started, and 2-2 is just after the jump has ended. So, this is specific force 1-1, and this is specific force at 2-2. So, Q squared divided by $g\ A_1$ plus $A_1 X_1$bar is equal to Q squared divided by $g\ A_2$ plus $A_2 X_2$.

So, for a rectangular channel of bottom width B, we can write the above as: see A_1 will be b y_1, A_2 will be By_2, and X_1 1 and X_1 bar will be $y_1/2$, and X_2 bar will be $y_2/2$. If we use all these values and put them in this equation, we are going to get $Q^2/g\ By_1 + A_1\ is\ By_1 X_1$ bar is $y_1/2$ is equal to Q^2/Bgy_2. This is A_1, this is X_1 bar, g this is, this is again A_2, and this is X_2 bar.

So, now we can write $Q^2/g\ By_1 + B(y_1^2)/2$ is equal to $Q^2/g\ By_2 + B(y_2^2)$. So, now if we multiply both sides by $2/B$, what we are going to get is $2Q^2/2g(B^2)y_1$ plus y_1^2 is equal to $2Q^2/2g(B^2)y_2$ plus y_2^2. What are we going to do? We are going to bring this one to this side, this one to this side, or the other way around. So, first what we do is we write *Q/B* as q. So, Q^2/B^2 becomes therefore, this is 2 $q^2/gy_1 + y_1^2$ is equal to 2 $q^2/gy_2 plus\ y_2^2$.

So, I have already told where small q is capital Q/B, which is the discharge per unit width. Again, writing $2q^2\ /gy_1$ plus y_1^2 is equal to $2q^2\ /gy_2$. by gy_2^2 plus y_2^2, or we bring y_1 to this side. So, it becomes y_2^2 minus y_1^2 is equal to $2q^2$ divided by gy_1. $2q^2$ divided by gy_2.

So, we can again write, take q^2 common from one side: q^2/g, $2q^2\ /g$. So, what we can do is $y_2 - y_1$ divided by $y2 + y_1$ is equal to 2 $q^2/g\ (1/y_1 - 1/y_2)$, something like that. So, y_2^2 minus y_1^2 is equal to $2q^2/g\ (1/y_1\ minus\ 1/y_2)$, same as I have written, or $(y_2 + y_1)$ into $(y_2 - y1)$ is equal to $2q^2/g$. So, $(1/y_1 - 1/y_2)$ can be written as $y_1y_2\ (y_2 - y_1)$. And this is put in place of this, and we get this particular equation. So, you see $y_2 - y_1$ can get cancelled; this $y_2 - y1$ can get cancelled. So, we can write $y_1\ y_2$ and y_1 and y_2 is sent here.

So, $y_1\ y_2$ into $(y_2 + y_1)$ is equal to $2q^2/g$. Or we can write $y_1\ y_2^2$ plus $y_2\ y_1^2$ minus $2q^2/g$ is equal to 0. So, this is a quadratic equation in both y_1 and y_2. You can have a quadratic if you consider y_2 as a variable, right? Then it is quadratic in both y_2 and y_1. Equation with its quadratic either in terms of y_1 or y_2.

Now, if you solve this quadratic equation in terms of y_2, so what happens is y_2 is the downstream depth. So, in most of the cases we will know what the condition is in upstream. So, initial condition we will know what is the depth before the jump. So, it could be when asking a numerical there might be conditions where you will be asked to calculate y_1 as well no doubt about it. But, in most of the cases we are generally asked to estimate the depth after the jump.

So, we will write and solve the equation in terms of y_2 that is the downstream depth. And, we shall also neglect the negative root we are going to write y_2 is equal to $[-y_1^2 + (\sqrt{y_1^4 + 8y_1q^2/g})]/2y_1$. Or if we simplify this y_2 is equal to $1/2\,[-y_1 + \sqrt{(y_1^4\ + 8y_1q^2/g)/y_1^2})]$. So, this equation, this equation can also be written in terms of $y_2\ /y_1$. So, you see this equation we are going to write simplify it further y_2.

So, how to simplify it further you see this y_1^2 and y_1^4 , it will get cancelled. So, it will be y_1^2 and here it will be $8q^2/gy_1$ as in this. or we take y_1^2outside it becomes. So, let us see this one term $y_1^2 + 8q^2/gy_1$. Let me take y_1^2 outside then $1 + 8q^2/gy_1^3$.

If you remember, this can be written in terms of yc as well: q^2/gy_1^3, if you recall from the previous class. So, this y_1 will come out. So, it will be $y_2 = ½[-y_1 + y_1(\sqrt{1 + 8q^2/gy_1^3})]$. Or, you see y_1 is here also; we can take it outside, and the next step would be. The next step would be we can write y_2 in terms of y_1.

But let us go and see what the Froude number before the jump can be written as. So, the Froude number before the jump can be written as: Froude number = $(Q^2)T/gA_1^3$ or $(Q^2)B/g(B^3)(y_1^3)$, this is for a rectangular section by putting $A = By_1$ in the previous equation. So, Fr_1 will now be $(Q^2)/g(B^2)(y_1^3)$. Or, $(Q^2)/(B^2)$ is nothing but small q, or this q^2/gy_1^3 can

be written as Froude number 1, Fr_1^2. So, you see this term and go to this equation before this one. So, we might be able to replace this one with the Froude number as well. So, if we substitute this, we can get $y_2 = ½[-y_1 + y_1(\sqrt{1 + 8Fr_1^2})]$. All right?

or we take the next step that I was saying we can take this y_1 and y_1 outside, it becomes y_2 is equal to $y1/2\ [-1 + (\sqrt{1 + 8Fr_1^2})]$.. Next step will be we will bring y_2 and y_1 . So, this will go down here and becomes y_2/y_1 is equal to $1/2\ [-1 + (\sqrt{1 + 8Fr_1^2})]$. So, beauty of this equation is depends only on upstream Froude number that is Fr_1 and this y_2/y_1 is called the sequent depth ratio.

and this particular equation is called the Belanger equation. So, this is nothing but Belanger equation. So, this particular equation is Belanger equation a very famous equation and this equation gives us the sequent depth ratio for hydraulic jump for a plane. So, Belanger equation you have to remember.

So, this equation Belanger equation and you can also write this equation is in terms of Fr_1 . and notice it is written as y_1/y_2. So, now important thing to note is here y_1 is the depth before the jump, y_2 is after the jump, Froude number 1 is Froude number before the jump which is greater than 1. this is super critical flow and this is after the jump that is sub critical ok. Now, we have calculated y_2 when we know y_1 it is and we have been talking about and stating that the hydraulic jump is accompanied by huge energy loss and that huge energy loss can be beneficial to us in many ways.

So, it is imperative and important to calculate the loss of energy in the jump, and for this, we are going to use the energy equations. So, let us say E_1 and E_2 are the specific energies before and after the jump. So, E_1 is at section 1, and E_2 is at section 2—before the jump and after the jump, respectively. Then, we can say the energy loss is nothing but $E_1 - E_2$

because E_2 will be less than E_1 since energy has been lost from the state of E_1 . Hence, we can write delta E as E_1 minus E_2, and E_1 is nothing but y_1 plus $V_1^2/2g$ minus y_2 plus $V_2^2/2g$.

And let us say, for a horizontal rectangular channel, what are the conditions? So, let me write down V_1 , A_1 is already there. So, I mean, the area is By_1, and we are going to use that area is equal to By_1, and s is equal to 0, but that is not required here. So, y_1 plus $V_1^2/2g - y_2 + V_2^2/2g$, or V_1 we write in terms of Q/A_1 and Q/By_1 , and V_2 will be Q/A_2, that is Q/By_1. So, it becomes Q^2/gA_1^2, and this is also.

Now, if the channel width is B, here. We can write $Q^2/2g(B^2)(y_1^2) - (y2 + Q^2/2g(B^2)(y_2^2))$, this one and this one. Or we can write $y_1 - y_2 + q^2$. So, Q^2/B^2 can be written as q, q^2. is capital Q^2/B^2.

Or we can write $y_1 - y_2 + q^2/2g\ (1/y_1^2 - 1/y_2^2)$. $y_1^2, y_2^2, y_2^2 - y_1^2$. This is what is done here. So, this $y_2^2 - y_1^2$ is written as $(y_2 - y_1)(y_2 + y_1)$, this one divided by $(y_1^2)(y_2^2)$ square, and this is here. Then we will take this as the equation.

So, when you are analyzing the jump in a rectangular section in a rectangular channel, we have seen that y_1, if you remember, we came up with this equation: $y_1\, y_2^2 + y_2\, y_1^2$. This was for the sequent depth, you remember. So, what we are intending to use is instead of $2q^2/g$, we will use this term. That is what our idea is. So, we can write $y_1\, y_2^2 + y_2\, y_1^2$ is equal to $2q^2/g$, or $y_1 y_2$ into $y_2 + y_1)$ is equal to $2q^2/g$, or q^2/g is half of $y_1\, y_2 * (y_1 + y_2)$. This is simplification only. And this is equation number 5. We are going to use this equation, replacing q^2/g from here in equation 4. We can write simply delta E as $y_2 - y_1 + 1/2\, y_1\, y_2\, (y_1 + y_2) * 1/2\, (y_2 - y_1)(y_2 + y_1)$ and $(y_1^2)(y_2^2)$. So, what is going to happen is

this becomes $y2 + y_1$, this and this multiplied together $(y_2 - y_1)/4y_1\, y_2$, and this $y_1 - y_2$ can be written as $-(y_2 - y_1)$. Now, $y_2 - y_1$ can be taken out: it is $-1 + [(y_2 + y_1)^2)/4y_1\, y_2]$. Or $(y_2 - y_1)[-1 + (y_2^2 + y_1^2 + 2y_1\, y_2)]$, and this will, if we take the whole denominator, become $4y_1\, y_2\, y_2^2 + y_1^2 + 2y_1\, y2 - 4y_1\, y2$. So, this minus 1. Now, you see this will become $y_2^2 + y_1^2$, so this can be written as $y_2^2 + y_1^2 - 2y_1\, y_2$. In the next step, this particular part will be written as $(y_2 - y_1)$^2.

So, delta E is $y_2./y_1$. That is $(y_2^2 + y_1^2 - 2y_1 y2)/4y_1 y_2$. , and then this can be, in the previous slide, written as $(y_1 - y_2)^2$, and then this multiplied with this will become

$(y_2 - y_1)^3$. So, this becomes $(y_1 - y_2)^2$. Now, this will become $[(y_2 - y_1)^3.]/4y_1y_2$, and this is the important equation for energy loss in a hydraulic jump.

So, what you do is, y_2 can be—you see here y_2 is there as well. So, y_2 can be estimated from the Belanger equation. Once y_2 is known, delta E can be easily estimated. So, $[(y_2 - y_1)^3.]/4y_1y_2$—this is an important equation.

Now, we talked about energy loss. What is relative energy loss? Relative energy loss is nothing but the ratio of delta—that is, the energy lost—divided by E_1. Delta E is the energy lost, and E_1 is the initial energy before the jump. That is, the ratio delta E/E_1 is defined as the relative energy loss during a hydraulic jump.

So, for a horizontal rectangular channel, the relative energy loss is given simply by this. You do not need to remember this equation because, using the Belanger equation, you can find y_2 and y_1. Then, you will be able to find out delta E once y_2 is known. y_1 should be given initially, and then you can calculate delta E and E_1. You already know $y_1 + y_1^2/2g$. E_1 can be expressed as $y_1 + y_1^2/2g$. So, the plot, or if we draw a plot of $delta\ E_1/E_1$. So, delta $E\ /E_1$ expressed as a percentage against Froude number 1, according to this equation. For your sake of convenience, I have plotted it here. It should look something like this. So, if you keep on increasing this, it is increasing.

Increasing the Froude number, and this is the relative energy loss. So, you see, if you keep on increasing. If you keep on increasing the Froude number, the maximum relative energy loss is approximately. Less than 90 percent. It becomes asymptotic.

Secondly, F_{r1} cannot be. Less than 1. Why? You see, it cannot be less than 1. A hydraulic jump will occur only if F_{r1} is supercritical. And the condition for supercritical flow is F_{r1} greater than 1, and in this case.

Energy loss, all right. So, you see, until this point, 50 percent, right? So, no matter how much you increase the Froude number, there will come a point after which the relative energy loss will almost become constant, as in the case of a Froude number greater than 20, and that makes sense as well. So, just revising some of the concepts that we have seen today, we have seen the derivation of the Belanger equation, that is, the sequent depth ratio. We started with that and saw what the energy loss is. Right? If we know a simple solution, then we saw what the relative energy losses are in this lecture.

So, in the next lecture, I think we will start with the efficiency of the jump and then proceed to further topics. So, that is all for today. Thank you so much.