Free Surface Flow

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Lecture 41

Welcome, students, to our new module, which is about rapidly varied flow. In this particular lecture, I am going to introduce you to the concepts behind rapidly varied flow. So, without further delay, let us introduce the concept behind rapidly varied flow and how it relates to the streamlines. We know that the streamlines in uniform and gradually varied flow are either parallel or may be assumed to be parallel, and this is the case for uniform, gradually varied flow, and spatially varied flow. Therefore, because the streamlines are parallel in uniform and gradually varied flow, the accelerations in these flows are negligible. So, there are no accelerations in GVF or SVF.

And the pressure distribution may be assumed to be hydrostatic. This was the key assumption for uniform flow and gradually varied flow. The analysis in which the pressure distribution is hydrostatic is referred to as the shallow water theory. So, what this means is that the shallow water theory is applicable

to GVF, SVF, and uniform flow. Many times, what happens is streamlines. So, the ideal case is that streamlines are either parallel or assumed to be parallel. However, many times, what is going to happen is the streamlines may have sharp curvatures. And therefore, we can no longer assume that the hydrostatic pressure distribution is present.

Therefore, the assumption of hydrostatic pressure is invalid. The assumption is invalid for cases where streamlines have sharp curvatures. In addition, the flow surface may become discontinuous, not just having sharp curvatures, but if the flow depth changes rapidly such that the surface profile breaks. So, the dy/dx

might be discontinuous. Now, due to the non-hydrostatic pressure distribution, rapidly varied flow means dy/dx is large. Non-hydrostatic pressure distribution means dy/dx is large, indicating rapidly varied flow or famously called RVF. So, due to this non-hydrostatic pressure distribution, rapidly varied flow cannot be analyzed using the same approach as parallel flow.

So, we have to apply a different approach. The rapidly varied flows have been analyzed based on Boussinesq and Fawer assumptions. So, what are the assumptions used to analyze rapidly varied flow? Rapidly varied flow signifies the special characteristic of having a non-hydrostatic pressure distribution. They are analyzed using Boussinesq and Fawer assumptions.

In the Boussinesq assumption or approximation, the vertical flow velocity is assumed to vary linearly from 0 at the channel bottom to the maximum at the free surface. So, normally So, the velocity has a different free surface profile, something like this, right? However, in the Boussinesq approximation, it states that the free surface velocity is assumed to vary linearly from the minimum at the channel bottom to the maximum at the free surface. However, in the Fawer assumption, this variation is assumed to be exponential.

So, the difference between the Fawer assumption and the Boussinesq approximation is that, in the Boussinesq assumption or approximation, the velocity is supposed to vary linearly from the bed to the free surface, whereas in the Fawer assumption, this variation is assumed to be exponential in nature—could be something like this exponent. So, rapidly varied flow usually occurs over a short distance. So, the depth of the flow varies largely over a very short distance compared to gradually varied flow. In gradually varied flow, there is a small change in depth over a large distance. Therefore, the losses due to shear.

So, since the length over which the water level is changing is short, which means that the shear at the channel boundaries is small. Because it will depend on how much length the shear is acting over. If the length is short, the shear is assumed to be small and therefore can be neglected in the typical analysis. So, this makes our life a little easier, as we can neglect frictional resistance in RVF, and the reason being the process happens over a short length.

However, because of sudden changes in the channel geometry, the flow may separate, and eddies and swirls may form. So, one good thing that happened is that because these changes happen over a short distance, shear is neglected. So, because of the sudden changes, there are other types of phenomena that are going to happen: the flow may separate, and eddies and swirls may form. This is typical turbulence, and there will be energy loss because of this. Now, these phenomena complicate the flow pattern, and it becomes difficult to generalize the velocity distribution at a cross-section because of turbulence.

It complicates the flow pattern and causes difficulty in generalizing the velocity. Now, one of the typical examples, or a very famous example of open channel flow and specifically rapidly varied flow, is the hydraulic jump. What is a hydraulic jump? The most basic definition, or the most scientific definition, is that a hydraulic jump is formed whenever the flow regime changes from supercritical to subcritical flow—one of supercritical to subcritical flow. That is the first thing that should come to your mind.

And when it happens, what are the things that happen next? So, in this transition from supercritical to subcritical flow, the important thing that happens is that the water surface rises abruptly, surface rollers are formed, intense mixing occurs, air is entrained, and usually a large amount of energy is dissipated. This is the effect of turbulence. Hence generated. So, the water surface rising abruptly can physically be seen, surface rollers being formed, intense mixing occurring, air entrainment, and a very large amount of energy is dissipated.

So, a hydraulic jump is accompanied by a huge loss of energy. Now, if we utilize these characteristics of a hydraulic jump. So, if we want to use this, a hydraulic jump can be used at different places to dissipate energy. For example, if there is a flow of water, a large velocity of water coming with great discharge, it might be a threat to our hydraulic structure. So, to protect our structures, we might plan a hydraulic jump such that it dissipates energy.

That will protect the structure. This can also be used to mix chemicals and to act as an aeration device because air is entrained in this process. So, a jump—so, a definition, the most basic one—a jump in a horizontal, that is, a slope of 0, a rectangular, simple geometry channel is referred to as a classical jump. So, what is a classical jump? A hydraulic jump occurring in a horizontal rectangular channel is referred to as a classical jump.

So, now what are some of the applications of a hydraulic jump? As we discussed in the previous slide before, a hydraulic jump is an energy dissipator. It dissipates the excess energy, which we can use to dissipate the excess energy of flowing water downstream of hydraulic structures, as told before. and prevent scouring downstream. So, energy is dissipated; if the velocity is reduced, the energy is lost, then the scour that happens downstream near the hydraulic structures or even near the downstream side of the river could be reduced.

Second use is to mix chemicals used for water purification or wastewater treatment. To aerate the polluted stream is the third one. It is also used for desalination of seawater. also to raise the water level on the downstream side of the metering flume and thus maintaining high water level in the channel for irrigation. You see when hydraulic jump is there in let us say it is section 1 is there and section 2 is there.

Here the flow is supercritical and here it is subcritical by definition hydraulic jump. So, in supercritical less depth, because Fr is $V/(\sqrt{g\,y})$. So, less depth here and for subcritical more water depth. So, if this jump can also be used at places where we know where we want more water depth where we need high water level.

For example, channels for irrigation for irrigation etc. We need higher water depth. Also, this also increases weight on the apron and thus reduce uplift pressure under a masonry structure by raising the water depth of the on the apron because the water level So, on the downstream side increases it reduces the uplift pressure under the masonry structure by how by raising the water depth on the apron. So, there are several applications of hydraulic jump which are very useful in nature and has been used maybe which has been used for many, many years for the protection of our structures.

Now, we are going to see the mathematical analysis of a hydraulic jump. So, this figure is here. So, this is the upstream side. This is the downstream side. At section 2, this is section 2, this is section 1, and the water depth y_1 is where it is a supercritical flow, and this is a subcritical flow. This entire process is

This y_1, y_2, here the velocity will be V_1, here the velocity will be V_2, and this is nothing the length of the jump. If you remember in SVF and in GVF, we actually in gradually varied flow last lecture when we solved the problem, we calculated the distance between the toe of the jump up till the vena contracta, and this is nothing but the length of the jump. over the bed where the hydraulic jump is happening. So, as I told before, this is supercritical flow where the Froude number is greater than 1, and the other is subcritical flow where the Froude number is less than 1.

Now, the mathematical analysis of hydraulic jump is based on the momentum equation. The energy equation is not taken into consideration, as the hydraulic jump is associated with a lot of energy loss. That is correct. However, in the end, when we are going to estimate the energy loss, we will be using the energy equation. However, in the beginning, to come up with the sequent depths and other things, we are going to use the momentum analysis or momentum equation for the analysis of hydraulic jump.

Now, as with any derivation, we need to have some assumptions. So, going through the length of the jump, we assume that the distance—the length of the jump or the distance over which water depth— changes is small. That is one of the assumptions: that the water surface changes fairly a large amount over a small distance. So, the length of the jump is that distance.

So, this is assumed to be small, and hence we are going to neglect the frictional resistance; it can be neglected. F_f is ignored. And why? Because Lj is small. The other thing we have considered right now is the channel is horizontal, such that theta is equal to 0. And, okay, the one important thing we said: non-hydrostatic pressure is there. However, we say that before the jump is happening and also after the jump is happening, the area or the location before the jump

and after the jump these two places we will assume hydrostatic pressure distribution. I will go to this slide; you can see. So, this area here and this area here both will have hydrostatic pressure conditions. Now, also, we assume that the flow is uniform before and after the jump. And this is the reason we are actually able to assume hydrostatic pressure distribution: because just before the jump and after the jump has taken place, the flow is considered to be uniform.

Depth is y_1 before the jump and depth y_2 is after the jump and they remain constant. They do not change the depth before and after the jump. This is another assumption. Now, starting with the analysis of the hydraulic jump. Now, if you recall the concepts of momentum equation specific force which we have already covered in detail in our previous lectures over past several weeks.

So, the momentum equation based on these and these above assumptions is given by F_1 minus M_2 is equal to M_2 minus M_1 and F_1 and F_2 are the hydrostatic pressure force at 1-1 and 2-2 since there is no friction and as the channel bed is assumed to be horizontal. Since, there is no frictional force F_f is gone since the bed is horizontal there is no weight component in the momentum equation and this is momentum flux at section 1 1 and 2 2. So, F_1 minus F_2 will be see this is F_1 is $\rho g A_1$ X_1 bar. This is the hydrostatic X_1 bar is the centroid and same is X_2 is centroid it is $\rho g A_2$ X_2 and M_2 minus M_1 is W/g or $\rho g/g$ $Q *$ $(V_2 - V_1)$. So, ρg can be written as w and then A_1 is A_1 X_1 bar is X_1 bar minus ρg is again written as WA_2 X_2 bar and this ρg is also written as W. So, just have a look.

So, this equation is WA_1 X_1 bar minus WA_2 X_2 bar is equal to W/g $Q(V_2 - V_1)$. Now, this there are several terms that can get cancelled w will get cancelled right here. So, we can write A_1 X_1bar minus A_2 X_2 bar is equal to capital Q/g and $V_1 V_2$ can be written as Q/A_2 and V_1 can be written as Q/A_1.

So, instead of this previous equation V_2 and V_1, this V_2 and V_1 we write Q/A_1 . A_2 and Q/A_2 or this will be all the terms that have A_1 is brought this side on the left hand side or let me just to explain. So, it is A_1 X_1 bar minus A_2 X_2 bar is equal Q^2 divided by gA_2

minus Q^2 divided by gA_1. So, what we do is we bring this one this side and we take this one this side.

So, this comes to be A_1 X_1 bar plus Q^2 divided by gA_1 is equal to Q^2/gA_2 plus A_2 X_2 and this is the same equation as this. and this is nothing but a specific force. So, these individual terms that is Q^2 divided by gA plus Ax bar is called specific force. So, in hydraulic jump we equate specific force rather than specific energy.

Now, starting with the equality of specific forces at 1, 1 and 2, 2, we can write Q^2 divided by $gA1$ plus A_1 X_1 bar is equal to Q^2 divided by gA_2 plus A_2 X_2 bar. This is also called conservation of specific force. Then what we do is we bring $x1$ this side and this one this side. So, it becomes A_2 X_2bar minus A_1 X_1 bar Q^2 by g will be common 1 by A_1 minus 1 by A_2 or A_2 X_2bar minus A_1 X_1bar is equal to Q^2 by g and this 1 by A_1 minus 1 by A_2 can be written as A_1 A_2 A_2 minus A_1 and this is here.

And then what we do is we take A_1 X_1bar common from here, and then we can write A_2 X_2bar divided by A_1 X_1bar minus 1 is equal to Q^2 by gA_2 minus A_1 divided by A_1A_2. Or this particular part A_1 X_1, we take it on the right-hand side of the equation. Then we can write A_2 X_2 bar divided by A_1 X_1 bar minus A_1 minus 1 is equal to Q^2 divided by gA_1 X_1 bar Q^2 minus A_1 divided by A_1 A_2. Or if we multiply the numerator and denominator of the right-hand side with T_1 $/A_1^2$. So, let us say if we multiply this one by T_1/A_1^2, both numerator.

So, this one is also multiplied by T_1/A_1^2, and this one denominator is also divided by T_1/A_1^2. So, what is going to happen? This one will become $(Q^2\ )T_1$ divided by gA_1^3 A_1^2/T_1 $x1$, and this will remain the same. So, we will try to segregate some of the terms, for example, Q^2 divided by gA_1^3, for example, and this is the Froude number at 1, that is whole square. So, we can write A_2 X_2bar divided by A_1 x1 bar minus 1 is equal to Fr_1^2, and then this particular term we can write A_1/T_1 into x1 bar and 1 A_1 that is remaining will come here A_2 minus A_1 divided by A_1 - A_2, and what is A_1/T_1?

So, we can write this A_1 will get cancelled with this A_1, and then our equation will be A_2 X_2 bar divided by A_1 $x1$ bar minus 1 is equal to $Fr_1^2A_1/T_1$ same, this got cancelled, so A_2 minus A_1 divided by A_2 . Now, we again write like this same, I mean, here the next step what has happened is this A_2 by A_2 becomes 1, and this is A_1 minus A_1 by A_2 . And this is the thing that is shown here. In the next step, we say that this is the A general, here we have a very less general equation to have hydraulic jump for any arbitrary.

Channel cross section. So, this is an equation which is valid for all types of cross sections. The next step would be to fix the channel cross section and to find, for example, let us say we have a rectangular section, a trapezoidal section, a circular section, or a triangular section. Then, we are going to further simplify these types of equations. And, we are going to obtain specific hydraulic jump sequent depth ratios or the energy losses, among other things. So, what we are going to start in the next lecture is the hydraulic jump in rectangular channels.

So, before I mean, I think this will be enough for today, this particular part. And at the beginning of the next class, we will directly go ahead. We will see this particular equation once more, and then we are going to solve our hydraulic jump in rectangular channels. So, that is it for this lecture today, and see you in the next class.