Free Surface Flow
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Lecture 4

welcome students to this fourth lecture of first module of our course free surface flow So, continuing from where we left last time, we will see the continuity equation for open channel flow. In the last lecture, we derived common for steady and you know, the 1D flow A_1V_1 is equal to A_2v_2. And for a three dimensional flow for a steady case, we derived $\frac{du}{dx}+\frac{dv}{dy}+\frac{dw}{dz}=0$ as continuity equations. But now we are going to see continuity equations for open channel flow.

So, this is a typical cross section or you know a sketch of open channel flow. Here in general which is shown that there is a lateral inflow q it might or might not happen, but this diagram you see there is lateral inflow q. This is the free surface initial free surface here and then we have after dt after small time the our free surface has moved here. I will remove this so as to make the figure very clear just to show you this is a general figure. So, this is the top width T.

And yeah, rest of the terms I will explain as we just go on. The continuity equation of the unsteady flow in an open channel. So, this is valid for all whether it is steady flow, whether it is unsteady flow. The difference of mass influx into and mass efflux out of the control volume must be equal to the rate of increase in the fluid mass within the control volume. So, the initial freeze let me just yeah.

No sorry yeah so let me okay so it's better if i just go one by one okay this is the lateral inflow so keeping this figure in mind here the initial free surface is shown by solid line this line okay while the final free surface after a small interval of time is shown by dotted line okay the flow is analyzed through a space between two sections having an elementary distance dx apart. So, our two sections this is one section this is this is one section this is the other section okay and this is the this they are apart by. apart by dx distance. Just trying to explain the figure here a little bit.

Now, the flow in the channel is fed laterally. We are also having an assumption that we are feeding the channel laterally that is q_L. So, the mass influx in time dt into the control volume $\rho UAdt + \rho q_L dxdt$. So, this is the main flow and this is the lateral flow if it is given q_L our simply $\rho q_L dxdt$ and this is the mass influx in certain time and what is *U*, *U* is the area average flow velocity through the left section and A is the flow area on the left section. So, now, the mass efflux in time dt out of the control volume will be simply, see if we assume at after distance out of the control volume that means, after that distance dx and at time *t* + *dt* you might have gone to *U* + *dU*, A must have gone to *A* + *dA*. So, this $\rho UAdt$. So, instead of *U* we write $\rho(U+dU)$ and instead of A we write *A* + *dA*. So, this comes to be this. This is mass influx mass efflux in time *dt*.

And this is the area average flow velocity through the right section and this is flow area of the right section that is the having that is that is actually is at distance dx apart. The rate of increase in fluid mass in time dt within the control volume is nothing but ρT dh/dt dtdx, right. And now what we are going to do, we are going to equate this to the difference of these two terms. And *T* is the top width of the flow, *h* is the initial flow depth.

So, from the statement of continuity equation, we can write $\rho UAdt + \rho q_L dxdt$, that is the one that is going in this is the one going out and this is the one resulting change inside the control volume, very simple. So, now I think I will remove at least this and write it here going out. So, if you simplify the above equation you get $U\frac{dA}{dx} + A\frac{dU}{dx} + T\frac{dh}{dx} = q_L$. And if we use *Q* is equal to *UA* and so this is *q* discharge is *U* into a what is and what is top width but nothing but $\frac{dA}{dh} = T$ at a given section if we use this in this particular equation we come up with $\frac{dQ}{dx} + \frac{dA}{dt} = q_L$ and this is the important continuity equation for open channel flow very simple to derive.

Using the key step is this particular figure. Now, if we further use hydraulic depth what is hydraulic depth A/T and $\frac{dA}{dh} = T$ at a given section. we can write

$U\frac{\partial h}{\partial x} + h_d\ \frac{\partial U}{\partial x} + \frac{\partial h}{\partial t} = \frac{q_L}{T}$ and this is the continuity equation again for open channel flow. It is simple in different terms. The above equations are also are the two different forms of the continuity equation for unsteady flow in an open channel.

This is common for unsteady flow. This one And the one I showed before, the one which is more general, right, but if you put in terms of hydraulic depth and hydraulic radius and stuff like that, you need to come up with this equation. So, for let us see the our channel is rectangular with no lateral flow. So, we say you let us and this is an example we have we are trying to simplify our equation before with no lateral flow and defining the actual geometry of the channel that is rectangular.

This equation will simply reduce because then area will be Byo and you know the height is *h* and then q_L is 0 this will turn out to be $\frac{\partial q}{\partial x} + \frac{\partial h}{\partial t} = 0$. where *q* is the discharge per unit width or equal to *U*h*. This equation was first introduced by Saint Venant. It is a very famous equation Saint Venant equation $\frac{\partial q}{\partial x} + \frac{\partial h}{\partial t} = 0$. So, now, we have seen the continuity equation. Similarly, we will see what happens to the momentum equation for the open channel flow.

Momentum equation you see this is the diagram. So, what does momentum equation states? Momentum equation states that the algebraic sum of all external forces acting in a given direction on a fluid mass equals equals the time rate of change of linear momentum of the fluid mass in this direction. So, in a steady flow the rate of change of momentum in a given direction will be equal to the net flux of the momentum in that direction.

These are the simple definitions valid for I mean every system that we are considering not just for this system. Just trying to explain this you see this is the if this is the flow these are let us say hydrostatic forces I mean the forces this is the weight and this is the weight component acting as a body force. I mean main force that is leading because it is in the direction of the flow. The forces that are perpendicular to the direction of flow we might not be concerned that much, but the forces that are in the direction of the fluid flow direction we are concerned with that. So, the various forces acting on the control volume in the longitudinal directions are as follows as I told you these were the hydrostatic forces, these are the body forces and things like that F_3 is the frictional force, but let us see in a slide it is written properly.

So, F_1 is pressure forces acting on the control surfaces are F_1 and F_2. F_3 is tangential force on the bed that is the frictional force. Body force that is the component of the weight of the fluid in the longitudinal direction is F_4. Now, if you apply the Newton's second law of motion, it is simple F_1, you see this direction $F_1 - F_2 - F_3 + F_4$ will be equal to the change in momentum $M_2 - M_1$. Very simple.

Now, we need to know what M_2 and M_1 is. So, F_4 is $W \sin \theta$, M_2 is momentum flux leaving the control volume. It is very standard definition. We have seen, I mean, we know that from Reynolds transport theorem as well that the momentum flux leaving the control volume is given by $\beta_2 \rho Q \times V_2$. Average velocity at and V_2 is average velocity at section 2 and momentum correction factor $\beta = \frac{\int_A (V^2) dA}{V^2 A}$ we know from before as well and this cross sectional velocity profile V^2 that is the average I mean no I mean the yeah.

And similarly, momentum flux entering the control volume is b_1, b_1 means $\beta_1 \rho Q \times V_1$ and V_1 is the average velocity at section 1. So, basically using these equations here and solving it will give us the momentum equation. I mean we are not solving a general one this is the we are not solving for individual cases, but as soon as in the questions or problems then we will have a defined geometry defined you know lateral and I mean if there are any components we are going to solve it. So now the energy equation for open channel flow. So we saw continuity that is the mass, the other was momentum and just seeing the energy equation for open channel flow.

So similar to momentum, we have this energy. So according to Bernoulli's equation, the total energy head at upstream section 1 should be equal to the total energy So, what does Bernoulli's equation says that the total energy head at upstream section 1 should be equal to total energy head that downstream section 2 plus whatever energy has been lost by the fluid flowing from section 1 to 2. So, plus head hf between the two sections very simple definition. So, $z_1 + h_1 + \frac{\alpha_1 (U_1^2)}{2g}$, this is section 1 is equal to this is section 2 and this is energy loss.

α is energy correction factor given by this equation for small values of θ, cos θ is equal to 1. For, you know, short reach of prismatic channel, so if you have very short reach, this hf will be negligible and α1 and α2 will be 1. And then we have simple you know $z_1 + h_1 + \frac{U_1^2}{2g}$ is equal to $z_2 + h_2 + \frac{U_2^2}{2g}$ very conservation of Bernoulli you know Bernoulli's equation where we do not consider energy losses. Now, values of alpha and beta. the coefficients of an *α, β* are both unity in case of uniform velocity distribution you should remember this α and β will be both one if we have a uniform velocity distribution.

For any other velocity distribution *α* and *β* are greater than 1 the higher will be the non-uniformity of the velocity distribution the greater will be the values of the coefficients. So, you do not need to know exactly, but you should know for the case of uniform you uniform *α* and *β* are 1, but this will always be more than 1 and higher the non-uniformity of velocity distribution *α* and *β* will be high even much more higher than 1. So, if you keep on increasing they will keep on increasing. Generally, the large and deep channels of regular cross section and with fairly straight alignment exhibit lower values of coefficient.

Conversely, small channel with small channel with irregular cross section contribute to larger values of alpha and beta. So, key concept is uniformity and non uniformity in the values of alpha and beta mostly you will be given that or you will be able to give you will be given to find out what that is. Generally one can assume alpha and beta as value one when the channels are straight, prismatic and uniform flow or even in case of gradually varied flow also you can take that. In local phenomenon it is desirable to include estimated values of this coefficient in the analysis. So, but if you have for example, natural channels following values of alpha beta suggested to be used.

What are those values? Natural channel, you see these are values, but you do not need to remember that. I mean, if you have, while solving the assignment and problems, you can look at it when you solve the weekly assignments. However, for exams, most of these variables will be given unless and until the whole question is about solving and finding out alpha and beta . Now, we will solve I mean we will see how one question can be solved in this I mean this particular lecture.

The next lecture number 5 will be dedicated to entire problem solving where I will solve the questions by hand. So, the question is in the measurement of discharge in a river it was found that the depth increases at the rate of 0.5 meter per hour. So, whenever you solve a problem you see try to underline the important things. Depth means if you write by *h* then *dh/dt* is 0.5 meter per hour. If the discharge at the section is 15 m^3/s, it should be 15 m^3/s and the surface width of the river is we have also given the river width as 15 meter.

Estimate the discharge at a section which is 1.2 km upstream. So, the key is to understand the question first we are given the values at first section, we are given how far the section 2 is from the first section and we are also given how much increases per hour *dh/dt*. So, now we look at the try to look at the solution. So, what is the continuity equation for open channel flow because there is no lateral inflow we have $\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = 0$.

This is very important equation. But we also know that $\frac{\partial A}{\partial y} = T$ where T is the top width and y is the flow depth. Thus, if we try to put this, this equation become $\frac{\partial Q}{\partial x} + T\frac{\partial y}{\partial t}$ right. or $\frac{\partial Q}{\partial x} + T\frac{\partial y}{\partial t}$ is equal to 0 very simple equation or dQ means $\frac{Q_2 - Q_1}{\partial x} = -T\frac{\partial y}{\partial t}$ ()/dx is equal to if we take this is written $-T\frac{\partial y}{\partial t}$ or $Q_2 - Q_1$ is equal to minus this we take this side $-T\frac{\partial y}{\partial t} \times \partial x$. You see we have to find Q_2. I mean $Q_1 = Q_2 - T\frac{\partial y}{\partial t} \times \partial x$ or Q1 is you see values are given 15 top width was given *dy/dt, t* was given per hour we can convert it into per second and *dx* was 1.2 kilometers.

Very simple to apply and then Q_1 becomes 15 plus 2.5 that is 17.5 m^3/s. Very simple approach to solve this problem. So, I think it is time to end this particular lecture here. The next lecture will be a little you know maybe 30-35 minutes and we will solve some problems at least 5 problems. So, that you get a good practice and these are the some of the references.

And thank you so much for this particular lecture. And I will see you next class when we solve the problems with hand. Thank you so much.