Free Surface Flow

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Lecture 39

Welcome back, students, to this 39th lecture, or basically the fourth lecture on spatially varied flow. So, what we are going to do is start with the point where we left off last time, and we said that we are going to start with De Marchi's equation for side weir. This is the definition sketch. Definition sketch. This is the height *S*, and this is the width *B*. We can keep coming back to this later as well if we need any clarifications.

Now, again, there are certain assumptions that we need to take into account. The channel is rectangular and prismatic. The other assumption is that the side weir is of short length, and specific energy is taken to be constant between sections 1 and 2. Now, this is equivalent to assuming that S_0 minus S_f is equal to 0, or both S_0 and S_f are equal to 0. Experiments have shown that this is a reasonable assumption.

The side weir is assumed to be a sharp-edged weir with proper aeration of the nappe and to be discharging freely. And here, the kinetic energy correction factor alpha is taken as unity. So, alpha is taken as 1. Now, the SVF equation differential equation with the above assumptions would become $dy/dx\ Q\ -\ dQ$ because S_0 minus S_f, so sorry $-dx\ dQ/dx$ divided by $g(B^2)(y^2)\ /\ (1\ -\ (Q^2)g(B^2)(\ y^2))$, or if we further simplify, we can write dy/dx as

So, the outflow rate is equal to discharge over the side weir per unit length and that is equal to minus dQ/dx is equal to we can assume 2/3 cm $(\sqrt{2g})\ (y-s)^{3/2}$ and this is a standard. standard equation. Here, what is C_m? C_m is a discharge coefficient known as the de Marchi coefficient. C_m is de Marchi coefficient.

Also, since this specific energy is assumed to be constant, the discharge in the channel any cross section is given by Q is equal to By C_m. $(\sqrt{2g(E-y)})$. And from the previous figure, if we substitute all these, we will get dy/dx is equal to $4/3\,C_m/B$ $\sqrt{(2g(E-y)(y-S^3))}$ divided by $3y-2E$. this is an important equation. So, if you assume that C_m is independent of x, that the C_m is what? Demarchi coefficient and if we assume C_m

does not depend on x. And if we integrate this one, we get x is equal to $3B/2C_m$ a function of y, y is the depth, E is specific energy and S that is the height + constant. in which this function is y,E ,s is equal to $(2E - 3s)/(E - s)$ $(E - s)$. This is specific energy also constant, - 3 sin inverse of. Now, this equation, this equation here, this equation A7 is known as De Marchi equation. Now, and the function φ_m is also known as de Marchi variate flow function. Now, if we apply this particular equation that is a De Marchi equation to both the sections that is section 1 and 2, we can write $x_2 - x_1$ is equal to the length is equal to $3/2\ B/C_m\ (\varphi_{M2} - \varphi_{M1})$.

Now, if we know the length s, Q, and y at, let us say, either section 2 or 1, the discharge over the side weir Qs can be computed by this equation. And we also use the continuity equation: Qs is Q_1 minus Q_2. Now, something about the De Marchi coefficient C_m. So, experimental and theoretical studies by Subramanian and Awasthy have shown that in the subcritical approaching flow, the major flow parameter affecting the De Marchi coefficient is the Froude number of the approaching flow. And this is one of the equations that is different.

C_m is a function of and if you further simplify it down, you can write this C_m as a function of the Froude number? However, for supercritical approach flow, the effect of the approach Froude number is insignificant in the variation of C_m. So, if F_1 is greater than 2, we can obtain for F_1 greater than 2, and these two equations are for subcritical flow.

So, there have been many studies on the side weirs in rectangular channels, and the majority of these studies have been done on subcritical approach flow conditions, as this is the most common situation in practice. I mean, there are other scientists who have studied the effect of different parameters like s/y_1, L/B, and proposed this particular coefficient for subcritical flow approach. So, this equation, $C_M = 0.7 - 0.48\,F_1 - 0.3\,s/y_1 + 0.06\,L/B$, is an equation for subcritical flow approach that also includes the effect of s/y_1 and the aspect ratio L/B. So, a more advanced equation. compared to Subramanya. So, they both assume that this C_m depends only on the Froude number, but Borghi and others have studied different other parameters as well and tried to experimentally derive it for subcritical flow.

However, an important equation for a sub-supercritical flow or for F_1 greater than 2 is given by this particular equation: C_m is equal to 0.36 - 0.008 F_1. Talking about uniformly discharging side weirs. So, in many applications of side weirs, such as in irrigation systems and in the disposal of effluents, it is sometimes necessary to have side weirs in which the discharge rate is constant. So, $-dQ/dx$ is constant along the length. And from the previous equation, we know that in SVF, dQ/dx is equal to q*.

It is equal to a constant now. So, in spatially varied flow, we know that dQ/dx is equal to q_*, which is constant now. From this, and $y - s$ is a constant along the weir. If s is kept constant, the water surface elevation will be constant for such weirs. If the specific energy is assumed to be constant, the water surface must be parallel to the energy line, meaning the velocity of flow will be constant for uniformly discharging weirs. Hence, we can write V is equal to V_1 is equal to V_2, or Q_1 is equal to q_*x is equal to Q in this particular case—this is for uniformly discharging side weirs—or $A_1 - q * x$.

V_1 is equal to A, and $q * / V_1$ is equal to $(A_1 - A_2) / L$ or $(A_1 - A_2) / L$. So, instead of this, we put this. Thus, the uniformly discharging side weir can be achieved by linear reduction of the area of the flow, and this can be achieved in two ways. The first one is by contracting the channel side and also by countering the channel bed. I mean, if you want to study more, you can go and look at it in the different literature. Now, for a uniformly discharging side weir, the lateral outflow Q_S is therefore given by this standard formula, and C_m is the de Marchi coefficient, depending on the nature of the approaching flow. So, this is for uniformly discharging side weirs, where Q_S is nothing but $(2/3)$ C_m, where this is de Marchi.

Coefficient $(\sqrt{2g})L \times (y - s)^{3/2}$, ok. Using continuity, we can write $q_1 - q_2 = q_s$. So, we want to do the computations for type 1, 2, and 3 flows. We must follow the same assumptions as indicated in the previous section. Now, the last theoretical topic of this particular spatially varied flow is bottom racks.

What is a bottom rack? A bottom rack is a device provided at the bottom of the channel. It is used for diverting part of the flow. This device consists of an opening in the channel bottom covered with a metal rack. It prevents the transport of unwanted solid material through the opening.

Bottom racks find many applications in hydraulic engineering as intake structures. Examples are trench weirs and curb outlets. Trench weirs are used as water intakes in mountain streams, and bottom intakes prevent gravel entry into the water. So, this is more like what exactly bottom racks are. It is important to understand that they also stop the transport of unwanted material.

Solid material, and these are provided at the bottom of the channel. They are used for diverting part of the flow and are generally used in intake structures. Now, bottom racks can be broadly classified into four categories. One is longitudinal bar bottom racks. What is that? Here, the bars are laid parallel to the flow direction. This is the most widely used type of rack arrangement.

The other is transverse bar bottom racks. The first was the longitudinal bar. The other is the transverse, in which bars are placed transverse to the direction of the flow. The third one is perforated bottom plates, in which a plate with uniformly spaced openings forms the rack. And the fourth is bottom slots.

This is actually the limiting case of transverse bar bottom rack without any rack. So, these are the four different types of bottom racks that can be classified. Further, the above types can be either horizontal or inclined with reference to the approach and bed of the channel. Like this, it can be inclined as well. The trench weir, which finds considerable use as an intake, is an important term.

I mean, you can get a question like, which weir is used as one of the intake structures most frequently in the mountainous streams? So, the trench weir finds considerable importance as the intake structure in mountainous streams. And especially for mini and micro hydel projects, it contains a sloping longitudinal bar bottom rack made up of round steel bars as its chief component. The inclination of the rack, which is of the order of 1 in 10, is provided to facilitate easy movement of the bed sediment load of the stream over the rack. So, this is used in

used in mountain streams. for small hydel projects. So, I mean, this particular figure before it shows this particular figure shows the definition sketch of a longitudinal bar bottom rack. The bars these bars are of circular cross-section and are laid along the direction of the flow. The flow over the bottom rack can attain a variety of water surface profiles depending on the nature of the approach flow.

The state I mean, there are a lot of factors: what type of profiles will appear depends on many parameters the approach flow, the Froude number, the state, the rack, the tailwater conditions. There are different classifications which Subramanya and Shukla have proposed for the flow over the bottom racks, and there are five types: A_1, A_2, A_3, B_1, and B_2. So, in the first one, when the approach is subcritical, the flow over the rack is supercritical. If this is the case, the downstream state may be a jump.

In the other case, the approach is still assumed to be supercritical and flow over the rack is partially supercritical, then the downstream state would be subcritical. If the approach is also subcritical, flow over the rack is subcritical, the downstream side state would also be subcritical. Now, what happens if the approach flow is supercritical and the flow over the rack is also supercritical, then there may be a jump again. And if the flow if the flow at the approach is also if that is B_2 supercritical and it is partially supercritical above the flow over the rack is partially supercritical, then definitely downstream state would be subcritical. So, basically five types based upon

studies by Subramanya and Shukla. So, that are A_1, A_2, A_3 corresponding to subcritical flow in approach. and B_1 , B_2 depending on supercritical approach flow. Now, this particular figure shows the characteristic features of these 5 types of flow. Out of the above type A_1, A_3 and B_1 are of common occurrence, A_1, A_3.

So, A_1, A_3 and B_1. and also of significance from design consideration. You see this one, this is B_2 type, B_2 type was supercritical, partial supercritical and then subcritical. So, here the flow is supercritical and this is the flow over the is partially, so B_2 is partially supercritical. and here it is subcritical.

B_1 type is also that means the approach is supercritical. This should also be supercritical in B_1. And therefore, this is a jump. The A_3 type, so A type means I mean subcritical approach flow. So, the flow here would be subcritical flow.

And if you go and see, the flow over the rack is also subcritical. And therefore, this is also subcritical. A_2 again this is subcritical flow and this one you see here, this one A_2 is partially supercritical. So if it is partially supercritical then to go this is subcritical and to go from supercritical to subcritical the way is whether a weak but at least some hydraulic jump would be required and the last one here it is. Subcritical flow and this one here is definitely supercritical flow but just check.

When you see after that there is a jump, see from supercritical jump could be of any type. So, five types based on the studies of Subrahmanya and Shukla. That is the flow over the bottom racks. Now, we talked about bottom racks. So, there should be some equations also for bottom racks.

So, there is a very famous equation called the Mostkow equations for bottom racks. So, what Mostow derived in expressions—I mean, he derived expressions for the bottom water surface profile for spatially varied flow. So, we derived expressions for the water surface profile for SVF over bottom racks by making some assumptions. The first assumption, as always, is the channel is rectangular and prismatic; the kinetic energy correction factor is equal to alpha, which is equal to 1.0.

The other is that specific energy E is considered constant along the length of the bottom rack. And the effective head over the rack causing flow depends upon the type of rack. So, important assumptions. The effective head over the racks, which causes the flow, depends on the type of rack such as for racks made of parallel bars, the effective head is equal to the specific energy, and for racks made of circular perforations, the effective head is equal to the depth of flow. So, it could be of two types: the head could be equal to the specific energy E for parallel

the head could be equal to specific energy, and this head could also be equal to the depth of flow for circular The differential equation for SVF with lateral outflow under the assumptions would become this is q_*, but let me write it down again: dy/dx is equal to

$qy\ (-dQ/dx).q\ *$, and this is $q * \ g(B^2)(\ y^3)\ -\ Q^2$. So, this equation can be solved mostly using all the methods that are employed for any spatially varied flow.

SVF equation and gradually varied flow equations. For example, the Runge-Kutta method (RK). Third or fourth order, Merson method, trapezoidal method. All these three methods can be used. So, I think this actually concludes the theoretical part of spatially varied flow.

In the last and final lecture on spatially varied flow, we will dedicate 30 minutes especially to solving one or two problems. If time permits, we can solve more. But we will see how the problem of spatially varied flow is solved. So, that is it for today, and see you in the next class.