Free Surface Flow
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Lecture 38

Welcome back, students, to the eighth module, the third lecture of this particular module. In the last class, we discussed the control points. We saw how to identify those control points, which were determined by estimating the critical points and other factors. We also saw how to calculate the transitional profile and how to solve this profile. Additionally, we talked about different methods. We estimated or rather concluded that the methods are exact. If you adopt the same method as for the computation of gradually varied flow profiles, you will obtain good solutions. Although we did not, we also mentioned that when it comes to this type of course, it is important to note that these types of solutions are very complex and can be solved using computers.

Solutions using hand calculations might not be very feasible, but to make the points complete, we saw the method and the solution procedure to tackle these problems. We also started with the assumptions for deriving the differential equation of spatially varied flow with decreasing discharge, but then we decided to continue that today in this particular class, starting from the beginning right from the assumptions. So, that being said, without further ado, I think we will begin the discussion. I mean, the derivations and the assumptions of the differential equations of SVF with decreasing discharge. As you may recall from the last class but repeating again here the pressure distribution is hydrostatic, which is an important point.

This means we have a smooth curvature; we do not have any rapid rise or fall of water. This is the same assumption as adopted in uniform flow and also in gradually varied flow. The second assumption is that the solution we are going to derive or the method of analysis we are going to use will be one-dimensional in nature. We also say, similar to uniform flow, where Manning's equation was used. We are also going to use Manning's equation to adequately represent friction here, as in the case of gradually varied flow.

And, we also said that, like the differential equation of SVF with increasing discharge, where the addition of water was not contributing to the momentum. Here also, we say the withdrawal of water does not affect the energy content per unit mass in the channel. So, momentum is not being affected. And the flow is steady. The channel is of a small slope and is prismatic in nature.

These were some of the assumptions that we started our discussion with. I mean the derivation with. Also, we mentioned that the total energy at a section, at any section in SVF, is given by $z + y + V^2/2g$. I also said that we might adopt the energy approach for the derivation of the differential equation this time for the solution of SVF with decreasing discharge. Now, if we differentiate with respect to x, what can we get?

dH/dx here will be equal to $dz/dx + dy/dx + d/dx$ of this term, which is this one here. Now, we know that this dH/dx, this is the energy slope -S_f, and why minus? I think you should remember from our previous classes. That is a question that I leave up to you: why it is -S_f, and dz/dx is -S_0, and why? Here also, y. Also, now we see we are looking at this individual term d/dx of $V^2/2g$, we can write V as Q/A. So, this we can write d/dx of $V^2/2g$ is equal to d/dx of $Q^2/2gA^2$. Now, we take this outside $1/2g$, and we can write $1/2g\ d/dx$ of Q^2/A^2. Or if we differentiate this, we can write d/dx of $(Q^2)/(A^2)$ as $2QA^2\ dQ/dx + -2(Q^2)\ dA/dx$ divided by $A^3. -2Q^2 dA/dx\ A^3$.

If we put this here, this is what we get. d/dx of $V^2/2g$ is $-2Q^2\ 1/2gA^3\ dA/dx + 2Q/2gA^2\ dQ/dx$. Or d/dx of $V^2/2g$ is equal to $-(Q^2)T$. This is because we write A in terms of top width and y. So, this becomes dy/dx. So, A is d/dy, and if we substitute it here, we get $-9(Q^2)T/gA^3\ dy/dx$, and this dQ/dx is q*. Lateral exit because these two get canceled, these two get canceled.

Or, and this is, yeah. Now, equation 1, if you put this here in the main equation, we can rewrite it as $-S_0 - S_f$ is equal to $-S_0 + dy/dx - (Q^2)T/gA^3$ into $dy/dx. + q*/gA^2\ I$ think we will take dy/dx common from this one, and other terms we will shift to one place. And we can write $S_0 - S_f - Qq*/gA^2$ is equal to dy/dx, and this will be taken down this side in the next step.

And we can write dy/dx is equal to $S_0 - S_f - Qq*/gA^2$ -. So, if you remember, if we take out this term, the only term remaining will be this one. So, that means it will look like it will be gradually varied flow GVF equation when q* is equal to 0. Anyways, this is not the main message.

You will also observe how this equation is different. This is, if you remember, in the increasing discharge there was a term 2 here. Now, this is the differential equation of SVF with decreasing discharge. Important equation. Do not worry, and this slightly, so I will cut this one because there is no 2 here.

So, let me, for the sake of writing, because this has become too dirty there. So, let me write $S_0 - S_f - Qq*/gA^2$ Okay, let me, oh no, I think I need a different separate place. 1 $-(Q^2)T$. So, this is the equation. We will come across this particular equation later as well.

So, let us proceed. We will see with one typical type of derivation or the problems for this one. So, the question is a horizontal frictionless rectangular lateral spillway channel of length L and width B has a free overfall. There are certain things that means it is given horizontal that means s is equal to 0 frictionless no friction. And rectangular, it is that is the shape.

The length and width is also given length L and width. Now, we have to show on this one that is free overfall means that we are having a decrease in the discharge. First, you have to estimate what type of SVF you are talking about spatially varied flow. Here it is an equation with the decreasing discharge because there is a free overfall. As I said, this given case is a spatially varied flow with decreasing discharge.

And we know that this q_* is dQ/dx or dQ can be written as $q * dx$ or capital Q is $q * x$. And we also know the top width *T* is given as *B* and area is *By* because it is rectangular Now, the dynamic equation of SVF with decreasing discharge is given like this, which I have written in the analytical problem while writing down this equation because of this pen. Anyways, so now you remember this is an important equation where dy/dx is equal to $S_0 - S_f - Qq*/gA^2$ / $(1 - (Q^2)T/gA^3)$. And since the channel is horizontal and frictionless, so we have $S_0 - S_f$ both 0. So, this will not be 0.

So, I think this is at the wrong location. So, this is going to be 0. Anyways, thus we get dy/dx is equal to minus $Qq*/g\,A^2$. by $(1-(Q^2)T/gA^3)$. And then we can write, instead of we can write Q is equal to $q*x$ because that is a free overfall divided by gA is By to $g(B^2)(\,y^3)$. or q is $(q*2)(\,x^2))/g(B^2)(y^3)$.

Now, this one we solve, we write $g(B^2)(y^3)\;-\;q*(\,x^2)/g(B^2)(\,y^3)$. or dy/dx is equal to $-(\,q*2)x\;g(B^2)(\,y^2)$. This one divided by this one multiplied by $(B^2)(y^3)$ divided by $g(B^2)(\,y^2)(\,q*2)(\,x^2)$. You see this, if this gets cancelled, only y will remain. Or this becomes

$(q^2)x\times y$ divided by $g(B^2)(\,y^3)\;-\;(q*^2)(\,x^2)$ or now we do dx/dy, we just divide or I mean divide by 1. This is we simply reverse the equation. So, the numerator becomes denominator and denominator becomes Other way of saying is we bring dx this side, dy this side, this one this side and this one this side.

So, we get dx/dy is equal to $-(\,g^2)(\,B^2)(\,y^3)-\;q*x,\,q*\;xy$. or dx/dy is equal to this - becomes $+\,q*\;x^2$divided by $q*xy\;-\;g(B^2)(\,y^3)$. So, certain things can get cancelled, this one get cancelled, this one get cancelled. This becomes $x/y\;-g(B^2)(\,y^3)$ divided by twice, here one y will get cancelled, this will become y square and this y gets cancelled. or $x\;dx/dy$ so multiplying by x. We get x dy $x\;dx/dy$ is equal to $x^2/y\;-\;(g^2)\;(B^2)(\,y^2)$ divided by $q*2$.

or xdx is equal to x^2. I mean you bring on left hand side, take care that change the of the left hand side. So, it becomes $x\;dx/dy\;-\;x^2y$ is equal to$-\;(g^2)\;(B^2)(\,y^2)$ divided by $(q*^2)$ and this is the thing that we needed to prove. So this was one of these examples that we solved.

Now we will start with something on side weir. So side weir is a hydraulic structure. First thing to understand is that it is a hydraulic structure. So, side weirs other name is also lateral weir and what is that is a free overflow weir set into the sides of the channel which allows part of the liquid to spill over the side when the surface of the flow in the channel rises above the water crest. So, it is a hydraulic structures which controls the flow of the water.

It stops the water up to a certain depth, and if the water level rises above that, then it allows for the free overfall or the discharge of the water. Now, side weirs are extensively used as a means of diverting excess stormwater from urban drainage systems and as water control devices in flood channels or flood control works. In irrigation engineering, side weirs of broad crest are used as head regulators of distributaries and escapes. So, weirs are very important and are also called lateral weirs. So, what I mean is, side weirs are usually short structures with—this is what you see.

If this is the length L and this is the width P, the aspect ratio L by B is less than 3. So, the length is less than 3 times the width and is therefore called a short structure. It is obvious, I mean— There are certain things that can be concluded: the specific energy consideration, the longitudinal water surface should increase in the downstream direction when the main channel flow is subcritical throughout. Similarly, the water surface profile would be a decreasing curve for supercritical flow in the channel, this one.

And this is the side weir. This is meant for diverting the flow. You will see, if you go and see hydraulic structures in Uttarakhand, there are a lot of them, and you will see the side weirs or the lateral weirs being used a lot. So, there are different types as well, you know, and we are going to discuss or talk a little bit about different types. Here, what happens is the channel is on a mild slope, and the weir height S here—this S is greater than the critical y c1, which is the critical depth corresponding to the incoming discharge q1 at section 1.

So this is section 1, this is section 2. So, the type 1 would correspond to this S is greater than the critical depth and critical depth which is corresponding to the incoming discharge Q_1 at section 1. At the downstream end, the normal depth corresponding to discharge Q_2 will prevail. Thus, y_1 will be equal to y_t that is the tailwater depth.

At section 1, the depth y_1 will be such that y_1 is greater than y_{c1} and that is greater than is less than y_0. So, y_{c1} will be less than y_1 and y_1 will be less than y_0 where y_0 is the normal depth for q_0 is equal to q_1. Along the weir, the depth increases from y_1 is equal to y_2. You see this depth is increasing. Upstream of the section 1, there will be an M_2curve from y_0 to y_1. the control for the SEF will be downstream depth y_2 is equal to y_2. As I said, this is subcritical, so the control for the SVF will be downstream towards the end.

This is type 1. There is another type, type 2. Here also the channel is on a mild slope that is y_0 is greater than y_{c1}.

This is the condition for mild slope and with s less than y_{c1}. This is type 2. Here, this s is less than y_{c1}. If the weir and this happens if the weir is long, flow below the critical depth is possible.

At the upstream end of the weir, the depth $y_1$can be considered equal to y_{c1}. This y_1 will be equal to y_{c1}. At the downstream depth, y_2 will rise rise to the tail of the water depth, and there will be a hydraulic jump here. So, the control of this type is at section 1; control of type 2 is at section 2. section 1. So, this is another type of. Now, talking about type 3 here, the channel is on a steep slope. That means y_0 is less than y_{c1}. and width is, and so here this S is again less than the critical depth.

The upstream depth y_1 is equal to y_2, decreasing depth here. The decreasing water depth profile starts from section 1, and at section 2, the depth reaches a minimum value. In the downstream channel, the water surface rises through an S_3 profile to meet the tailwater depth yt. This is the tailwater depth. And this is also here, which is the control for type 3. This is also the control for type 3.

So, in the first two, it was mild slope type 1 and type 2. In the first case, s was greater than the critical depth; in the second type, it was less. Now, in the third case, s is less than the critical depth, but the slope is steep. And the water here is decreasing. In the earlier two cases, the water was rising in the first case, and in the second case, it was decreasing slightly before a hydraulic jump occurred. Then, it rises through an S_3 profile. So, I think it is better to stop this lecture here because, in the next lecture, we will start with the de Marchi equation for side weirs.

We are going to talk about different types of equations that are used for solving different types of weirs. And that being said, I will see you in the next lecture.