

Free Surface Flow
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Lecture 37

Welcome, students, to yet another lecture on spatially varied flow. In the last class, what we did was derive the governing equation, or the dynamic equation, for spatially varied flow with increasing discharge. Today, we are going to continue on that a little more, talk about control points, and if time still remains, we will start the derivation of spatially varied flow with decreasing discharge. So, as I mentioned about the control point, what is a control point? Let us say if the flow is subcritical everywhere in the channel, the control point of the profile will be located at the downstream end of the channel.

So, if the flow is subcritical everywhere. However, for all the flow situations other than the above, the determination of the control point is necessary to start the computations, as we have also seen that point in our gradually varied flow theory and calculations. So, in spatially varied flow with increasing discharges, the critical depth line is not a straight line parallel to the bed as in GVF, but is a curved line. Depending on the combination of the bottom slope, channel roughness, and channel geometry, the critical depth of spatially varied flow can occur at a location somewhere between the ends of the channel, giving rise to a profile which may be subcritical during the first part and supercritical in the subsequent part of the channel. And this is specific to

So, this is specific to spatially varied flow. The basic differential equation governing the motion in SVF with increasing discharge, as we have seen in the last lecture. So, our equation can be modified because our last equation was $S_0 - S_f$ minus—I mean, it was a slightly modified form—but $gA^2 S_0$ divided by $1 - Q^2T/gA^3$. $1 - Q^2T/gA^3$. So, there was no S_0 here.

But yeah, this was from our previous lecture. But anyways, so now Q is let us define Q as equal to $K\sqrt{S_f}$, which is the actual discharge, and Q_n is nothing but $Q_n = K\sqrt{S_f}$, which is the normal discharge in the channel at depth y . So, at any point, Q is equal to $K\sqrt{S_f}$, Q_n is

given as $K\sqrt{S_f}$, and this Q_n is nothing but $(gA^3/\beta T)$, and this is critical discharge modified by β , this β . Now, this equation—this equation A₁ reduces to So, you see here: $1 - S_f/S_0$ is $-K^2Q^2/K^2Q_n^2 - Q^2/Q_n^2 \times 2\beta K^2q^*/gA^2Q / (1 - Q^2/Q_c^2)$, or here, K square K square gets canceled, and this becomes Q/Q_n .

Right, and yeah, and So, this becomes a common term, and then this is $1 + 2\beta K^2q^*/gA^2Q$ and $1 - Q^2/Q_c^2$. Now, if we redefine Q_{n1} as the modified normal discharge Q_{n1} , we can write $Q_n/(1 + 2\beta K^2q^*/gA^2Q)$ Then, the location of the transitional profile at any depth at any distance x would be determined by the condition $Q_{n1} = Q_c$. The intersection of the transitional profile with the critical depth line will satisfy the condition $Q = Q_{n1}$. This is important and hence would locate the control point.

This is the standard method of locating the control point in SVF with increasing discharge. That is the section at which the actual flow would pass at critical depth. At transitional depth, as we have seen, $Q_n + Q_{n1} = Q_c$ or Q_n divided by Q_n —what is Q_{n1} ?

We have seen from this previous equation here, this is Q_{n1} . So, $Q_n/\sqrt{1 + 2\beta K^2q^*/gA^2Q}$ is equal to critical this is Q_c . Then we simplify this and we can write $2\beta K^2q^*$. So, this is So, what we can actually do is we can square root this whole side and simplify it in this format. So, it becomes $\sqrt{1 + 2\beta K^2q^*/gA^2Q}$ is equal to β

$Q_n^2T/gA^3 - 1$. And we substitute Q_n is equal to $K\sqrt{S_0}$ and simplify this is the equation that we are going to get q^*/Q is equal to half of $S_0T/A - gA^2/\beta K^2$. And now this is the equation of the transitional profile for SPF with increasing discharge. transitional for spatially varied with increasing S_0 , in general, in SVF, with increasing discharge, we can write Q is equal to $Q_i + q^* dx$.

This is the lateral entry term. lateral entry term, where Q_i is the channel discharge at x_0 is equal to 0 for the SVF in a lateral spillway channel Q_i is equal to 0 and q^* is equal to constant that is Q is equal to q^*x . So, if we use this Q is equal to q^*x which simplifies our equation to $1/x$ is equal to half of $(S_0T/A - gA^2/\beta K^2)$. The suffix t denotes the transitional profile in this one here. It is interesting to note that in an SVF due to side

channel spillway the transitional profile is independent of the rate of lateral inflow. And now how to locate the control point to locate the control point the critical depth line is the I mean that is the first thing that is needs to be calculated and plotted to scale.

No, I mean, we have to note that the critical depth line for SVF with increasing discharge is to be calculated by using this relationship. This is the procedure for finding the control point. Now, the transitional profile is then calculated by our previous equation here. That equation A3, this one, is an important equation. And we can plot it in the same figure as well.

Now, this method is of general use and can be easily incorporated into a numerical method algorithm to compute SVF profiles using a digital computer. So, this entire method can be solved numerically using computers. So, this is how it looks like—you know, this is the control section, and if we try plotting the water surface profile. So, this is, you know, the transitional profile indicated by this, and this will be the control point R . So, the first step is always finding that particular point, which is Q_c , this one. Now, the classifications and solutions.

So, unlike gradually varied flow, spatially varied flow with lateral inflow is not—I mean, that is not so well-known. I mean, people have not studied it that much. And therefore, a lot of detailed classification analysis is not available—I mean, in the literature or research papers. So, by assuming, first of all, we assume zero friction and the value of beta as one, that is the factor. There are people who have made a detailed study—Li, there is a scientist called Li who has made a detailed study and classified the flow into the following categories.

So, flow classification of the SVF. was given by I mean, we are going to discuss different types. So, type A. Here, the flow is subcritical throughout the channel, and the Froude number increases continuously in the downstream direction. This is one type of SVF.

The velocity will increase because of the increasing discharge. Therefore, what is going to happen? The Froude number will increase, but the flow will remain subcritical throughout. There is another type where the flow is subcritical throughout, but the Froude number will

first increase, reach a maximum value less than unity, and then decrease. So, this is another type of SVF classification: type *A* and type *B*.

Now, there is another type, *C*. Here, the flow is subcritical initially and passes through a critical section to become supercritical in the downstream portion of the channel, then terminates in a jump. So, in type *C*, the flow is subcritical initially and then passes through a critical section where the Froude number is equal to 1, becoming supercritical in the downstream portion of the channel. And because the Froude number will keep increasing, it will terminate in a jump due to downstream conditions—for example, if there is a structure. The last is type *D*, which is the same as *C*, but the jump is not formed in the channel; the outlet is free. So, basically, four types in which.

Initially, it will start with subcritical flow only. In the first two types, it will still remain subcritical. In the first one, the Froude number will keep on increasing. In the second one, it will reach a maximum value which will be less than 1, but then keep on decreasing. In the third one, the subcritical will change to critical and then supercritical.

The fourth one is the same as *C*, but there is no hydraulic jump being formed. So, these four types of flow can be determined by a study of the transitional profile and the critical depth line along with the downstream end conditions. In general, type *C* and *D* solutions—where do type *C* and *D* occur in a side spillway channel design?—and type *A* and *B* can occur in wash water trough and gutter design problems. So, these are some of the examples where type *A*, type *B*, type *C*, and type *D* can happen. So, this is one of the most common ones—spillway.

So, type *C* and type *D* are quite common there. Now, similar to gradually varied flow, we know we have to compute the profile, right? As already indicated, the basic differential equation of SVF with lateral inflows is nonlinear, and no closed-form solutions are available for a general problem. A host of numerical techniques are, however, available for its solution. The computation proceeds from a control point.

How do we do that? We proceed from a control point where the flow parameters are known. So, control points are also said to be the points where flow properties are known beforehand. for example boundary conditions, boundaries, anything, any boundary.

Now, regarding the frictional formula exclusively for SVF, a convenient uniform flow, I mean, we can use the Manning's formula. There is some evidence that the value of the roughness coefficient is likely to be higher in SVF than in uniform flow. Till conclusive results are available, it is prudent to use. So, we are going to use uniform flow variables. Experimental studies have shown that the assumption of is unrealistic and a proper selection of beta will.

So, what we need to do is we need to select a proper beta because beta is not equal to 1 in SVF. And if we choose a proper beta, then we can enhance the quality and the accuracy of the prediction of the spatially varied flow profile. n formula is still used. However, in reality, friction in SVF is a little higher than uniform Now, talking about different numerical methods that we use for solving the SVF equations, they are more or less very similar to what the gradually varied flow.

As I said, I have mentioned this many times—the advanced numerical methods discussed in connection with gradually varied flow computations are all eminently suitable for, you see, GVF computations are all eminently suitable for SVF computations also. So, this is the main equation. Here, in this particular equation, R is a function of x and y and can be written as, you know, so this is the main equation. We adopt two methods: SRK and KM method.

And we have seen that also in GVF; now we are going to see that a little more in detail in the next slides. So, SRK is nothing but the standard fourth-order Runge-Kutta method, or SRK stands for standard Runge-Kutta—it could be third-order as well, but here what we see is we define that in terms Fourth-order means there are four different steps or four different stages; the accuracy is also fourth-order. So, what we generally do is, for example, if this is the x , we say this—so this is, you know, the value at x , x_0 , x_1 , x_2 , x_3 . So, let us say

the value at x is called x_i , is called y_i . So, at the boundary, that is y_0 is known, because that is the boundary condition. So, how we do it is $y_i + 1$, that is y_1 , is equal to $y_0 + 1/6 (K_1 + 2K_2 + 2K_3 + K_4)$. This is an advanced method where we calculate $K_1 = \Delta x f(x_i, y_i)$, $K_2 =$

$\Delta x f(x_i + \Delta x/2, y_i + K_1/2), K_3,$ and K_4 . In the third-order Runge-Kutta-Merson method, there are three steps.

So, these are the same as the one that we have discussed in gradually varied flow. This is also the same Kutta-Merson method. So, now here is K_1, K_4, K_5 , but we need to calculate for K_4 and K_5 , we need K_3 s as well, K_1, K_2 , for K_5 , we need K_2 s as well. So, we need K_1, K_3, K_4

For K_4 , we need K_1, K_3 , for K_3 , we need K_1, K_2 . So, we need to calculate. So, this is more iterative in procedure. The good part is, we obtain one solution instead of going into a lot of iterations.

This is another method which is called the trapezoidal method for SVF. This also we have seen in GVF. If time remains, we will try to solve one particular problem, but these problems are so lengthy and cannot be generally solved using pen and paper. We need a computer for that. So, probably we might not, but if time remains, we will see.

So, as I said, the details of these particular methods—the three methods. The standard Runge-Kutta method, the Mersenne method, and also the trapezoidal method. There is a trapezoidal method for SVF. So, trapezoidal means you see f of x_i, y_i and $f(x_i + 1)$, and $y + I$; half of it multiplied by Δx is the value that we are going to put at the other one. It is more like—it is like a trapezoid.

So, I mean, they are almost the same as we did for gradually varied flow computations. Now, we have discussed a lot about the differential equation of spatially varied flow with increasing discharge. Now, what we are going to do is see the differential equation of spatially varied flow with decreasing discharge. So, we will adopt a similar strategy for the derivation. Now, what are the assumptions?

Assumptions are the same: the pressure distribution is hydrostatic, and this amounts to assuming again that the water surface curvature has to be moderate. No sudden jump or fall in water surface curvature. Profile. Best is smooth curvature. And here also, the one-dimensional method of analysis is adopted.

The frictional losses are adequately represented by Manning's formula, same as the one with increasing discharge or the GVF. Withdrawal of water also, we assume that when the water is being withdrawn, it does not affect the energy content per unit mass of water in the channel. And the flow is steady. So, these are some of the assumptions. And here, the channel is prismatic and of very small slope.

So, again, I think the assumptions we are going to state are: one is hydrostatic pressure, the other is the *ID* method is adopted, frictional losses by Manning's n , withdrawal of water does not affect the momentum of the system, the flow is steady, which means $\frac{d}{dt}$ is equal to 0 . And the channel is prismatic and is of a small slope. These are some of the conditions that we need to consider for deriving the differential equation of SVF with decreasing discharge. Now, the total energy at a section is given by H is equal to $z + y + V^2/2g$. Here, we are going to use the energy method to analyze

with decreasing discharge. That being said, I think it is because we do not have enough time remaining in this lecture to complete the derivation. So, we will start again from the assumptions in the next lecture to maintain continuity. But just remember certain assumptions again: hydrostatic pressure, *ID* method, frictional losses represented by Manning's equation, withdrawal does not affect the momentum in the flow, the flow is steady, and the channel is prismatic and of small slope, same as before.

So, same as those of the differential equation of SVF with increasing discharge. So, I think I will close the lecture a little bit early today, and we will start again in the next class with this particular slide, where we will discuss again the assumptions that are required for the derivation of the differential equation of a spatially varied flow with decreasing discharge. Thank you so much, and see you in the next class.