Free Surface Flow

Dr. Mohammad Saud Afzal
Department of Civil Engineering
Indian Institute of Technology Kharagpur

Lecture 36

Welcome students to our new module today that is about the spatially varied flow. In today's lecture, we will try to introduce the topic to you and then further continue with other lectures about more details. Maybe we will cover 2 to 3 lectures about the concepts. And in another two lectures, we will try to solve some of the problems. That is the plan.

It could change as well. If we finish our lectures a little early, then we can start the problems before. So, what is spatially varied flow? So, the steady spatially varied flow, what it represents? It represents a gradually varied flow with non-uniform discharge.

So, important message is that spatially varied flow is nothing but a variation of gradually varied flow. We will come to more details soon. Important to notice that when a gradually varied flow, if the flow is either taken out or added in laterally, okay, withdrawal or lateral addition, because of that the discharge in the channel varies along the length, all right. and thus spatially varied flow is formed. So, spatially varied flow is nothing but a gradually varied flow with non-uniform discharge in a way that either the flow is added or removed.

One of the typical examples of one in the gradually varied flow where the flow is being added is for example, in the drains and canals where the water comes from the home, or even the rainwater comes from different places and gets added to the main canal. And the withdrawal a similar thing where you know because of the supply line at the house. So, the water is flowing in the main line and water is taken away and supplied to different places in the building. Based on this, the spatially varied flow can be classified in two categories.

The first one is spatially varied flow with increasing discharge. The second one is spatially varied flow with decreasing discharge, two classifications. of spatially, spatially means in space or famously known as SVF. Now talking about a spatially varied flow with increasing discharge. So, spatially varied flow with increasing discharge finds a lot of practical applications as I just mentioned a little bit in the previous slide.

So, for example, flows in side channels, spillways, wash water troughs in filter plants, roof gutters, highway gutters and some of the typical instances. So, for example, side channel, spillway, wash water troughs in filter plants, roof gutters, because in that the water is added, highway gutters. So, these are one of I mean, some of the examples where you can see spatially varied flow and the discharge being added to the main line. Now, differential equation of a spatially varied flow with increasing discharge.

So, what we are now going to see is this differential equation. We have seen the differential equation of GVF before. So, how that equation is modified or how we actually come across and solve the spatially varied flow right from the beginning—that differential equation is of primary concern here in this particular lecture. So, first of all, there would be some assumptions. The assumptions would be, first, that the pressure distribution is hydrostatic.

So, this is the same as uniform flow and also gradually varied flow or GVF. Now, what does this assumption mean? It amounts to assuming that the water surface curvature is moderate. The water surface curvature is not too random, not like a rapid one.

It is like, you know, very—I mean, this is also rapid. Let us say like this. So, this is like a GVF; this is an RVF, for example, the water surface curvature. The other assumption is that the one-dimensional method of analysis is adopted. We will use the 1D method, which we will see in the next slide.

The frictional losses in SVF are assumed to be adequately represented by uniform flow. It is the same assumption that we have made for the gradually varied flow as well. So, here also, same as in gradually varied flow and uniform flow, we have the same resistance equations such as the Manning's formula, and Chezy's can also be used. We also neglect the effect of air entrainment. It is also assumed that the lateral flow does not contribute any momentum.

It is important because if you contribute, if you send in some lateral discharge, it will have some momentum in the longitudinal direction. But our assumption is that we do not, I mean, this lateral flow will not contribute any momentum in the longitudinal direction as such. Now, one of the other assumptions is, as always, the flow is considered to be steady. So, that means del/del t is equal to 0 for any quantity, and the channel is prismatic and of small slope, small slope, and the channel is prismatic. This is another assumption.

So, there are different assumptions for the derivation that we are going to see. So, let me explain this diagram to you a little bit. This is a channel, this is the bed, and We assume two sections, section 11 and section 12, sorry, 11 and 22. So, at section 11, we see what are the things; there is the pressure force or F_1due to the hydrostatic pressure, basically.

And similarly, at section 22, it will be in this direction. If we assume the velocity is V here, we say it might have changed to a value $V + dV$ here. The weight component between section 11 and section 22 is here. This is the frictional resistance. The total length that we are assuming is L, and this is a small part dx.

So, most of the forces we have written, we say that the discharge is Q. And, we also assume that the discharge is being added laterally. So, we say at section 11, this is Q, and at section 22, the discharge becomes $Q + dQ$. I mean, we are assuming it could change as well, and the same for velocity, that is V and $V + \Delta V$. And Qw, as I said, is the lateral inflow per unit width per second.

It is meter cube per second per meter. Now, we try to calculate the momentum flux. At section 11, this section. It will be M_1 is equal to a very simple $w/g\ Q \times V$. This is the standard formula. w is nothing but the unit weight of water.

ρg. Similarly, the momentum flux at section 2, M_2, is equal to w/g, right. Instead of Q at the section, we have $Q + dQ$, and instead of V, we have V plus dV because this is nothing. This is the discharge at section 2, and this is the—sorry, this is the $V + dV$ is the velocity at section 22. Similarly, this is the discharge at section 11, and this is the velocity at the section 11.

All right. So, hence the rate of change of momentum between 11 and 22 is normally given by $M_2 - M_1$, that is, the flux from this $M_2 - M_1$ will be the difference. This is the rate of change of momentum, which will be $M_2 - M_1$. And simply, $M_2 - M_1$is w/g $(Q + dQ) \times (V + dV)$ minus this M_1, which is $w/g\ Q \times V$. Or we can write w/g. Now, we multiply these two quantities. This will come to QdV—sorry, first it will come to $Q \times V, + QdV + VdQ + dQ\ dQ\ dV$. Four terms will be there. and in this one. And this $QV\ w/g$, this w/g will cancel out with this one. So, in the end, we will have only three terms remaining: this one, this one, and this one. So, $w/g\ QdV + VdQ$ and $dVdQ$.

But also, we know that the product of the differentials $dQ\ dV$ is very small and can be neglected. So, what we are going to do is we are going to neglect this one as well because $dQ \times dV$ is very, very small. Hence, $M_2 - M_1$ can be written as $w/g\ (QdV + VdQ)$. Now, writing the momentum equation, so now we are going to use Newton's second law, which is the rate of change of momentum is equal to the net force.

And we know that we already have so many forces here that we have seen. You see, we have P_1 forces, P_1, P_2. I mean, due to P_1, P_2, $W\ sin\ \theta$, $F\ of\ F$. So, there are certain forces that are already there. Now, so that will $M_2 - M_1$ will also be equal to $P_1 - P_2 + W\ sin\theta - F_f$. If you look into the force balance diagram, let me go once again there so that it is easier for you. You see, P_1 is in the positive x direction, P_2 is in the negative x direction, F_f is in the negative x direction, and $W\ sin\ \theta$ is in the positive x direction.

$M_2 - M_1$ is equal to $P_1 - P_2 + W\ sin\theta - F_f$. And what is P_1? P_1 is the pressure force at 1-1, which is equal to nothing but w, that is, the unit weight of water A into z bar, where z bar is the depth of the centroid of area A from the free surface. And this pressure force at 2-2 is nothing but $w \times (\bar{z} + dy)$ because we have assumed at section 2-2, the depth has changed by dy, and w is nothing but weight, and F_f is frictional resistance. This is what we have. Now, the difference in pressure forces between 1-1 and 2-2 is given by $P_1 - P_2$, which is given by $wA\ \bar{z} - w\ (\bar{z} + dy) \times A$. This is for$P_1$, and this is $P_2\bar{z} + dy$.

So, this will finally come out to be $-wAdy$. So, the difference between pressure 1 and pressure 2. Now, if we use this equation here, on the right-hand side of this one, if we put $P_1 - P_2$ here, then we get $M_2 - M_1$, which we know from before, and this is $M_2 - M_1$. This is $P_1 - P_2$; the rest of the things are the same. Now, let's talk about the frictional force F_f. So, the frictional force $F_f$along the channel wall is equivalent to the pressure due to the friction head multiplied by the average area.

So, instead of simplyF_f being the unit weight of water $\times\ (A + dA/2) \times h_f$, where $h_f$is the frictional head. Or, $h_f$can be written as $S_f\ dx$. So, hf can be written as S_f dx. Now, if we neglect the product of the differentials, this h_f can be written as $wAS_f\ \ dx + w\ dA/2\ S_f\ \ dx$. If we neglect the

So, dA multiplied by dx will go to 0, and this will come out to F_f being equal to $wA\ S_f\ dx$. Now, what is the next step? We will also substitute F_f in our previous equation. Which was the previous equation? This equation. And Sf is nothing but the energy slope, which can be obtained from Manning's equation.

We have seen a lot, F_f is $n^2Q^2/A^2R^{4/3}$ or $n^2V^2/R^{4/3}$. We have seen this a lot, this particular equation. Now, the component of weight along the longitudinal direction is $w\ sin\ \theta,$ we are going to see, nothing with $\rho\ gA \times dx$ and $sin\ \theta,$ this is weight. It is a very simple volume, $\rho gA\ dx\ sin\ \theta$ is the volume, g is the acceleration due to gravity, and this is the component. Since for a small bed slope, $sin\ \theta$ goes to So, $W\ sin\ \theta$ can be written as $WA\ dx\ S_0$.

Use this in the main momentum equation. Now, if we put all those things, the value of $W\ sin\ \theta\ F_f$ in our main equation, we can get this was $M_2 - M_1$, this one was $P_1 - P_2$, this is the weight component, this is the frictional component. Or *w* can get canceled from each side. And then we can write $1/g\ (QdV + VdQ)$ is equal to $-Ady + Adx(S_0 - S_f)$.

And this one, you see, not just that, we can take out a dx common here from here and here. So, it becomes $S_0 - S_f$. This is the equation that we are going to get. Or we can write 1/g, we bring a down. So, we take a common from here and here and bring it later here, it will give us $1/g(QdV + VdQ)/A$ is equal to $-dy + dx(S_0 - S_f)$.

Or, instead of *dy*, what we are going to do is we are going to take this one on this side. Sorry, so what we are going to do is we are going to take *dy* on this side and bring it to this side. So, our equation will become *dy* is equal to plus $dx(S_0 - S_f) - 1/g\ (QdV + VdQ)/A$. So, what we have done here, just to reiterate, is we are taking dy to this side and this one to the right-hand side of the equation to obtain this particular equation. Or, *dy* can be written as, so Q we divide these equations by so $QdV/A + V/A\ dQ$. And Q/A is nothing but $VdV + V/A\ dQ$.

So, this is by dividing A with this or we take so V and V are common here. So, we take it $-V/g\ dV + dQ/A + dx(S_0 - S_f)$. If you remember, we had a similar equation $S_0 - S_f$in the GVF as well. Now, we know since *Q/A* is equal to *V* and $(Q + dQ)/(A + dA)$ is nothing but equal to $V + dV$. The above equation can take the form which form we are going this one is going to take.

So, $-V/g$ will remain the same. dV can be written as $(Q + dQ)/(A + dA) - V$. So, this is, if you see, this is $V + dV - V + dQ/A$ or so what we have done is we have replaced dV by this term, you see here. dV is written as $(Q + dQ)/(A + dA)\ V + dQ/A + dx(S_0 - S_f)$. And now, what we are going to do is we take this denominator $A^2 + AdA$ and we try to solve this.

So, it becomes $Q \times AdQ$ or $2AdQ - Q\ dA$ and $-\ dA\ dQ$ because V is also written as Q/A. And if we neglect probably we will neglect this for simplification—after neglecting AdA in the denominator, this is small and $dA\ dQ$. So, we neglect this and we neglect this. What we are going to get is *dy* is equal to $-\ V/g(2AdQ - QdA)/A\text{^}2 + dx\ (S_0 - S_f)$. Or we take V and g inside; it becomes $-2VdQ/gA$, this A and this A will cancel, or $QVdA/gA^2 + dx(S_0 - S_f)$. Or, in other terms, we write $-2V/gA\ dQ/dx + QV/gA^2\ dA/dx$.

dQ/dx can be written as Q_*, and this can be written as $QV/gA^2 *$, so basically area can be written in terms of top width, and Q can be written in terms of velocity. So, what we are trying to do is replace *Q/V* now. So, it becomes $(V^2t * \ dy/gA\ dx$. Because xx we have taken out here from here, it becomes dy/dx, that is dQ/dx, and this dA is equal to $T\ dy$, as I have already told, or $-2VQ */gA\ +\ Q^2T/gA^3$. This is a very famous term.

Round number, or we will take $dy/dx\ \times\ gA^3$ on this side in the next step. So, we take this one to this side, so it becomes $dy/dx * (1 - Q^2T/gA^3)$ is equal to $2Vq * /gA\ (S_0\ -\ S_f)$ or dy/dx. So, instead of Q, we write V as Q/A in this step. Here. So, $dy/dx\ *\ (1\ -\ Q^2T/gA^3)$ is equal to $-2Vq */gA\,(S_0\ -\ S_f)$, or dy/dx is $-2Vq * /gA^2\ +\ (S_0\ -\ S_f)/\,(1\ -\ Q^2T/gA^3)$.

or dy/dx is $(S_0\ -\ S_f - 2q */gA^2)$. Same dy/dx is you can write Q^2/gA^2, it is the same thing. And now this is the dynamic equation of a spatially varied flow with increasing discharge. Let us say this q_* is equal to 0 that is lateral then it becomes gradually varied flow equation.

So, this will become to be *dy/dx* is equal to $(S_0 - S_f)/\,(1\ -\ Q^2T/gA^3)$. And what is this equation? This is gradually varied flow equation. So, this is the derivation of dynamic equation of spatially varied flow with increasing discharge.

This is an important equation to remember. So, I think this is a right point where I should stop this lecture and in the next class we will start with control points. Thank you so much. See you in the next class.