Free Surface Flow
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Lecture 35

Welcome students to this last lecture of our module 7 which is problems on gradually varied flow. Today we are going to solve one particular question on the step method, direct step method. Although it will be a very long question but it will give you fairly an idea of how to solve this type of question and if time remains we will proceed to another question in next 30 minutes. So, without further wait, we should. So, as always, let me write down the question so that you are able to understand.

So, a sluice gate discharge a stream of depth 0.15 meter at the Vena contracta. Vena contracta that is 1.40 meter cube per second per meter. The channel can be taken wide rectangular horizontal channel.

The channel is wide rectangular horizontal channel and the discharge intensity is 1.40 meter cube per second per meter. Now, it says if the hydraulic jump is formed at a depth of 0.25 meter. the distance from the toe of the jump to the vena contracta and it says.

Take two steps and use the direct step method. It is also given that Manning's n is equal to 0.015. So, the question In general, it says that in the presence of a sluice gate, there will be a formation of hydraulic jump. Just below the sluice gate, that area is called vena contracta, and the hydraulic jump is formed. This depth where vena contracta is present is 0.15 meter, and the hydraulic jump is formed after a certain distance.

Right. And it says that the hydraulic jump is formed at a distance where the depth is 0.25 meter. So, we have to estimate the distance between the vena contracta and the place where the hydraulic jump is formed, all right. So, how to—I mean, there are several other things that have also been given, right. So, what we are going to do is and it says that we have to take two steps. So, in the direct step method, there are several dx's that we have to take. It is given: take two steps. So now, as per the question, we can write: consider two steps with depths of 0.15, 0.22, and 0.25 meter forming the ends of the.

So, basically, what it says is that the d here is 0.15. So, it is vena contracta here; this is 0.25 meter. So, all these units are in meters, and here it is the tow of hydraulic jump, and they have taken this as 0.22 meter. So, that being said, we can say in this case, the distance of the water surface in the Δx is obtained using our standard formula of energy differential, the energy difference differential equation, $S_0 - S_f$. So, here q is 1.40 meter cube per second per meter, Manning's number is 0.015, and the channel is horizontal, which implies So is equal to 0. Now, we say, assume the channel to be wide rectangular channel with bed slope B horizontal, that is $S_0$is equal to 0.

So, velocity V can be written as q/y for a wide rectangular channel. Hydraulic radius R is equal to nothing but y, and specific energy E is equal to $y + V^2/2g$. Energy slope S_f is given as $V^2\ n^2/R^{4/3}$. So, now using these three formulas, using these three equations, we will solve the steps.

So, just showing, however, in the direct step method, what are the important things. First, we will draw a table indicating what are the quantities that we need. So, first is we know y and we are given 3 ys. All, no need to draw this, we will 0 point that is in meters right there are 3 steps one is 0.15 between 0.22 and 0.25 that part we know. So, we will need to calculate velocity right, we will need to calculate E right, we will need to calculate energy difference ΔE.

Then, we need to calculate Sf bar, that is the average friction. We need to calculate other quantities $S_0 - S_f$ bar. Then we need to calculate Δx and then we need to calculate x and x is the required answers we need to. Determine, ok. So, in the beginning, we say assume x is equal to 0 and whatever the value will come here, this is going to be our.

Answer that is the distance, alright. So, we start calculating the values at different different points. So, we start, let us say at we will start first at y is equal to 0.15 meter right. We are given q is 1.40 meter cube per second per meter that is wide. Rectangular channel.

And so, this is, let us say, This is, let us say, y_2, and this is y_3. So, we are saying y_1 is equal to 0.15, and we also say that a hydraulic jump is formed. At which one? y_3.

So now, with the equations that are shown here in these ones, we are going to utilize and solve for the value at the first step. So, our first step is y_1 is equal to 0.15 meters. So here, V_3 is going to be q/y_1, which means 1.4/0.15. And which means V is 9.33 meters per second. So, we have obtained 9.33 meters per second, correct?

So, where will this be put? This will be put as 9.33. Secondly, we need to calculate energy 1 as here: $y_1 + {V_1}^2/2g$ is equal to y_1 was 0.15, velocity came as $9.33^2/(2 \times 9.81)$, and this comes to be 4.59 meters. And where is this going to be put? 4.59 is going to be put here.

Now, the next step is S_f . or S_{f1} that is $V^2\, n^2/R^{4/3}$ and that will come $9.33^2 \times 0.015^2/\ 0.15^{4/3}$. So, S_{f1} comes to be 0.2459 and where is this value going to be put? Here 0.2459.

So, similarly, we are going to solve this for y_2 at y_2 is equal to 0.22 meter. So, we are saying here that V_2 is nothing but q/y_2 that is 1.4/0.22 and this will come out to be 6.36 meters per second. And where is this value going to be put?

In the previous table below, here it is going to be 6.36. Similarly, we will find E_2 as well which is $y_2 + {V_2}^2/2g$ is equal to $0.22\ +\ 6.36^2/(2\ \times\ 9.81)$ that is 2.284. right meter and where is this going to be put here 2.284 and in a similar pattern we are going to find S_{f2} or that is $({V_2}^2)(n^2)/R^{4/3}$ which means $6.36^2 \times 0.015^2/(0.22^{4/3})$ and this will come out to be 0.0686 ok and where is this going to be put in the table here 0.0686 ok.

Now, we can actually go to the third part as well, but let us solve some of these values ΔE, so minus Sf bar and things like that. So, for the first step, ΔE will be $E_2\ -\ E_1$. Let me just give one more gap otherwise. So, ΔE will be $E_2\ -\ E_1$, and that will be 2.284 - 4.284. Or $E_1\ -\ E_2$, whatever you want to call it, or yeah, that is 2.306. And where is this 2.306 going to be? I am just talking about the magnitude, the difference. This is going to go here, or $E_1\ -\ E_2$ also you can say.

Right. And what about this value S_f, $S_f$bar? So, this was S_{f2}. So, now we are calculating S_{f1} bar. S_{f1} bar is $(S_{f1}\ + S_{f2})/2$, this is the average value $(0.2459\ +\ 0.0686)/2$, that is coming to be 0.1573. And where is this going to be? So, we have S_f. Sorry, this was not Sf. So, this was S_f, S_f bar, and this was S_f bar.

So, S_f bar and then it was Δx and this was x. So, S_f bar is coming out to be here 0.1573. So, 0.1573. And therefore, $S_0 - S_f$ bar. $S_0 - S_{f1}$bar is 0 - 0.1573, that is S0 was 0 - 0.1573. And where is this value going to go?

Here, -0.1573. And using the formula, ΔX_1 is equal to ΔE_1 divided by $S_0 - S_{f1}$ bar, which is equal to 2.306 - 0.1573. That will give us -14.7. So, $X_2 - X_1$ is equal to -14.7, which implies $X_2$14.7.

So, ΔX_1 is 14 - 14.7. So, what we are going to do here is only -14.7, and this is also -14. So, X_2 has come out to be 14.7 using the first step. So, now we are going to solve for At y_3, the last step, which is equal to 0.25 meters, which means V_3 is q/y_3, that is 1.4/0.25, that is 5.60.

And E_3 is going to be $y_3 + {V_3}^2/2g \; 0.25 + 5.60^2/(2 \times 9.81)$, and this value will come out to be 1.848. Are we going to put? So, velocity this one and this value where we are going to put, we are going to put in this table here that is 5.60 and 1.848. And then we will similarly calculate E_2, that is $E_1 - E_2$, or one point—I mean, the sign is not that important here right now—2.284, that is 0.436, and where are we going to put this here. In the centre, 0.436, and then we will calculate.

Was $({V_3}^2)(n^2)/R^{4/3}$, which is equal to $(5.6^2)(0.015^2)/(0.25^{4/3})$. 0.048, and this comes out to be 0.048, and we are going to put this—we will put that together, that is 0.048, not here, 0.048. And then, a simple calculation: Sf2 bar is going to be $(S_{f2} + S_{f3})/2$ which is $(0.0686 + 0.0448)/2$, that is 0.0567. Similarly, we are going to put it. Here in the centre, that is 0. $S_f$bar is 0.0567, and similarly, we will calculate S_f - S_f bar.

It is $S_0 - S_{f2}$ bar is equal to 0 - 0.0567, that is -0.0567. And we are going to put it again here: -0.0567. Now, the calculation of ΔX is required. ΔX_2 is nothing but $\Delta E_2/(S_0 - \overline{S_{f2}})$, that is $0.436/-0.0567$. That is -7.7 meters, and this we are going to put it: -7.7, and finally.

What we are going to calculate is X, $X_3$is $X_2 + \Delta X_2$, so -14.7 + -7.7, and that will give us -22.4 meters. And where are we going to put this? We are going to put -22.4. So, continuing the distance, the distance between two depths, 0.15 meters and 0.25 meters, that is between the Veena between the vena contracta and the toe of the hydraulic jump, is found to be 22.4 meters and this is the answer. So, you see, we used the direct step method here and calculated different values, and I think this is good enough for today's lecture. So, we will end it, and I will meet you again at the beginning of our next module, which is module 8, and that is about spatially varied flow. Until then, bye-bye.