Free Surface Flow
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Lecture 34

Welcome, students, to our seventh module, which is about problems on gradually varied flow. And we are going to see at least two problems in this session, in this lecture today. So, let me write down a question. This is more like a theoretical part, this particular question. If Q_n represents normal discharge at a particular depth y, and Q_c represents the critical discharge at the same depth, y, then what do we have to prove? dy/dx is equal to $S_0(1 - \sqrt{Q/Q_n})/(1 - \sqrt{Q/Q_c})$ So, this is what we have to prove.

So, what do we know? We know that the differential form of the GVF profile we know. We know that dy/dx is equal to $(S_0 - S_f)/(1 - Fr^2)$, or we can also write dy/dx is equal to, if we take So common, $(1 - S_f/S_0)/(1 - Fr^2)$.

Manning's, from Manning's equation. What is Manning's equation? Manning's equation is Q = $1/n\ AR^{2/3}\sqrt{S}$. If the depth of flow is y_1, then we can also write $Q = K\sqrt{S}$, which is also something that we know.

So, for depth y, normal discharge Q_n is equal to $K\sqrt{S_0}$. or depth y, Q will be equal to $K\sqrt{S_f}$, and why so? The reason is that one of the assumptions for gradually varied flow was that we are going to change the uniform flow equations—or, I mean, where we have the bed slope, we are going to use the energy slope, and that is why we have Sf. I will write it in a much better way.

Q is equal to $K\sqrt{S_f}$. So, we are going to divide—I mean, these two equations, 1 and 2, which were the first and second equations, respectively. But what we are going to do is divide Q/Qn here. These two $\sqrt{Q/Q_n}$ are equal to S_f/S_0. We also know that Fr^2 is equal to $V/\sqrt{gA/T}$, or Fr^2 = $Vr^2/(gA/T)$, or $Fr^2 = Q^2T/(gA^3)$, which we also know. And we also know that for critical flow condition Froude number is equal to 1 and Q is equal to Q_c. So, ${Q_c}^2T/(gA^3)$, is equal to 1 or $gA^3/T = Q_c^2$. So, the Froude number equation we put as 3 and this equation we put as 4. So, what we do is we put the value of 4. in 3 that is use gA^3/T is equal to Q_c^2 in Fr^2 is equal to $Q^2T/(gA^3)$. We get Fr^2 is equal to $\sqrt{Q/Q_c}$. Now, put the value of Fr and S_f/S_0 in equation 1, we are going to get $dy/dx. dy/dx$ is equal to $S_0(1 - \sqrt{Q/Q_n})/(1 - \sqrt{Q/Q_c})$. Okay.

And therefore, hence proved. So, this was a simple problem, quite simple to do. What we had to do? We had to do some manipulation in for Froude's number and then we had to use the critical flow conditions where Fr is equal to 1 and Q is equal to Qc and using that we were successfully able to transform the gradually varied flow that is GVF equation 2D desired.

So, now moving on to the next question. So, the question is about a gradually varied flow. In a rectangular channel with a bottom width of 3 meters. The discharge is 8 cubic meters per second, and the depth of the flow changes from 1.4 meters. At section M to 1.05 meters at section N.

So, there is a gradually varied flow happening in a rectangular channel where the bottom width is 3 meters. The discharge is 8 cubic meters per second, and the depth of flow changes from 1.4 meters at section M to 1.05 meters at section N. The question is to calculate. The average energy slope between these two sections, and we have to assume that n is equal to 0.018. So, I mean, just drawing a cross-section, a simple cross-section, this is the depth, right? The bottom width is given as 3 meters, and this is varying from section to section. The discharge is given as 8 cubic meters per second, this is the discharge, okay, and Manning's.

n is 0.018. y changes from 1.4 to 1.05 meters. Anyways, so in gradually. Varied flow, the Manning's formula for any. Section is V is equal to $1/n\ R^{2/3}\ \sqrt{S}$..

This is how the Manning's formula is written. Here, it is important to note that Sf is the energy slope. So, in the previous lecture, I have spent a dedicated I mean, a considerable amount of time explaining that if the Manning's equation is adopted for a gradually varied profile, the bed slope So must be replaced by S_f. This is one of the major assumptions. S_f is going to be $n^2V^2/R^{4/3}$.

Average energy slope between 2 sections is how we can write the average energy slope. S_f bar is nothing but the slope at 1 plus the slope at 2 divided by 2. That is the average—that is the simplest. And these calculations we are going to show in the form of tables for sections M and N, or let us do section M first no need.

Area is $B \times y$, that is 3×1.4, which is 4.2meter square. And let this be section N. Here, the area is $3\ \times\ 1.05$, which is 3.15 meter square. The perimeter is $(B\ +\ 2y)$, that is equal to $(3\ +\ 2) \times 1.4$. That is 5.8 meters. The perimeter here is $3\ +\ 2 \times 1.05$, which is 5.1

meters. The hydraulic radius for section 1 is A/P, that is $4.2/5.8$, and that is 0.724. Here, the hydraulic radius is $3.15/5.10$, which is 0.6176.

About the other properties, Velocity is 8/4.2 because that is the discharge. So, that is 1.9048 meters per second divided by area. Here, *Q/A*. Here it is 8/3. So, velocity here will be faster: 2.5397 meters per second, and now S_f will be

So, for S_f, we are going to use this particular formula: $n^2V^2/R^{4/3}$ for each of these. So, n squared is 0.018. V is 1.9048, whole square divided by $0.724^{4/3}$. That is the hydraulic radius. And similarly, S_{f2} this is S_f, S_{f1}, S_{f2}, or S_f let it be S_f, is 0.018^2.

2.5397^2 / $0.6176^{4/3}$. So, S_{f1} comes out to be 1.8077 $\times 10^{-3}$, whereas S_{f2}comes out to 3.973 $\times 10^{-3}$, and average S_f bar is the sum of these two: 1.8077 * 10^-3 + 3.973 $\times 10^{-3}$and divided by 2, and that comes to be 2.89×10^{-3}. So, S_f bar is nothing but 2.89×10^{-3}.

So, this question was quite simple, and what we had to find was the average energy slope. So, the key step here was to first write down the energy equation sorry, the Manning's equation for gradually varied flow and find S_f using that Manning's equation and the rest for different reaches. We know *n*, we calculated *V*, we calculated *R*, we calculated *A*, we calculated *P*, and then the sum of the average this was another important step that the average energy slope between two sections can be found using this particular formula. So, I think this should be enough for today's lecture, and I will see you in one more lecture to solve another problem of gradually varied flow. Until then, thank you.

See you in the next class. Bye.