Free Surface Flow
Dr. Mohammad Saud Afzal
Department of Civil Engineering
Indian Institute of Technology Kharagpur

Lecture 33

Welcome students to yet another session of problem solving in gradually varied flow. So, last class we solved a problem where we were asked to calculate the surface profile for a trapezoidal channel and in that we use different tables and we were able to draw the surface profiles and we were able to see the change in grade as well. So, starting with a new problem now. So now, what we have? We have a rectangular channel.

Earlier with the trapezoidal channel, M and N are in series with reach M being upstream of reach N. These, so here we have only two reach and the difference is it is a rectangular channel. These channels, so these channel reaches have the following characteristics. width, discharge, slope and Manning's number and the reach are M and N. Width is 5 meter in the first reach, in the second reach is 4 meter. Discharge is same 15.0, 15.0 and slope is 0.0004, this is 0.0003 and n is 0.025 and this is 0.015. the question is sketch the resulting gradually varied profile due to change in the channel characteristics as above.

So, this is the question. So, two reaches are given, M and N, with different parameters: width 5, 4; discharge is the same; slopes are different; and Manning's number is also different. So, what we have to do is sketch the resulting gradually varied flow profiles. So, this is a rectangular channel. So, this might not be that difficult to calculate different y_c and y_0 and other things.

So, we know capital Q, right? So, right, and therefore, we can calculate small q_m. In reach M, this demonstrates the reach, and that is Q/B. So, Q_m/B_m or 15/5, that is 3 meter cube per second per meter, and similarly, if we have q_m, then y_{cm}, that is the critical depth in this particular stretch, is going to be $\left(\frac{q^2}{2g}\right)^{1/3}$, that is $\left(\frac{3^2}{2\times 9.81}\right)^{1/3}$. And that gives us 0.9717 meter.

Similarly, for stretch N, so $\boldsymbol{Q_n/B_n}$, that is 15. Discharge was the same, but the width is different. So, it is 3.75 meter cube per second per meter, which means ycn is going to be $\left(\frac{q^2}{2g}\right)^{1/3}$ or $\left(\frac{3.75^2}{2\times 9.81}\right)^{1/3}$. or 1.1275 meter. So, $\boldsymbol{y_{cm}}$ is equal to 0.9717 and $\boldsymbol{y_{cn}}$.

$\boldsymbol{y_{cm}}$ and $\boldsymbol{y_{cn}}$ is 1.1275 meters. This we have found out very easily since it was a rectangular channel. So, now the important part is the normal depth calculation. This is quite important. So, for normal depth calculation, we start with one reach.

So, let us say reach M here. $\boldsymbol{B}$ is given as 5 meters, $\boldsymbol{Q}$ is given as 15 meter cube per second, So is given as 0.0004, and $\boldsymbol{n}$ is given as 0.025. For a rectangular channel, $\boldsymbol{m}$ can be given as 0. So, we do almost the same procedure: $\boldsymbol{\varphi}$ is equal to $\frac{nQ}{\sqrt{S_0}B^{8/3}}$, that is $\frac{0.025\times 15}{\sqrt{0.0004}\times 5^{8/3}}$, and that will give us 0.25665.

For $\boldsymbol{\varphi}$ is equal to 0.2565 and m is equal to 0, we again look at Table 1. So, we go and start looking at Table 1, and the value we have to look for is 0.2565. 0.25, so 0.2565. So, you see one value is here, m is equal to 0, and the other value is here, 0.2565. So, interpolation between these two values.

Will result for m is equal to 0. See, we will see this value; I have already written it down. At Table 1, we get εo is equal to $\boldsymbol{y_o/B}$ is equal to 0.6071. It was between 0.6 and 0.610. yo is equal to $\boldsymbol{B}\times 0.6071$, and $\boldsymbol{B}$ was 5×0.6071.

So, yo comes out to be 3.036 meters. And similarly, for reach N, we calculate φ is equal to $\frac{nQ}{\sqrt{S_0}B^{8/3}}$. And this is equal to 0.015 and 15 divided by under root 0.0003 and into 4 raised to the power 8 by 3, and this value is going to come to 0.3222. We know that for this particular one, $\boldsymbol{B}$ was 4 meters. $\boldsymbol{Q}$ was 15 meters cube per second, So was 0.0003, and n was 0.015. So, if we look at this table,

Again, for 3222 for m is equal to 0. So, the value was 0.322. M is equal to 0, 3, 2, 2, so somewhere between here, and it should come out to be between here. So, it is around 0.725 or something. That for φ is equal to 0.322 and m is equal to 0, $\boldsymbol{\eta_o}$ is equal to $\boldsymbol{y_o/B}$ is equal to 0.7254.

That means y_0 is equal to 2.902 meters. Now, we will draw the table, and the table will be filled out. Reach, y_0, y_c, slope, classification. Nature of break in grade and nature of gradually varied flow profile due to break in grade. Now, there are two reaches, M and N. y_0 came out to be 3.3036 in the first case, and here it came out to be 2.902.

Critical was 0.9717, and this was 1.1275. So, you see y_0 is greater than y_c. So, this y_0 is greater than this y_c. So, that means it is a mild slope. Here also, y_0 is still greater than y_c.

This one is greater than this one. So, this is again a mild slope. So, the nature of the break from this section to this section is mild to steeper mild. Here, it is an ***M2*** curve in reach, and here it is M. So, what happens is the channel slope

changes from mild slope to steeper mild slope and an ***M2*** curve is formed in the reach M. The curve has the upstream asymptote of yom is equal to 3.3036 meter. And this thing we will get it clarified when we draw the figure and ends at the depth of 2.902 meter at the junction of the two reaches.

The figure is below. that is below. This is channel M, this is channel N. NDL NDL and this is ***M2*** curve. this is CDL, this is CDL 0.9717.

Of course, this is not according to the scale 3.306 meter and here it is 1.1275 and this is this is not up to the scale. So, this is the rough figure, how it will look like. So, we will solve yet another problem. A 2.0 m, wide rectangular channel that is n is equal to 0.015 carry the discharge of 4 meter cube per second. the channel is laid on a slope of 0.0162.

A downstream sluice gate raises the water surface to 7 meter immediately behind it. question is find the transitional depth. So, the question is there is a wide 2 meter wide rectangular channel that carries a discharge of 4 meter cube per second. The channel is laid on a slope of 0.0162.

A downstream sluice gate raises the water surface to 7 meter immediately behind it. Find the transitional depth. So, in the theory, what we have seen is from uniform flow topic for critical slope in case of wide rectangular channel the limit value of S_{*c} is equal to $\frac{S_{Lc}B^{1/3}}{gn^2}$ and that is equal to 2.667 and the limit slope S_{LE} sorry S_{Lc} $\frac{2.667\times9.81\times0.015^2}{2^{1/3}}$ is equal to

0.004672 since the actual slope So is greater than S_{Lc}. This is the thing to check if transitional depth is possible or not. So, since So is greater than S_{Lc} that is the limit slope, transitional depth is possible.

The normal depth yo for given So, n, B, and Q is found using Table 1, which we have been using in our PPT a lot. So, φ is $\frac{4\times0.15}{\sqrt{0.0162}\times2^{8/3}}$. is equal to 0.07424. And if we look for φ 0.07424 for m is equal to 0, that is a rectangular channel. I have already looked at it, and you can see that y_o/B is equal to 0.2466. y_o/B is equal to 0.2466, which implies 0.2466×B, 0.2466×2, that is 0.493 meters. So, the normal depth here is 0.493 m. Similarly, we shall also find the critical depth, and the critical depth formula y_c is $\left(\frac{16}{4\times9.81}\right)^{1/3}$by 0.742 meters. y_o is less than y_c is less than y, the channel is a steep slope channel, and the GVF profile is an S1 curve. So, at the transitional depth that is S_{*0c} is $\frac{S_0B^{1/3}}{gn^2}$, which is $\frac{0.0162\times2^{1/3}}{9.81\times0.0015^2}$. And that is 9.247, and that is equal to $\frac{(1+2\eta_t)^{4/3}}{\eta_t^{1/3}}$ So, how do we find that? How do we solve this? By using trial and error.

The error method: the two depths are found as η_{t1} is equal to 2.985, or y_{t1} is 5.970, and η_{t2} is equal to 0.00125, or y_{t2} is equal to 0.0025, both in meters, OK. So, yeah. So, the second transitional depth, y_{t2}, is not of any significance in this problem.

So, the ***S1*** curve starting after a jump from the normal depth will continue to rise till y is equal to y_t, which is equal to 5.970 meters, at which point it will become horizontal. Beyond y, the Froude number at y is equal to yt is F_t is $\frac{4/(2\times5.97)}{\sqrt{9.81\times5.97}}$, that is 0.0438.

All right, so we were asked to find the transitional depth, which we found out, and this is the answer. It was quite a complicated and long question. Complicated in the sense that it was lengthy; we needed to solve for many things, and we needed to use the tables for the normal depth. The critical depth was simple because it was rectangular. In the end, this is how you are going to approach this type of problem. So, I will end this lecture now, and I will see you in the next class again.

Until then, goodbye.