Free Surface Flow

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Lecture 31

Welcome, students, to the beginning of a new module today. This module is about problems on gradually varied flow, GVF. In these approximately five lectures, we are going to solve some problems. Many of the problems will be lengthy because, when it comes to gradually varied flow, the most important thing is to solve for the profiles. To solve for the profiles.

In any case, if you want to find the profiles, you will need to calculate the critical depth and the normal depth. So, for the calculation of GVF, both critical and normal depths are required, and that was the reason we solved a question for the most efficient channel section for a trapezoid, just so that you can have a revision. So, without further ado, I think we should proceed now and start solving some questions on gradually varied flow. There are different tables which we may or may not use for solving our problems. But I have included them for our easy reference.

So, I am just going one by one, and for critical depth in trapezoidal channels, we have covered many of these problems in our critical depth chapter and also the uniform depth chapters. But still, we also have this particular type of GVF profile so that we can always go back and refer to what type of flow profiles we are talking about. You remember we have the M_1 profile, M_2 profile, M_3 profile, S_1, S_2, S_3, C_1, C_3, H_2, H_3, A_2, and A_3. So, there are a total of 12 profiles.

So, that being shown, I think we will take up our first problem of the day. The question is about a rectangular channel. So, our channel cross-section has already been defined. A rectangular channel with a bottom width of 4 meters. And a bottom slope of 0.008 has a discharge of 1.50 meters cubed per second in a gradually varied channel. The depth at a certain location is found to be 0.30 meters. If we assume Manning's n is equal to 0.016, determine the type of gradually varied flow. So, this is the question.

It says that the channel is rectangular, the bottom width is 4 meters, and the bottom slope is given. So, this is one of the most standard types of problems that you are going to solve. These types of problems can also appear in your exercises during the weekly assignments. So, instead of first, first we should do what we actually should do, I mean, although there is not much to draw, but still, we will draw the sketch. The depth is given as 0.30 meters, and this breadth is given x is 4 meters and y is 0.30. So, first, writing down the things that are given to us, we are given S_0 is equal to 0.008 discharge Q. Q is given as 1.5 meters cubed per second, n is given as 0.016. And the width, this is the bottom width, is given as 4 meters. So, the first thing for solving the normal depth of the flow.

So, this is a gradually varied flow question. So, as I told you before, what we need to do is we need to find both. The normal depth and the critical depth, and then try to judge what the profile is going to be. And for the calculation of normal depth, the equation that we normally use and we must use is Manning's equation. So, from Manning's equation, we can write Q is $\frac{1}{n}R^{2/3}$ and $S^{1/2}$. So, now calculating area A as $B \times y_n$ or y_o, that is $4y_n$. Perimeter P. So, needless to say, this is the wetted area, and this is the wetted perimeter, that is $B + 2y_n$, that is 4 plus $2y_n$. Radius R is equal to A/P, that is $\frac{By_n}{B+2y_n}$. $\frac{4y_n}{4+2y_n}$.

Now, we should substitute the values of n, A, R, S, and Q and try to solve for y_n. We can write discharge 1.5 is equal to $\frac{1}{0.016} \times 4y_n \times \left(\frac{4y_n}{4+2y_n}\right)^{2/3}$ and $0.0008^{0.5}$. The only quantity here is y_n. Now, how do we solve this? We have to use the hit-and-trial method. So, the hit-and-trial method is going to give us y_n equal to 0.426 meters.

So, how do we check if it is correct? You put this value of y_n in this particular equation, and it will satisfy. Now, this is the normal depth. of the flow. The second step is to calculate the critical depth.

So, The critical depth of the flow y_c is equal to $\left(\frac{q^2}{g}\right)^{1/3}$, and what is q? It is capital Q / B. So, y_c can be written as $\left(\frac{(Q/B)^2}{g}\right)^{1/3}$ or y_c is $\left(\frac{(1.5/4)^2}{9.81}\right)^{1/3}$ equal to 0.243. So, first we write down yn equal to 0.426 and yc equal to 0.243. We can also write it as y_0, y_n, or y_0.

So, first of all, since the y_o is greater than y_c, this is ***M*** type. Now, again writing down our water depth was 0.30 meter, y_o or y_n came out to be 0.426. 0.426 meter and yc came out to be 0.243 meter. Therefore, since y_o is greater than y is greater than y_c, this is a typical ***M2*** profile. Just to check, you go and see y lies between y_o and y_c gives us ***M2*** profile. So, this is the solution of the first one of the most simple problem. Continuing to our second problem, the question is a rectangular channel of 4 meter width has a Manning's coefficient of 0.025.

For a discharge of 6 meter cube per second in this channel. Identify the possible GVF profiles produced in the following break in grades. I hope you remember this topic during a break in grades. A part is S_{01} is equal to 0.0004 to S_{02} 0.015.

And another change in grade, break in grade is 0.005 to 0.004. So, first we solve, first when doing the solution, what we do is we first write the things that are given, and the given things are B is equal to 4 meters. And n is equal to 0.025, and discharge is also given as 6.0 meters cubed per second. So, the first step is the calculation of normal depth.

Let the normal depth of the flow be y or y_o, let us say, or let it be y. So, area A is By or $4y$. P is equal to $B + 2y$, that is $4 + 2y$. Hydraulic radius R is equal to A/P, that is $\frac{4y}{4+2y}$.

This is dimensionless. Now, the critical depth of the flow is that is y_c is given by $\left(\frac{q^2}{g}\right)^{1/3}$, or very simply because it is given $\left(\frac{(6/4)^2}{9.81}\right)^{1/3}$ = 0.612. So, our y_c is obtained very normally, okay. 0.612 meters is our critical depth.

So, now for the first part, from Manning's, we use Manning's equation. Q is equal to $\frac{1}{n} R^{2/3} S^{1/2}$. So, at S is equal to S_{01} is equal to 0.004. Normal depth of flow By_1. So, what are we going to do?

We are going to substitute these values in this equation. So, we know Q is 6.0 is equal to $\frac{1}{0.025} \times 4y_1 \times \left(\frac{4y_1}{4+2y_1}\right)^{2/3} \times 0.0004^{0.5}$. And the way to solve is hit and trial always.

If we solve this using hit and trial, we are going to get 1.906 meters. So, as y is greater than y_c. Slope is mild, slope important. Now, we do the same for S is equal to S_{02} is equal to 0.015 and say let the normal depth of flow $\frac{B}{2}$.

Therefore, we can write Manning's equation as $\frac{1}{n}R^{2/3}\ S^{1/2}$ or 6 is equal to $\frac{1}{0.025} \times 4y_2\left(\frac{4y_2}{4+2y_2}\right)^{2/3} \times 0.015^{0.5}$. Again, this one also has the solution done using the hit and trial method, and the value of y_2 comes to be 0.540 meters. And as y_2 is less than y_c, the slope is a steep slope. We can observe that type of change in grade is from mild to steep, and hence gradually varied flow profiles are plotted below. So, how does it look like? 0.015. This is CDL, the critical depth line, and this is the normal depth line. This is y_{o1}, equal to 1.906; the flow is in this direction. This is the ***M2*** curve; this is NDL, which is already drawn, and this is the ***S2*** curve, and this is y_{o2}, equal to 0.540 meters, y_{o1} and y_{o2}.

So, part B, that was the first part that we did. So, from Manning's equation, which is $q = \frac{1}{n}R^{2/3}\ S^{1/2}$. At $S = S_{01} = 0.005$. Let the normal depth of flow be y_1, that is $6 = \frac{1}{0.025} \times 4y_1 \times \left(\frac{4y_1}{4+2y_1}\right)^{2/3} \times 0.005^{0.5}$.

And solving by hit and trial, y_1 will be 0.780 meters. So, as y_1 is greater than y_c, the slope is mild. The second part at $S = S_{02} = 0.004$, let the depth of flow be y_2. So, again putting it into Manning's equation, $\frac{1}{0.025} \times 4y_2 \times \left(\frac{4y_2}{4+2y_2}\right)^{2/3} \times 0.004^{0.5}$. Again, solving using hit and trial, $y_2 = 1.906$ meters. Here also, as y_2 is greater than y_c, the slope is mild, but y_1 is less than y_2. Hence,

The slope is a milder slope. So, what can we observe? We can observe that type of The change in grade is mild to milder, and hence GVF profiles are plotted below.

So, mild slope, even milder slope, okay. Then, first draw the CDL line and This is S_{01} equal to 0.05, and this is S_{02} equal to 0.04. C is equal to 0.612, 1.906 This is the ***M1*** curve.

So, this is how it looks. I think with this, we will finish this particular lecture, and I will see you in the next lecture. Thank you so much.