Free Surface Flow

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Lecture 3

Welcome students to this third lecture for module 1 about the introduction to the basic concepts in free surface flows. So, last time we finished topics and we were about to just start about the velocity distribution in a channel section. So, the distribution of velocity in the channel cross section is not uniform. What it means is that the along the cross section the velocity is not having one particular quantity. It will vary from one place to the other.

And what is the reason? And this happens due to the presence of free surface and friction along the channel walls and the bed. You see there when there is a when there is a free this is the. channel right. So, this is bed and in that direction there will be walls as well right.

The wall come with the friction and this is because of the presence of friction you know this will velocity will be 0 here, but velocity will not be 0 here. So, if there is a difference in velocities at two different point that means the flow is non uniform and therefore, that means due to presence of the friction owing to the bed and the walls the flow velocity is non uniform. So, according to Chow, a very famous scientist, the measured maximum velocity in ordinary channel occurs below the free surface at a distance of 0.05 to 0.25 times of flow depth. You see this defect here, this defect, so Chow says it is of the order of 0.05y. all right ok sorry So, the flow velocity may have components in all the three directions.

This flow velocity as I said, it will have V_x, V_y, V_z. However, these component of velocity in the vertical and transverse directions are usually small and may be neglected. This also I have covered in the previous slide. like this and this is typical velocity variation with depth, 0 here maximum somewhere here and this is approximately 0.05 depth of y. Thus only the velocity in the direction of the flow needs to be considered. So, we consider the flow velocity in this direction that is the average velocity.

So, when we say we are going to calculate the average velocity, we are going to do an approach that is one dimensional method of flow analysis. Which means the flow properties such as the velocity and pressure gradient in a general open channel flow situation can be expected to have components in the longitudinal as well as the normal direction. This we have been talking a lot. that the in general the velocity is three dimensional in one, so x, y and z. However, we are going to consider only velocity in one direction. The analysis of such three dimensional and the reason is this analysis is a three dimensional problem and is very complex.

However, if we want to obtain engineering solutions, engineering solutions might not be mathematically exact solutions, but very close to it because of the difference or the you know the ratio of the velocity magnitude in x direction compared to y and z direction the velocity is more. We use 1D analysis that means we will analyze only in x direction where and what we use only the mean or representative properties of the cross section are considered. So, basically what is what I mean to say is we neglect the smaller the directions in which the velocities are small and consider only the dominant flow direction. This method when properly used not only simplifies the problem but also gives the meaningful result when it comes to the solving the engineering type problems.

Now, regarding velocity, a mean velocity V for the entire cross section is defined on the basis of the longitudinal component of the velocity. So, as I said, you see the there is a velocity that has that is 0 at the bed and maximum at almost at the top. We need to define the mean velocity V for the entire cross section. okay And how do we define it? We say we $\frac{1}{A}\int_A vdA$ and divide by the whole area that will give us the average velocity.

And, this velocity V is used as representative velocity at the cross section. Instead of using different velocity at different depths, we use this velocity V. The discharge past the section can then be, so if we have an average velocity, we can have a discharge that is $\int vdA$, very simple or V into A. These are two important things. So, we will see some problems almost towards the end of this particular module, where we will try to solve some of these type of problems. The following important features specific to one dimensional open channel flow is to be noted. A single elevation represents the water surface perpendicular to the flow. okay

Velocities in the direction other than the direction of the main axis of flow are not considered. okay So, important thing is we will use a single elevation okay water perpendicular to the flow right and velocities in the other directions not to be considered and not considered for and this one single elevation perpendicular to the flow. What is a vertical pressure distribution? So, we are going to see what the vertical pressure distribution is case one is when the fluid is static like this. So, the pressure distribution here is hydrostatic along a vertical section in a static fluid.

The pressure is given by ρgy very simple ρg and this is the water depth y for flow depth and ρg is nothing but unit weight of water. ρg is unit weight of water. Now, case 2. So, first case was when the fluid was static. Now, the real condition when the fluid is moving.

The static case you will know, you know, you will find when there is a dam. The water rests and stands against the dam. It is a practical significance as well. But when it comes to the problems, with mostly you will find the rivers moving. So, case 2 is when the liquid is moving.

So, the pressure distribution of the moving liquid is given by $P = wy + \frac{a_n}{g} wy$ and this we have seen studied in a lot of detail in hydraulics, where an is the normal acceleration also equal to $\frac{v^2}{r}$, v is the velocity at the streamlines. r is the radius of curvature of the streamlines. So, just summarizing this $wy \pm \frac{a_n}{g} wy$. And this was hydrostatic part as well, if you remember. And this is plus minus depending on how it acts, this is the dynamic dynamic part due to moving liquid.

So, this is a figure you see we have several sections 1-1, 2-2, 3-3, and 4-4. So, you see at section 3-3 this section and this section 4-4 the liquid is flowing with straight streamlines and the pressure distribution is simply hydrostatic ok. And what is the reason? For straight and parallel streamlines the radius of curvature is infinity and therefore, the component $\frac{a_n}{g}$ that is $\frac{v^2}{r}$ if the radius of curvature is infinity that goes to 0. However, this will not be the case at section 1-1 and 2-2.

So, for infinite radius the normal acceleration is 0 which we already told you right for let us say even at section you know the this was wy + a_n you see this let me show you this is $\frac{a_n}{g}wy$ and what is an $\frac{v^2}{r}$ what is r? radius of curvature. So, for straight and parallel streamlines the radius of curvature is infinity. Therefore, when r is infinity, r goes to infinity $\frac{v^2}{r}$ goes to 0. That means pressure is only the hydrostatic part.

Hence, the second term on of the pressure distribution equation for a moving liquid vanishes for parallel streamlines. So, for flows with straight and parallel streamlines, the pressure distribution is hydrostatic like this. Now, let us see at section 2-2 the streamlines are concave. So, the pressure intensity increases here due to centrifugal action therefore, this will be plus term. That is $1+\frac{a_n}{g}wy_2$.

The actual pressure intensity is more than the hydrostatic pressure. Again talking about the section 1-1 the streamlines are convex here. The pressure intensity decreases here. Convex. So, P1 is the hydrostatic part minus this is dynamic part alright.

The actual pressure intensity is less than the hydrostatic pressure. Now, we can summarize a little bit of the discussion that we did in this part. For flows with parallel and street streamlines, the pressure distribution is hydrostatic. for flow profiles having concave curvature of a streamlines the pressure distribution is more than the hydrostatic. And for the convex curvature the pressure distribution is less than the hydrostatic ok.

Parallel and straight streamlines is hydrostatic, concave is more than hydrostatic, for convex less than hydrostatic. So, these these things about the different classification and you know the pressure distribution everything is done. We are going to see a little bit about Reynolds transport theorem which normally has to be you know covered as a part of hydraulics. However, you know just to make the next 15 minutes or something we will try to cover Reynolds transport theorem to revise later revise. So, this is one of the most fundamental theorems therefore, that is the reason I am going to repeat this again.

The basic equation of fluid mechanics are applied to open channel flow with some modification due to the free surface. These equations that the basic equations are the continuity equation, momentum equation and energy equations. And, these equations cannot be derived directly from Reynolds, I mean, these equations actually come from the Reynolds transport theorem. And how if we apply this Reynolds transport theorem to a fixed control volume. So, these are the figure of control volume that we are seeing that is A is the arbitrary control volume, B is the stream tube and C is the river reach and D is the stream line.

So, the Reynolds transport theorem has a general formula that is given $\frac{dB}{dt} = \frac{d}{dt}\int_{cv} b\rho d\forall + \int_{cs} b\rho(V.n)dA$. All right. Now, let us see what different terms are, B is any system property, it could be anything, momentum, mass and t is time, small b the intensive value of V per unit mass m that is db/dm, ρ is the fluid density, v is the volume of the control volume, capital V is the velocity vector. n is the outward normal unit vector and a is the area of the control surface. The volume integral on the right hand side of equation sums up the values of the property per unit mass b over each mass element given by ρdV.

In the surface integral in the equation ρV.ndA represents the mass flux through an elemental area dA on the control surface. So, basically I am covering it in a brief because this topic is actually a part of hydraulics and actually not even just hydraulics it is a part of fluid mechanics and you can find those courses on NPTEL. What is the dot product? So, dot product of the velocity vector with the unit outward normal determines the component of the velocity perpendicular to the surface since only that component can carry property through the surface. Furthermore, the dot product is positive for an outward flux and negative for the inward flux into the control volume.

Thus, the surface integral sums up the product of the property per unit mass B and the mass flux over the control surface to give the net outward flux of the property. So, I am underlining the important terms that are there in those integrals. In summary, what does this Reynolds transport theorem equation states? The time rate of change of the system property is the sum of time rate of change of the property inside the control volume and the net outward flux of the property through the control surface. This is quite an important statement.

So, the Reynolds transport theorem can be applied to the properties of mass. So, B this B can be mass, it could be momentum. it could be energy as well to obtain the control volume from the corresponding governing equation conservation equations. The control volume forms of the equation can be simplified for the case of so if we try to simplify the above equation for what steady one dimensional flow and used in the analysis of many open channel flow. So, for most of the problems we use this particular equation and try to do wherever this time for example, if you are doing study that means, we are any term that is ddt that we are going to put as 0 right 1-d flow we are going to neglect the directions y and z for example,

So now applying these equations to the conservation of mass, so this is the control volume, this is mass of the fluid entering and this is mass of the fluid leaving, this is control volume in the fluid flow. So if you consider an enclosed region in the flow constituting control volume as shown, the equation of conservation of mass can be written in terms of mass flux as. Mass flux is nothing but mass of flux leaving plus change, so mass of flux leaving plus change of mass in control volume per unit time. So, for example, if this is also changing, so mass of flux entering will be mass of the fluid leaving plus mass of the change on this control volume. If it has increased or decreased, depending will add and subtract.

For the steady flow, there is no change of mass of fluid in the control volume and the relation reduces to mass flux entering is equal to mass flux leaving. Applying this principle to steady flow in a streamline, having an elementary cross sectional area through which the velocity to be considered as constant across the cross section, there can be no flow across the wall of the stream tube, right, between 1 and 2. So, for steady flow, this is the main thing. And if we apply this, we will see some equations, we will have some equations you know, A_1V_1 equals to A_2V_2 for example, these all comes from Reynolds transport theorem. The final equation that comes as I wrote before also is $\rho_1 u_1 dA_1 = \rho_2 u_2 dA_2$ or dm is equal to constant.

If you have applied Reynolds transport theorem to the mass conservation, the mass influx or mass entering per unit time at a section 1 equals to the mass efflux or mass leaving per unit time at section 2. So, in above figure u is the I mean the or figure on the above equation a figure also we have shown here right. You see in this figure we have U_1, we have U_2, we have small u_1, u_2, small u_1, dA_1 densities area A_1 and this area A_2. U is the velocity through the elementary cross sectional area dA. q is the mass density of the fluid and subscript denote sections like 1 is denote for section 1, 2 denote for section 2. Therefore, for a steady flow it implies that the mass flow rate termed as mass flux dm across any cross section of the elementary stream tube is constant.

And this is known as the continuity equation of the compressible fluid flow through an elementary stream tube. Therefore, the continuity equation of the fluid flow for the entire cross section of the stream tube can be obtained by integrating the above equation. That is $\rho_1 u_1 dA_1 = \rho_2 u_2 dA_2$ = m = constant and $U = \frac{1}{A}\int_A u dA$. This is the average where u is the average velocity through the cross sectional area A. If the fluid is incompressible, incompressible means density is constant.

Therefore, ρ_1 is equal to ρ_2 and above equation reduces to very very common equation $u_1 A_1 = u_2 A_2 = Q$ is equal to constant. where Q is the discharge or the volume rate of the flow. So, you see with the Reynolds transport theorem, we have come up with this particular equation $U_1 A_1 = U_2 A_2$. Of course, we have not derived the Reynolds transport theorem, but that is done in a smaller level course. So, continuity equation so, we have seen continuity equation in 1D right we have we can have continuity equation in 3 dimension.

So, you see all now let us I mean we see in all the 3 directions. So, differential mode of continuity equation is used to analyze 2 and 3 dimensional flow. Earlier we use simply u

$A_1 v_1 = A_2 v_2$, but when we are considering the three-dimensional, we have seen that before as well. And if we need to derive three-dimensional continuity equation of fluid flow, we must assume a control volume element that is having a box of dx dy and dz in a Cartesian coordinate system and having velocity u, v and w. So, the mass influx of the fluid flow through the back face of the control volume by advection in the x direction is given by you see this one here $\left[\rho u - \frac{d}{dx}(\rho u)\ \frac{dx}{2}\right] dydz$.

See, I am not going to derive it, but just to revise your terms. This is the term the mass influx of the to the back face of the control volume. So, in this above expression ρudydz is the mass influx through the central plane normal to the x axis. The second term this one $\left[\frac{\partial(\rho u)}{\partial x}\right]\left(\frac{dx}{2}\right) dydz$ is the change of mass flux with respect to the distance in x direction multiplied by the distance dx/2 the back face. Similarly, the mass efflux through the front face of the control volume will be it will be of course, the addition.

Because we are assuming at the center velocity u therefore, the before that it was negative now here it is plus. So, we will have $\left[\rho u + \frac{d}{dx}(\rho u)\ \frac{dx}{2}\right] dydz$. Therefore, the net mass flux out in the x direction through these two faces is you see $\frac{d}{dx}(\rho u)\ \frac{dx}{2} dydz$ if you subtract that the other. So, basically you see the difference here this minus this right this will give us this minus this is going to give us $\frac{d}{dx}(\rho u)\ dxdydz$ and that is in the x direction.

Similarly, other direction will give us d d. the other direction will give us $\frac{\partial}{\partial y}(\rho v)\ dxdydz$ the z will give us $\frac{\partial}{\partial z}(\rho w)\ dxdydz$ and if we add we are going to get $\left[\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial x}(\rho v) + \frac{\partial}{\partial x}\ (\rho w)\right] dxdydz$. So, basically I am very sorry in the previous line I said $\frac{\partial}{\partial y}(\rho u)$ no it will be $\frac{\partial}{\partial y}(\rho v)$ and $\frac{\partial}{\partial z}(\rho w) dxdydz$. So, from the concept of conservation of mass the net mass flux out of the control volume plus the rate of change of mass in the control volume, which is dρ/dt dxdydz equals the rate of production of mass in the control volume which is 0 or we say $\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho y)+\frac{\partial}{\partial z}(\rho w)=-\frac{\partial \rho}{dt}$. which must hold for every point this is very important.

For incompressible fluid flow rho is constant this particular will come to $\frac{du}{dx}+\frac{dv}{dy}+\frac{dw}{dz}=0$. Very famous equation. Otherwise, this is the general equation. and this is incompressible flow continuity equation in three directions. So, I think this is a nice point to stop and we will continue with this continuity equation for open channel flow in our topic lecture number 4 for module 1.

Thank you so much.