Free Surface Flow
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Lecture 29

Welcome back, students, to our sixth module and fourth lecture of this particular module, where we have been discussing the theory of gradually varied flow. We talked about profiles, we talked about gradually varied flow equations, and we talked about gradually varied. flow energy equations, the profiles like M_1, M_2, M_3, S_1, S_2, S_3, C_1, and C_3, and H_2, H_3, and A_2, A_3. And then we went on to see what the control sections are, and we are going to start with the analysis of flow profiles today. The process of identification of possible flow profiles as a prelude to quantitative computation is known as the analysis of flow profiles. So, what we do is we try to identify the possible flow profiles before we start any quantitative computations. We judge what the profile is, and this process is called the analysis of flow profiles.

A channel carrying a gradually varied flow can, in general, contain different prismoidal channel sections of varying hydraulic properties. There can be a number of control sections of varying locations. To determine the resulting water surface profile in a given case, one should be in a position to analyze the effects of various channel sections and controls connected in series. So, basically, talking about different types of control sections at various locations, we should be in a position to analyze different channel sections separately and add them in series. Something like that we need to do for analysis. Now, there is a term called a break in grade. So, simple situations of series combinations of two channel sections with differing bed slopes are considered. So, you see analysis where we see there are various channel sections and controls connected in series. So, you see here also

So, it is a simple situation of a series combination of two channel sections with differing bed slopes that are considered. This is the normal depth line, and this is the critical depth line. Here, the flow profile is M_1 because here y is greater than y_0, which is greater than y_c. This is a mild slope, and this is even milder, heading towards S_0 equal to 0. Another case is, you see, we have an S_2 profile. This is a steep profile, and this is a steeper profile. This is the control point here, and there is a control point here where they are meeting. The two different types of slopes are meeting.

In this one, we have, you know, this is an M_2 profile, and here we are matching with mild, and this is a control point again here. This is again two different milder and mild. Similarly, there is a steep and steeper, and this is a control point. Here we have a steep slope, we have a mild slope, and this is the control point here. We have a control point here.

So, basically, where there is a junction where they are meeting. Similarly, there is a conjoined between mild and steep, and this is the one, right? The joint between adverse slope and mild slope. Then a control point between the horizontal and the steep slope, a lot of figures are there. So, these are just to tell you that when there are two types of channel sections with differing bed slopes considered, we call this a break in grade, right? And that is determined using the control point and controls connected when they are connected in series.

So, now the important step is the computation of gradually varied flow profiles. Now, what is the meaning of gradually varied flow profiles computation? Computation of gradually varied flow profile means the solution of the dynamic equation, which is given by *dy/dx* is equal to $\frac{S_0 - S_f}{1 - \frac{Q^2 T}{g A^3}}$. And the solution involves the determination of the profile type, length, and shape of the profile. Profile type, length, and shape of the profile.

What is the profile type? M_1, M_2, M_3, S_1, S_2, S_3, C_1, C_3, H_2, H_3, and A_2, A_3. But the above differential equation cannot be integrated analytically as the dependent variable *y* is a function of both *x* and *y*. That is correct. No direct integration is possible. No direct integration is possible, and the reason is that the dependent variable *y* is a function of both *x* and *y* implicitly.

Then we can write *dy/dx*. So, basically, the equation was *dy/dx* for $\frac{S_0 - S_f}{1 - \frac{Q^2 T}{g A^3}}$, which is a function of *x* and *y*. Hence, the exact mathematical solution of this first-order differential equation is not possible. Then we will come up with different techniques, and we will talk about it later in our lectures. Now, something called transitional depth.

What is transitional depth? The transitional depth is defined as the depth at which the normal discharge Q_n is equal to the critical discharge Q_c, and the slope of the gradually varied flow profile is horizontal. For such a situation, here Q_n is equal to Q_c, and the slope of the gradually varied profile is equal to S_o. So, we remember this particular equation.

You remember this particular one. So, at the transitional depth, Q/ Q_n is equal to Q/ Q_c, or K_o/K is equal to Z_o/Z_c, or $\frac{Q}{\sqrt{S_0}}$. You see, we substituted the value of Q_n. And we substituted the value of Q_c here. What we are going to get is $\frac{S_0}{n^2 g}$ is equal to $\frac{A}{T}\frac{1}{R^{4/3}}$, or that is also equal to $\frac{P^{4/3}}{TA^{1/3}}$. For a trapezoidal section of side slope m, this is what we are going to get.

Now, this equation 1 is the same as the generalized flow relation equation 2 with Froude number 1. For a trapezoidal channel, the non-dimensionalized form of equation 1 will be this one. Where η_t is equal to y_t, that is the transitional depth. It may be noted that S_{*OC} is similar to S_{*C}, but with the bed slope So being used in place of S_C. This is quite complicated, but what you can understand is the transitional depth is defined as the depth at which normal discharge Q_n is equal to critical discharge Q_c, and the slope of the gradually varied profile is horizontal.

Okay? Now, we are coming to what the different methods of computation for the gradually varied flow profile are. These are five different methods. One is the direct step method, that is modified or standard step method, graphical integration method. Third, fourth one is integration with the help of varied flow functions, and the last one is numerical integration method.

Now, let us see what a direct step method is. This method was first suggested by Charmonsky in 1914. This method is suitable for field engineers, and we already know that H is $z + E$ where specific energy. So, we can write E as $H - z$ or dE/dx can be written as $\frac{dH}{dx} - \frac{dz}{dx}$ or $\frac{dE}{dx}$ is equal to, so instead of $\frac{dH}{dx}$, we write $-S_f$.

And instead of dz/dx, we get minus of $-S_o$. So, that dE/dx is equal to S_o-S_f. This is energy, energy, GVF differential energy equation. So, in difference form, the above equation can be written as $\Delta E/\Delta x$. So, ΔE can be written as, so dE can be written as ΔE and dx can be written as Δx. And what is $\overline{S_f}$? This is the average frictional slope calculated at x_n and x_{n-1} section.

So, what do we do? We divide our entire domain into different slopes. Sorry, different sections of length Δx, and SF bar is nothing but the average friction slope calculated in between; you calculate one section, you calculate two sections, and take the average in the direct step method. Now, we can write Δx as ΔE; you see this one, this equation here. So, we write

Δx is equal to $\frac{\Delta E}{S_0 - \overline{S_f}}$. Finally, we can write Δx as x_n - x_{n-1}, and E can be written as E - E_{n-1}, and S_o - $\frac{S_{f,\mathrm{n}} + S_{f,\mathrm{n-1}}}{2}$, a very simple mathematical differential equation. So, this is called the direct step method. Now, the second-order Runge-Kutta method for the calculation of the GVF profile. If we employ Manning's equation for the energy gradient Sf in the dynamic equation, we get dy/dx equal to $S_0 - S_f$. What was the original equation? If you remember, dy/dx was $\frac{S_0 - S_f}{1 - \frac{Q^2 T}{g A^3}}$.

S_o, instead of S_f, using Manning's equation in this one. In the above equation, the flow depth y is implicitly contained in the variables A, P, and T. So, for the solution of the above, appropriate boundary conditions should be provided at some control section, right? The control section or the boundary condition that we have talked about in the previous slides as well as in the previous lecture. So, starting from the control section or from the boundary conditions, the second-order Runge-Kutta method is applied as follows. So, if we start from the boundary, $f(y_i)$, a function of y of i, is this function dy/dx. S_o - $n^2 Q^2$ is evaluated at one particular node, that is, y equal to y_i.

Similarly, $\frac{T}{A^3}$ is evaluated at y of yi, and the reason is A and P contain y, and here T and A contain y. After selecting a spatial computational step Δx, an adjacent value for water depth is estimated as y_{i+1}^* equal to y_i. So, you see, after we select a spatial computational step Δx. The adjacent value of the water depth is estimated as $\frac{\mathrm{y_{i+1}} - \mathrm{y_i}}{\Delta \mathrm{x}}$ equal to $f(y_i)$. If you try to, you see, so if you see, what is that function? This function is nothing but dy/dx, and that can be written as $\mathrm{y_{i+1}} - \mathrm{y_i}$.

Yeah, divided by Δx, and since this is average, they are taking half of it in any. The function y_{i+1}^{*} is re-evaluated, and a corrected value is assigned to the water depth of the adjacent section, as this is standard. So, y_{i+1} is equal to y_i + y_{i+1}^{*} Δx. So, basically, what they are calculating is at half of the distance, not at the full distance here. The algorithm continues until $y_{i+1} - y$ is less than a very small quantity, very, very, that is a predefined small number. And this is the Runge-Kutta method.

Another method is the standard step method. So, the direct step method is suitable for use in prismatic channels and hence applicable to artificial channels. There are some basic difficulties in applying it to natural channels. So, for natural channels, applying the direct step method is difficult. As we have already seen, in natural channels, the cross-section shapes are likely to vary from section to section, and also, cross-section information is known only at a few locations of the channel.

So, the direct step method is very good for prismatic channels but not for natural channels. And the reason is that natural channels have varying cross-sectional areas from one part to the other part, and therefore, it is very difficult. Thus, the problem of the computation of the gradually varied profile for a natural channel can be stated as: how can it be stated, given the cross-sectional information at two adjacent sections and the discharge and stage at one section? It is required to determine the stage at the other section.

So, the process is, if we have been given the cross-sectional information at two adjacent sections and the discharge and stage at one section, we can determine everything for the other section. The sequential determination of the stage as a solution to the above problem will lead to the gradually varied flow profile, very simple. So, the solution to the above problem is obtained by a trial and error solution of the basic energy equation. Considering if you see this figure, which shows two sections, section 1 and section 2, in a natural channel, section 1 is downstream of section 2 at a distance Δx, see Δx. Calculations are assumed to proceed upstream by equating the total energy, that is, section 1 and 2.

We are going to see that, but this is the definition sketch of the standard step method. You see, we have these energy mentions. This is the bed, water surface, and energy line. So, at section 1 and 2, we are going to write down. So, we can write $z_2 + y_2 + \alpha_2 \frac{v_2^2}{2g}$ is equal to $z_1 + y_1 + \alpha_1 \frac{v_1^2}{2g} + h_f + h_e$.

Because you see, these are the energy losses that are going to happen, and therefore, we must account for them. What is h_f? hf is the frictional loss, and h_e is the eddy loss. The frictional loss h_f can be estimated as nothing but h_f is $\overline{S_f}\ \Delta x$, that is, $\frac{S_{f1}+S_{f2}}{2}$. And what is the formula for S_f in general?

n^2V^2, and this is using Manning's equation for GVF by replacing So with Sf in Manning's equation of uniform flow. of flow conditions. So, there is no proper method for estimating the eddy loss, but it is usually expressed as $h_e = C_e \frac{\alpha 1 v_1^2 - \alpha 2 v_2^2}{2g}$. where C_e is the coefficient having the values as below.

So, if there is no transition, the values of C_e will be for expansion and contraction: when there is no transition, it is 0; for gradual transition, for expansion case, it is 0.3; for contraction case, it is 0.1. For abrupt transition, it is 0.8 for expansion and 0.6 for contraction. An alternative practice of accounting for eddy losses is to increase Manning's n by a suitable small amount. Another way could be to increase Manning's number n. This procedure simplifies our So, if we denote the stage $z + y$ as h and the total energy as H, then using suffixes 1 and 2 to refer to the parameters at the appropriate section, we can write capital $H = h + \alpha \frac{v^2}{2g}$, and h_2 is nothing but $H_1 + h_f + h_e$, where these are the energy losses.

The problem now can be stated as: knowing *H1* and the geometry of the channel at sections 1 and 2, it is required to find *H2*. Very simple. This is achieved in the standard step method by a trial and error procedure outlined below. So, for doing this, we use See, this particular part is a calculative part, very lengthy, which you will not be given in your assignments because it is too lengthy and too complex, but to make the subject complete in itself, we have to cover that.

So now we are going to the procedure for trial and error. So if we select for this particular case, we select a trial value of *H2* and calculate capital *H2*, h_f, and h_e, and check whether equation number A If there is a difference, we improve the assumed value of *H2* and repeat the calculation until the two sides of equation A match to an acceptable degree of tolerance. This is the overall procedure for trial and error. Based on the *i*-th trial, the *i+1* trial of *H2* can be found by following the procedures suggested by Henderson.

Let he be the difference between the left-hand side and the right-hand side of equation A. This is equation A. In the i^{th} trial. So, H_E can be written as H_2 - $(H_1 + h_f + h_e)$ in the *i* th trial. And our objective is to make this H_E vanish by changing the depth y_2. Hence, what we can do is $\frac{dH_E}{dy_2}$; we can differentiate it.

We can differentiate it, and since y_1, z_1, z_2, and V_1 are constant, we can compute $\frac{dH_E}{dy_2}$ as d/dy_2 of this complicated particular part and substitute the values, and this *F* is nothing but $\alpha_2 \frac{Q^2T_2}{gA_2^3}$. So, these equations are actually outlined. I am not covering this in a lot of detail because, more or less, when it comes to problem-solving for your assignment or exam, it is sort of out of scope. And for a wide rectangular channel, dS_f/dy is nothing but, so S_f, what is this S_f? Using Manning's equation for GVF profile by using S_f in the Manning's equation of Uniform flow. So, hence we, hence I mean, after using all this, dS_f/dy was $3.3S_f/y$, and this is what we are using here. And if $\frac{dH_E}{dy_2}$, you see $\frac{dH_E}{dy_2}$ after substituting from the previous equation, we substitute ΔH_E is equal to H_E, and we get this step, and this is the This step that we need to Δy_2, and the negative sign denotes that Δy_2 has the opposite sign to that of H_E. It may be noted that if the calculations are performed in the downward direction, as in supercritical flow, the third term in the denominator will be negative.

The procedure is illustrated in the following example, which we are not going to cover. I mean, maybe we will have this one particular example to try in the exercise class. So, this last part is actually quite complicated. So, I am not expecting you to remember this, but yeah, it is a good way to put it, you know, in this slide so that if somebody is interested in further research, since this is a higher-level course, somebody interested in doing a PhD or your master's thesis work can make use of this particular lecture and the topic. So, I think

I will end the class here today, and in the next class, I will continue the lecture, and we will see how direct integration of GVF differential equation is done, and then we will later proceed to our next module and try to solve some problems, maybe 10, 15 problems for the gradually varied flow. Thank you so much. That is enough for today. Bye.