

Free Surface Flow
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Lecture 28

Welcome students to this lecture, which is the third lecture in gradually varied profiles. Today, we are continuing with flow profiles and how these profiles look. In the previous lecture, we saw that there are actually five different types of slope conditions. One is mild slope conditions, one is steep, one is critical slope condition, one is horizontal bed, and one is adverse. We also saw different types of flow profiles. Based on the M_1 , M_2 , M_3 , S_1 , S_2 , S_3 , C_1 , C_3 , H_2 , H_3 , and A_2 , A_3 . There were 12 profiles.

So, now we are going to continue. In the last class, we said that in the next lecture, we are going to start with a solved example, a sort of derivation. Therefore, we are going to continue that. It says that for a wide rectangular channel, the slope is mild or steep, according to S_0 being less or greater than this particular quantity. Where q is the discharge per unit width, and n is Manning's roughness coefficient.

So, we have to show that a different condition for mild and steep slope, and this channel has been given as a wide rectangular channel. We know that for a mild slope, the condition is y_0 should be greater than y_c , or alternatively, S_c should be greater than S_0 . For a steep slope, y_c , which is the critical depth, should be greater than the normal depth, where S_c is the critical slope. Hence, we need to determine the critical slope S_c . This is our target. Now, discharge per unit width q for a flow depth y is given by small q is equal to capital Q , that is discharge and width or breadth Q is AV , and this is B . For a rectangular channel, we can write A as Vy . So, it becomes ByV/B or finally, q is $V \times y$ or q can also be written as when we use Manning's formula for velocity V is $\frac{1}{n} R^{2/3} S^{1/2}$. Now, hydraulic radius R for a wide rectangular channel is R is equal to area by perimeter or wetted area by wetted perimeter. Area for a rectangular channel is by rectangular channel only, and perimeter is $B + 2y$.

But if you have a very, very wide rectangular channel, that means B is much, much, much larger than the depth. In that case, hydraulic radius is y . And the question is, we have to show that for a wide rectangular channel. This means we have to assume that B is much,

much greater than y . So, hydraulic radius is y here. Now, this equation, equation 1, this becomes $q = \frac{1}{n} y^{5/3} S^{1/2}$. You see hydraulic radius is y .

So, q is $\frac{1}{n} y^{5/3} S^{1/2}$ or $S^{1/2}$ is equal to and y comes this side. $S^{1/2}$ is $\frac{nq}{y^{5/3}}$ and if you square this equation. S becomes $\frac{n^2 q^2}{y^{10/3}}$. Now, for the slope given by the previous equation to be critical, we have to replace y by y_c here. So, in this and for a rectangular channel, we know that y_c is $\left(\frac{q^2}{g}\right)^{1/3}$.

or $y_c^{10/3}$ is also equal to $\left(\frac{q^2}{g}\right)^{10/9}$. If we use this equation in this, we can write S_c as equal to $n^2 q^2$, and we replaced $y^{10/3}$ with this and write $\left(\frac{q^2}{g}\right)^{10/9}$. Then, S_c is equal to $n^2 q^2$. This one is $q^{20/9}$ if you take this becomes. Thus, the slope is mild or steep according to S_0 being less or greater than S_c , and that S_c value is given by this. That is one complete solution to our problem. Another question: a rectangular channel with a bottom width of 4 meters and a bottom slope has a discharge of 1.5 meters cubed per second. In a gradually varied flow in this channel, the depth at a certain location is found to be 0.3 meters. Assume $n = 0.016$, determine the flow profile. So, we are given a rectangular channel with a bottom width of 4 meters, slope as 0.0008. Three.

Oh, three zeros, yeah. Q is 1.5 meters cubed per second, it is GVF, and y is given as 0.3 meters. Manning's roughness is given as 0.016. Now, we have to determine the flow profile, either M_1, M_2, M_3 , or S_1, S_2, S_3 . So, basically, first we will have to find out the corresponding normal depth and the critical depth. So, if we are given b is equal to 4 meters, I have already written it down here. Q is 1.5, y is 0.3, and n is 0.016.

So, if we apply Manning's equation, which states that $Q = A \frac{1}{n} R^{2/3} S_0^{1/2}$. So, Manning's equation is used for uniform flow. This means the depth obtained using Manning's equation will be the normal depth, as straightforward as that.

So, area is given as $By_0(1/n)$, and R is the wetted area divided by the wetted perimeter raised to the power of $2/3$, and then $S_0^{1/2}$. Q is given as 1.5. B is given as 4, so we have to find out, and this is again $\frac{4y_0}{4+2y_0}$. We substitute all the values here and use trial and error.

Solve the equation. The important thing is that in the first part, we used Manning's equation because it gives us the uniform flow depth, and the answer we obtain for y_0 is 0.426 meters. The second step will be to obtain the critical depth.

So, for critical depth, we find the unit discharge Q , which is Q/B , that is $(1.5/4)$, which is $0.375 \text{ m}^3/\text{s}$. The critical depth formula we already know is $\left(\frac{q^2}{g}\right)^{1/3}$. And in this case, $\left(\frac{0.375^2}{9.81}\right)^{1/3}$, it comes to be 0.243. So, our normal depth was 0.426. And the critical depth is 0.243. We see that $y_0 > y_c$, which means it is an M profile already.

And what was our water depth? 0.3. So, what we are seeing is that our water depth lies between y_0 , which is normal depth, and y_c , which is critical depth. And y_0 is greater than y , which is greater than y_c .

And which profile is that? It is the M_2 profile. If you want to check, we can go to the previous slides. Yes. Yes, this one.

So, you see y_0 , y lies between y_0 and y_c . That means it is the M_2 profile based on our solution to the problem. So this was a very good and classical example and a very simple problem to solve for our type of profile. Now, we are going to talk about some features of our flow profiles, the type M profile that we have seen. So in the M profile, the M profile is the most common of all gradually varied profiles.

And among that, the most common is the M_1 type, which is a subcritical flow condition. Obstructions to the flow, such as weirs, dams, control structures, and natural features such as bends, produce M_1 backwater curves. These extend to several kilometers upstream before merging with normal depth. M_1 is the most common one. Which means y is greater than y_0 , which is greater than y_c . This is the M_1 profile you see: y , y_0 , and y_c . So, y greater than y_0 greater than y_c , and examples are obstructions to flow, such as weirs, dams, control structures, and natural features such as bends, produce M_1 backwater curves. The M_2 profiles occur at a sudden drop in the bed of the channel, at a constriction type of transition, and at the canal outlet into the pools. One of the examples. So, it occurs at a sudden drop in the bed of the channel, at a constriction type of transition, and at the canal outlet into the pools. So, you see our water surface y_0 , this is sorry

Hour is y , this is y_0 , and this is CDL is y_c . So, y_0 is greater than y , which is greater than y_c . This is the M_2 profile, and these are the characteristics. Where does it occur? It occurs at a sudden drop in the bed of the channel and at a constriction type of transition. So, where a supercritical stream enters a mild slope channel, the type M_3 profile will occur where a supercritical stream enters a mild slope channel, which is mild slope M . A flow leading from a spillway

A sluice gate to a mild slope forms a typical example. The beginning of the M_3 curve is usually followed by a small stretch of rapidly varied flow, and the downstream is generally terminated by a hydraulic jump. Compared to M_1 and M_2 , M_3 curves are relatively short in length. Something like this. You see, this is M_3 , and this is after having a hydraulic jump. This is CDL, this is y_c , this is y_0 , and this is our y . So, you see y_0 is greater than y_c , which is greater than y , which means M_3 . It is important to know when a supercritical

Supercritical flow enters a mild slope channel, the M_3 , and this is a sluice gate, right? And just you see, the beginning of the M_3 curve is usually followed by a small stretch of rapidly varied flow. This is rapidly varied flow, and the downstream is generally terminated by a hydraulic jump. This is a hydraulic jump. Now, different types of S profiles.

The S_1 profile is produced when the flow from a steep channel is terminated by a deep pool created by an obstruction. So, from a steep channel, S is terminated by a deep pool created by an obstruction. Such an obstruction could be a weir or a dam. At the beginning of the curve, the flow changes from normal depth, that is, supercritical flow, to subcritical flow through a hydraulic jump. The profiles extend downstream with a positive water surface slope to reach a horizontal asymptote at the pool elevation. So, something like this, you see, this is where

This is a steep slope, anyways, right? And this is because of this. Because of this, there is a deep pool, correct? Created at the beginning, the flow changes from the normal depth here. You see normal depth to the subcritical flow through a hydraulic jump. This is This is a hydraulic jump. Now, the profile extends downstream with a positive surface slope to reach the horizontal asymptote, and this is our water level y , and this is y_c . And this is y_0 . So, y is greater than y_c , greater than y_0 , is S_1 profile.

So, we have to understand the cases and conditions where these types of profiles are occurring. Now, the profiles of the S_2 type occur at the entrance region of a steep channel leading from a reservoir and at the break of grade from mild slope to steep slope. Generally, S_2 profiles are of short length. How? This one.

You see, this is a steep channel. Right, and this is our water profile region of a steep channel leading from a reservoir and at the break of grade from mild slope to steep slope, right. M profile, and they are of very small length. Now, S_3 profiles: free flow from a sluice gate with a steep slope on its downstream is the S_3 type. The free flow from a sluice gate with a steep slope on its downstream is the S_3 type.

You see something like this. This is A steep slope, a steep slope. S_3 curve you see from this sluice gate, y_0 , y_c , y is less than y_0 , and this is also one of the cases where you will see the steep slope. And it tries to reach the normal depth line.

Now, different types of C profiles. So, C_1 and C_3 profiles are very rare and highly unstable. A critical horizontal channel can be considered as a lower limit reached by a mild slope. As its bed slope becomes flatter, it is obvious that there is no region 1 horizontal channel as y_0 goes to infinity. The H_2 and H_3 profiles are similar to the M_2 and M_3 profiles, respectively.

However, the H_2 curve has a horizontal asymptote. So, H is on the horizontal bed. It is similar to the M_2 and M_3 profiles. The difference is that the H_2 curve has a horizontal asymptote. Now, the adverse type of profile, adverse slopes are rather rare, and A_2 and A_3 curves are similar to the H_2 and H_3 curves, respectively.

These profiles are of very short length. So, the adverse slope you see there is only one. So, this is y_0 , y_c . Yes, y_0 , not y_0 , sorry, this is y . So, y is greater than y_c , indicating an A_2 profile on an on an adverse slope.

So, what is a control section? A control section is defined as a section in which a fixed relationship exists between the discharge and the depth of the flow. Weirs, spillways, and sluice gates are some typical examples of structures which give rise to a control section. What is a control section? It is defined as a section in which a fixed relationship exists between the discharge and the depth of the flow.

Examples include weirs, spillways, and sluice gates. The critical depth is also a control point. However, it is effective in flow profiles which change from subcritical to supercritical flow. In the reverse case of transition from supercritical flow, to subcritical flow, a hydraulic jump is usually formed by passing the critical depth as a control point.

So, supercritical flow to subcritical hydraulic jump, critical depth as a control point. Subcritical flows have controls at the downstream end, while supercritical flows are governed by the control sections existing at the upstream end. This is also important. So, critical depth serves as a control point. How?

One, when the profile changes from subcritical to supercritical; another, in hydraulic jump, where the flow changes from supercritical to subcritical. So, subcritical flows have controls at the downstream end. Whereas, in supercritical flow, subcritical flow is at the downstream, and supercritical flow is at the upstream section. This is now seen in figures a and b for the M_1 profile, the control indication; this is the control section. Profile.

And this is just upstream of the spillway. And here also, this is the control. Just upstream of the. So, the previous slide was about the control section. So, first, we defined what the control section is, and then we got the control point.

This heading came a little bit later. So, you see now in figures *C* and *D*, for M_3 and S_3 profiles respectively, the control point is at the vena contracta of the control point. In subcritical flow reservoir, which is figure *C*, even though the discharge is governed by the reservoir elevation, the channel entry section is not strictly a control section. The true control section will be located downstream of the channel, this one. So, you see this one *B* here.

See D_1 , M_3 , and S_3 profiles are there right here. And then what we see is S_1 , the control point is at the vena contracta of the sluice gate flow. And similarly for figure number *C*. So, for a supercritical canal intake, that is figure *E*, the reservoir water surface falls to the critical depth at the head of the canal, and then onwards the water surface follows the S_2 curve. The critical depth occurring at the upstream end of the canal is the control for this flow.

And for a mild slope channel discharging into a pool of variable surface elevation, it is indicated in this particular figure. So, I think we will close the description of the flow profile now. And in the next lecture, we will go and try to do the analysis of the flow profile. Thank you so much.