Free Surface Flow

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Lecture 27

Welcome back, students, to the second lecture of gradually varied flow. The theoretical concepts that we are covering, we have seen differential equations of gradually varied flow in the first lecture. The basic concepts that we began with, and now we are going to continue with the differential equations and other details of the gradually varied flow. So, now, we have seen just the differential equations in terms of S_0, S_f, Froude number, Q^2T/gA^3, also in terms of Q_n, Q_0, Q_c. Now, we start with the differential energy equation of gradually varied flow. So, we know that the total energy H of a gradually varied flow in a channel of a small slope is given by $H = Z + E$, where Z is the datum head and E is the specific energy.

If we again just try to repeat, so this is the bed slope, this is the water surface. This is the energy line. This is the velocity head. This is the datum head, and this is S_f. Energy slope, which is to be used in the GVF equation. All right, if we apply Manning's formula, all right.

So, we differentiate this: $\frac{dH}{dx} = \frac{dZ}{dx} + \frac{dE}{dx}$. So, from this particular equation, the previous equation, we differentiate $H = Z + E$. So, $\frac{dH}{dx} = \frac{dZ}{dx} + \frac{dE}{dx}$. So, what $\frac{dH}{dx}$ represents? $\frac{dH}{dx}$ represents the energy slope *(*S_f*)*. Since the total energy of the flow always decreases in the direction of motion, it is common to consider the slope of the decreasing energy line as positive.

So, $\frac{dH}{dx}$ is -S_f, which we have seen in the previous lecture as well. $\frac{dZ}{dx}$ represents the bed slope, So. It is common to consider the channel slope with bed elevations decreasing in the downstream direction as positive. So, $\frac{dZ}{dx}$ will also be -S_o. This particular part we have also covered in a lot of good detail in our previous lecture as well.

So, we can write $-S_f = -S_0 + \frac{dE}{dx}$. But here our idea is different; we are going to write the differential energy equation for a gradually varied flow profile. So, $\frac{dE}{dx}$ is nothing but $S_0 - S_f$. So, this particular equation you take this side, and this turns to be $S_0 - S_f$. And this particular equation is called the differential energy equation of gradually varied flow to distinguish it from the gradually varied flow differential equation.

This is the differential energy equation. This energy equation is very useful in developing numerical techniques for the gradually varied flow profile computations. So, two equations we have seen: one is the gradually varied flow differential equation. And it looks like $\frac{dy}{dx} = (S_b - S_f)/(1 - (Q^2)T/gA^3)$. This is the gradually varied flow differential equation.

Second is GVF. Differential energy equation that is gradually varied flow differential energy. Equation is given by $\frac{dy}{dx} = (S_0 - S_f)$. So, two different equations. Now, classification of flow profiles.

So, in a given channel, the normal or uniform depth is given by y_o and critical depth y_c. So, normal depth and the critical depth y_c are two fixed depths when the discharge *Q*, Manning's coefficient *n*, and bed slope s are fixed. So, if discharge and channel properties are fixed. Then, in this case, y_0 is fixed and y_c is fixed.

You can also write y_n for normal depth. There are three possible relationships between these fixed values, and there could be an arbitrary water depth y. So, now there are three possible relationships between y_0 and y_c. Among y_0 and y_c, it could happen that $y_0 > y_c$, meaning the normal depth is greater than the critical depth. That could be one of the conditions. Another could be that the critical depth is greater than the normal depth, or the third could be that these depths could be equal. The first one is normal depth greater than critical depth.

The second one is critical depth greater than normal depth. Critical depth and normal depth are equal. Further, there are two cases where y_0 does not exist. What are the two conditions where y_0 will not exist? The first condition is a horizontal channel bed where there is no bed slope, that is, $S_0 = 0$.

And also in a case when a channel has an adverse slope on the opposing side. So, for these two cases, there does not exist a normal depth, right? And based on all these three, 1, 2, 3, and horizontal channel bed and adverse channel slope, $S_0 = 0$ and $S_0 < 0$. The channel bottom slopes are classified into five categories, namely mild slope, steep slope, critical, horizontal, and adverse. This horizontal and adverse slope we have seen where there is no normal depth.

So, what are these classifications? So, the first letter of these names refers to the type, that is, *M*. S_0, for mild slope, we shall use *M*, right? For a steep slope, we shall use *S*, for critical slope, we shall use *C*, and *H* for horizontal and *A* for adverse slope. So, for each of these five categories of channel lines, channels, lines representing the critical depth and the normal depth can be drawn in the longitudinal section.

These would divide the whole flow space into 3 regions. The first region is the space above the topmost line, the space between the top line and the next lower line, and the space between the second line and the bed. So, I will show you what I mean. So, anything is there. Let us say there is one line here, and one line is there.

So, how many regions? This is one region, this is two regions, this is three regions. So, the region space above the topmost line is one region, the space between the topmost line and the next line is another region, and the space between the second line and the bed is the third region. So, this is what it means. Now the classification.

A mild slope is defined as when $y_0 > y_c$, or the normal depth is greater than the critical depth, which means if y_0 is greater than the critical depth. Right, which also means if the depth is higher, the flow will be subcritical at normal depth. Very simple to understand, right? Second is a steep slope, which means that the critical depth is greater than the normal depth. Right, so a depth which will be lower than the critical depth will have a supercritical flow at normal depth. So this also means supercritical flow at normal depth and is indicated by the symbol *S*. A critical slope is where $yc = y_0$, and since $y_c = y_0$, we will have critical flow at normal depth and is indicated by the letter *C*. A horizontal bed we already discussed where the bed slope is 0, and therefore, it cannot sustain uniform flow. And also for $S_o < 0$, indicated by the letter A, this also cannot sustain uniform flow.

So, five classifications: *M, S, C, H*, and *A*. Now, different regions. So, this is a mild slope. CDL means critical depth line. NDL means normal depth line.

You see here what happens is the normal depth line, which means this is y_0 and this is y_c, and $y_0 > y_c$, which means this is a mild slope. Very simple to understand. So, here $y_0 > y_c$, indicating mild slope conditions. Second, you see this is the NDL. Critical depth line.

This is NDL, so this is y_0 and this is y_c; therefore, this is critical depth. So, you see $y_0 < y_c$, indicating that steep slope conditions. The third case, this is both the normal depth line And the critical depth line; they both are overlapping, they are at the same level, and therefore, $y_0 = y_c$, which means this is a critical slope condition.

Now, in the horizontal bed, there is only a critical depth line, no normal depth line, and this is the critical depth. This is a horizontal bed. And lastly, this is So, which is negative, that is, an adverse slope. This is the critical depth line, and therefore, this is critical depth, and therefore, no normal depth.

So, whether a given GVF profile will have an increasing or decreasing water depth in the direction of the flow will depend on the term dy/dx. So, what it means is whether the water surface elevation is going to increase or decrease in the direction of the flow will depend on the term dy/dx and is given by the dynamic equation of gradually varied flow. Now, the behavior of dy/dx at certain key depths can be summarized as follows; there are certain behaviors. So, if y goes to the normal depth, y goes to y_0, dy/dx will go to 0; that is, the water surface approaches the normal depth line asymptotically.

If the depth goes to critical depth, dy/dx will go to infinity; that is, the water surface meets the critical depth line vertically. If y goes to infinity, dy/dx will go to S_0; S_0 is the bed slope; that is, the water surface meets a very large depth as a horizontal asymptote. Now, based on this information, the various possible gradually varied profiles are grouped into

12 types, as tabulated below. So, among mild slopes, we see that there we had y_0 and y_c, but in a normal flow, we see then these are fixed. In a normal flow, the water surface elevations can be at any level y, and that y can vary.

However, with the determination of y_0 and y_c, our profiles are fixed. So, by knowing y_0 and y_c, we can determine whether it is a mild slope, steep slope, critical slope, horizontal bed, or adverse slope, y_0, y_c, and So. These three things are also fixed. We know this. Now, when there is a flow and a water level, that water level can be anywhere; it could be above normal depth, below normal depth, above critical depth, below critical depth, below both, or above one of them.

So, under that condition, you see, we have several types of sub-profiles within different types of channels. So, the channel type is determined based on. So, these three things are decided based on y_0, y_c, and So. These are fixed. However, with different values of y, we will have several other cases. So, if y, our water depth, is greater than normal depth and critical depth, we call this type the M_1 profile. If our water depth lies between the normal depth and critical depth, it is called M_2. See, one thing is for sure: if $y_0 > y_c$, it means a mild slope.

Now, the third condition could be that our y could be less than both of these, and this is the M_3 profile. In the second case, y_0 was greater than y_c; in the steep slope, $y_0 < y_c$. So, it could happen that our water depth in the steep slope case could be higher than both of these, then that will be called the S_1 profile. If our water depth lies between y_c and y_0, that is between the critical depth and the normal depth, we call it the S_2 profile. If it is smaller, if our water depth is smaller than both of these, this is called the S_3 profile.

For the critical profile, we have y_o equal to y_c. So, there are only two conditions that are possible. Either y is greater than both of these, and that is called C_1, or our water depth is less than both of these, in which case we have the C_3 profile. C_2 is missing because normally, 2 refers to when it lies in between.

For a horizontal bed, it is very simple; So is equal to 0, and there is no normal depth, right? So, it will have only two conditions: if y is greater than the critical depth, or y is less than; if y is greater, we call that the H_2 profile. If y is less than the critical depth, it is called the H_3 profile. Similarly, for an adverse slope, which is negative.

So, we have only one depth that is critical depth. So, again, only two conditions can be there regarding our water depth. It is either greater than critical depth or our water depth is less than the critical water depth. And in that case, it is called the A_2 profile and it is called the A_3 profile. So, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12.

We have 12 different types of profiles as in the table given on the left side. So, for mild slope, just to demonstrate what these different types of figures are, you see M_1, this is our water surface elevation, which means y is greater than y_o, which is greater than y_c. This is M and this is M_1. This here, this water surface this water surface lies between y_0 and y_c, implying that y_0 is y greater than y_c. So, indicating the M_2 profile, and here our water surface is less than both y_0. Second is steep slope. When this is the water surface elevation, you see y is greater than y_c, which is greater than y_0, and this is the S_1 profile. In S_2, y is lying between y_c and y_0, here S_2, you see here, and this is S_3.

It is less than both y_c and y_0, and therefore, the S_3 profile is present. The third one is a horizontal bed. This is a horizontal bed. So, this is the water surface. So, this is the H_2 profile; $y > y_c$, and here $y < y_c$.

Therefore, this is H_3, and this is the critical slope. Here, both NDL and CDL are the same. Therefore, y will be greater than y_0 if it is equal to y_c; this is C_1, and in this than y_0 and y_c, this is C. And the last profile is this adverse slope. So, the adverse slope is S_0, which is not positive; that is, S_0 is negative.

And here Also, NDL does not exist. Here, either our water surface is greater than y_c. Therefore, this is A_2. And the second case is $y < y_c$, which is again a case of adverse slope.

So, all in all, the important thing to notice is that there are 12 different profiles. You might be asked to calculate these y_0, y_c, and for certain conditions, try to estimate and draw what type of different profiles are there: M_1, M_2, M_3, S_1, S_2, S_3, C_1, C_3, H_2, H_3, A_2, and A_3. So, I think we should end this lecture here. It is a nice point, and when we meet in the next lecture, we will start with a solved example. It is more of, you know, not a numerical example but trying to show some sort of derivation, which is more like a theory. So, that is it for this lecture, and I will see you in the next one.