Free Surface Flow
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Lecture 24

Welcome back, students, to the last lecture of the fifth module. No, not the last lecture, but the second last lecture of the fifth module. Here, we are going to again solve some problems related to uniform flow. These will be a little shorter problems. There is a trapezoidal channel that is 10 meters wide and has a side slope of $1.5H: 1V$. The bed slope is 0.0003. The channel is lined with smooth concrete of n equal to 0.012. Compute the mean velocity and discharge for a depth of 3 meters. Simple question: the channel is trapezoidal, which is 10 meters wide, with a side slope of 1.5:1. The bed slope is also given, and it is given that it is lined with smooth concrete, whose Manning's number is 0.012.

It is asking us to calculate the mean velocity and discharge for a depth of flow of 3 meters. Very simple. Fastest We will need to draw a trapezoidal channel. This is a normal depth *y* naught, and this is 1 in 1.5, and this is 10 meters.

We say let y_0 is equal to uniform flow depth. The things that are given here are *B* is equal to 10 meters and side slope *m* is equal to 1.5. Area is very simple $(B + my)y$. So, area *A* is we just substitute in the values $(10 + 1.5) \times 3$ because normal depth is also given right 3×3 that is 43.50 m^2 ok.

Area has been calculated. Wetted perimeter *P* is equal to $B + 2y\sqrt{1 + m^2}$. So, $10 + 2 \times 3 \times (1 + 1.5^2)$ and this will come out to be 20.817 meters and hydraulic radius *R* is given by A/P. So, you see area was $43.5/20.817$ meter square meter and this comes out to be 2.09 meters hydraulic.

Now, mean velocity, this is as simple a problem can get in Manning's number. $1/n\, R^{2/3} S^{1/2}$. $1/0.012 \times 2.09^{2/3} \times 0.0003^{1/2}$ or V comes out to 2.36 meters per second. And therefore, discharge Q is area into velocity 43.50×2.36 that comes to be 102.36. 63 m^3/s.

So, the beauty of this problem is that it is one of the simplest problems that you can get. Normal depth is given, right? Width of the channel is given, side slope is given, bed slope is given, and Manning's number is given. So, it is very regular. So, what you need to calculate first is the area.

You need to calculate the perimeter, you need to calculate the hydraulic radius, and plug these values into Manning's equation. For discharge, you can calculate A_1 V_1 equals A_2 V_2. So, this problem was too simple. Now, a follow-up problem in the previous example, It says, find the bottom slope necessary to carry only 50 m^3/s of discharge at a depth of 3 meters. So, the setup is the same, just that we have been further asked to find the bottom slope. So now here the bottom slope is not known, right? But the discharge is given. So if we know the discharge, we know the

If we know the discharge, we know the velocity, or we can use Manning's equation just in terms of discharge. So, the solution to this problem is quite simple. We calculated. So, let us see what are the things that are required. First, let us see the formula for Manning's number.

V was $1/n\, R^{\,2/3} S^{1/2}$ or Q was A raised to the power. So, first $AR^{\,2/3} S^{1/2}$ or $1/n\, A(A/P)^{\,2/3} S^{1/2}$ or Q is $1/n\ (A^{5/3}/P^{2/3})\ S^{1/2}$. So, squaring both sides, $Q^{\,2}$is equal to $1/n^2\ A^{10/3}/P^{4/3}$ So implies S naught can also be written as $Q^2 n^2/A^2 R^{2/3}$ or $Q^2 n^2/A^{10/3} R^{4/3}$. This looks much simpler. So, we will choose this. $Q^2 n^2/A^2 R^{2/3}$ From our previous, we saw that 43.50 was the area, perimeter was 20.817, hydraulic radius was 2.09 meters, and therefore, So will be $Q^2 n^2/A^2 R^{2/3}$, $50^2 0.012^2/43.5^2 2.09^{2/3}$, sorry. So, it comes to be 0.0000712. So, the problem was slightly changed, but due to the change, they did not complicate the matter. Instead of the bed slope being given, we had to calculate it, but in the first problem, we were asked to find the discharge, whereas here the discharge was given. So, only one variable at a time, which is pretty easy to solve.

Another question, again a simple one. A slightly rough brick-lined channel, that is, Manning's value is given as 0.017. A trapezoidal channel carrying a discharge of 25 cubic meters per second is to have So, there is a rough brick-lined trapezoidal channel carrying a discharge of 25 cubic meters per second, and it is to have a longitudinal slope of 0.004. So, what do we have to do?

We have to analyze the proportions of an efficient trapezoidal channel section. having a side slope of *1.5H:1V*. So, that is the question. So, for an efficient trapezoidal section having a side slope of m.

We know from our theory that Ae is equal to $(2 \times \sqrt{1\ + m^2}\ -\ m)({y_e}^2)$, or so we have been given m already: $(2\sqrt{1\ + 1.5^2}\ - 1.5) \times ({y_e}^2)$, and then the R effective is $y_e/2$. Also, we are given q as 25 meters. This is dot. 25 meters cubed per second. Now, if we substitute these values into Manning's equation. What is Manning's equation? Q is equal to $1/n\ AR^{\ 2/3}{S_0}^{1/2}$. So, Q is 25, n is 0.017, and what is A? It is 2.1056 ${y_e}^{2}$)

And R is $(y_e/2)^{2/3} \times \sqrt{0.0004}$. The solution is done. We found out that the y_e solution is 2.830. So, now we know y_e; y_e comes out to be 2.830 m. The formula for B_e efficient channel was $2y_e(\sqrt{1\ + m^2} - m)$, that is $2 \times 2.83(\sqrt{(1 + 1.5^2)} - 1.5)$.

and B_e comes out to be 1.714 meters. So, y_e and B_e are known, and that solves our problem. So, we are going to solve yet another very simple problem. If we are given that g is the acceleration due to gravity and R is equal to the hydraulic hydraulic radius, find the relation between the Darcy-Weisbach friction factor and Manning's rugosity coefficient So, we know from Chezy's equation V is equal to $C\sqrt{RS}$. V is equal to $1/n\ R^{\ 2/3}S^{1/2}$. If we equate the above 2, we get $C\sqrt{RS}$ is equal to $1/n\ R^{\ 2/3}S^{1/2}$. or $C\sqrt{R}$ is equal to $1/n\ R^{\ 2/3}$. As we know, C is equal to $\sqrt{8g/f}$. That means, if we put it here, we can write $\sqrt{8g/f} \times \sqrt{R}$is equal to $1/n\ R^{\ 2/3}$ or $\sqrt{f}$ is $n\ \sqrt{8g}/\ (R)^{2/3\ -1/2}$ Squaring on both sides, f is equal to $8\ n^2g/R^{1/3}$, relation between, this is the relationship between Manning's n and Darcy's Weisbach friction factor. So, that is enough for today, and I will see you in one last problem-solving session for problems on uniform flow in the next class. Thank you so much.