Free Surface Flow
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Lecture 23

Welcome, students, and we are going to continue problems on uniform flow in this lecture as well. So, let us get started directly. A trapezoidal channel, most of the questions are given for trapezoidal channels because it is a little bit complex and not that simple as well, but not too difficult either. With a bottom width of 3.5 meters. And side slopes of *1H:1V* on the left and *1.5H:1V* on the right. So, there is a trapezoidal channel whose both sides do not have the same slope, with n equal to 0.016 and a bed slope of 2.6 in 10,000. It carries a discharge of 8 cubic meters per second. Determine the normal depth and the average shear stress on the channel bed. So, this is a question. Where we have a trapezoidal channel. With a bottom width. B is given as 3.5 meters.

And the side slope is 1:1. On the left. And on the right, it is 1.5:1. Manning's number is given. And the bed slope is also given.

The discharge is also given. Now it asks us to calculate the normal depth. And τ_o. So we start writing down. With what we are given.

Given Q is equal to 8 meter cube per second. S is given as 2.6/10000. n is given as 0.016. Given the channel. This is y, this is y is 3.5, this is 1:1.5, and this is 1:1.

My drawing is not very good, but anyway, the idea is to draw a sketch. From Manning's equation What is Manning's equation? Q is equal to $1/n\ AR^{2/3}\ \sqrt{S}$. These are the things that we know right from the question itself.

Q, S, n we know the diagram, and we know Manning's equation. So, the first step will be to find out the area. A will be $0.5 \times (B + B\ + 1.5y + y) \times\ y$, regulator episode because this is $m\, y$ and that in one case m is 1.5, the other side it is 1. So, the area is going to $be\ (B\ +\ 1.25y) \times y$. Similarly, perimeter P is equal to $B + y\ (\sqrt{1\ +\ 1.5^2}+\sqrt{1\ +\ 1^2})$ or P is equal to $B\ +\ 3.217\ y$.

Hydraulic radius R is equal to $area\ /\ perimeter$. 1.25/3.5 Just repeating what A is $(3.5\ +\ 1.25\ y)\ \times y$ and the perimeter again is $3.5\ +\ 3.217y$. Now, if substituting A and P in the Manning's equation that is 8 is equal to $1/0.016\,(3.5\ +\ 1.25y) \times y$ ((3.5+1.25y)

$\times$ *y*/ $(3.5 + 3.217y)^{2/3}$ $\sqrt{2.6/10000}$ or $(3.5 + 1.25y) \times y \times (3.5 + 1.25y) \times y)$ / $(3.5 + 3.217y)^{2/3}$ is equal to $8 \times 0.016 \times \sqrt{2.6/10000}$ So, the solution to do that is solving the hit and trial method. And if we do that, we get the value of *y* as 1.505 *m*. So, this is the normal depth. See, as the geometry gets complicated, the solution becomes more complicated, and the final way is to solve using the hit and trial method.

So, after that, the next part is the average shear stress on the bottom of the bed is equal to $\gamma R \times S$, that is τ is equal to $\gamma R \times S$. Gamma is ρg, $1000 \times 9.81\ \rho g \times (3.5 + 1.25 \times 1.505)\ \gamma R \times S$. $1.505\ /(3.5 + 3.217 \times 1.505) \times \sqrt{2.6/10000}$, and this τ_0 comes to be 2.434. Newton per meter square.

So, a long question because of the difficulty in the cross-sectional area. However, these are the regular things that are generally needed to be asked or calculated. So, the average bed shear stress comes out to be 2.434 N/m^2. Now, we will continue with another question. Show that the normal depth of flow in a triangular channel having a side slope *z:1* as *H:V*. Is given by what do we need to show1.189 $(Q_n/\sqrt{S})^{3/8} \times ((z^2 + 1)/(z^5))^{1/8}$. So, this is more like a derivation rather than a numerical. So, the question talks about a triangular channel.

So, first what we should do is draw a triangle. Let us say this is the water depth, this is the This is the normal depth and this is *1:z*. This is how it looks like. So, from Manning's equation Q is equal to $1/n\ AR^{2/3}\ \sqrt{S}$ or $Q_n/\sqrt{S}$ is equal to $AR^{2/3}$ very simple.

For this particular triangular section, the area is nothing but $Z(y_n{}^2)$. Perimeter is $2y_n\sqrt{(1 + z^2}$.Hydraulic radius R is equal to A/P or $Zy_n/2\sqrt{(1 + z^2)}$, i.e., the hydraulic radius. This is, let us say, equation a. If we use this in a. If we use this in this equation, substituting R in equation a, $Z(y_n{}^2). (Zy_n/2\sqrt{(1 + z^2)})^{2/3}$ by $Q_n/\sqrt{S)}$ or $(z^{5/3}y_n{}^{8/3})$ / $2^{2/3}(1 + z^2)^{2/3}Q_n/\sqrt{S}$ or $y_n{}^{8/3}$ is equal to $2^{2/3}$, taking all these terms.

Here and this one here. $2^{2/3}$ $((z^2 + 1)/(z^5))^{1/3} \times$ $Q_n/\sqrt{S}$ So, y_n is equal to $(2^{1/4})$ $((z^2 + 1)/(z^5))^{1/8}$, and $(Q_n/\sqrt{S})^{3/8}$. What we have done is we have raised this equation's power by 3/8. and this becomes $1.189((z^2 + 1)/(z^5))^{1/8}$ $(Q_n/\sqrt{S})^{3/8}$All right. So with that, this is what we needed to prove, and we proved it. Starting with another question. Probably the last one for this particular lecture.

Water flows in a channel of the shape of an isosceles triangle bed with width a. And sides making an angle of 45 degrees with the bed. Determine the relation between the depth of flow for the maximum velocity condition So, we need to apply the maximum velocity condition and the maximum discharge condition. And here we have to use Manning's formula.

And note that d is less than 0.5a. So, we have to find the condition for the maximum velocity condition and the maximum discharge condition. So, the first thing is to draw the figure. This is the bed, this is how it looks like, this is 45 degrees, this is a 1 in 1 slope, this is d. And this is a, this is x. So, Manning's equation $1/n\ AR^{2/3}\sqrt{S}$ or $1/n\ A(A/P)^{2/3}\sqrt{S}$or $1/n\ A^{5/3}P^{2/3}\sqrt{S}$. A is area is $a + a - 2d$ This is $a - 2d$, $((a + a - 2d)/2) \times d$. That is $(a - d) \times d$ and perimeter is $a + 2d\sqrt{1+m^2}$ or $a + 2\sqrt{2}d$ as n and s are constant. And for maximum discharge, dQ/dd is equal to 0.

So, dQ/dd is equal to d/dd. $A^{5/3}P^{2/3}$ is equal to 0 or 5/3 $A^{5/3}P^{2/3}dA/dd$ - *2/3* $A^{5/3}P^{2/3}$ dP/dd is equal to 0 or $5P\ dA/dd - 2A\ dP/dd$ is equal to 0. If we substitute the value of A and P, we are going to get y, $5(a + 2\sqrt{2}d)(a - 2d) - 2(a - d)d\ 2\sqrt{2}$ is equal to 0.

So, we can finally get, if we can get $5a^2 - 10ad + 10\sqrt{2}ad - 20\sqrt{2}d^2 - 4\sqrt{2}ad + 4\sqrt{2}d^2$. is equal to 0 or $22.627d^2 + 1.515ad - 5a^2$ is equal to 0. On solving the quadratic equation. So, d will be 0.438a or -0.505a, we neglect this. So, for d is equal to 0.438a.

Maximum discharge will occur for maximum velocity condition, very similar, very simple. V is $1/n\ AR^{2/3}\sqrt{S}$ or dV/dd is equal to $d/dd\ (A/P)^{2/3}$is equal to 0 or $P\ dA/dd - A\ dP/dd$ is equal to 0. or $(a + 2\sqrt{2}d)(a - 2d) - (a - d)\sqrt{2}d^2$ is equal to 0. or $a^2 - 2ad + 2\sqrt{2}ad - 4\sqrt{2}d^2 - 2\sqrt{2}ad + 2\sqrt{2}d^2$ is equal to 0. $\sqrt{2}d^2 + 2ad - a^2$ is equal to 0, and this will give us d is equal to the quadratic equation's two roots: 0.338a or -1.05a. This is neglected because of the negative root. So, d is equal to 0.338a. is the condition for maximum velocity.

So, in this question, we have seen the relationship between d and a that was asked for two cases: maximum discharge condition and maximum velocity condition. And in both cases, we try to maximize this with regards to the depth. So, I think this is the end for today's class, and I will see you in the next lecture. Thank you so much. Where will we continue?

More problems. Thank you.