Free Surface Flow
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Lecture 22

Welcome back, students, to our second session of problem-solving on uniform flow. So, last class, we solved almost three questions dedicated to finding the discharges. We saw Manning's number, we saw Chezy's number, and we saw how D is equal to $4R$ when we want to use the equations for pipe flow. So, today we continue with our problem-solving classes again. So, let me first write down the question: A lined channel of trapezoidal section has one side vertical. And the other side has a slope of *1H:1V*.

The channel The channel has to deliver 8 m^3/s when laid on a slope of 0.0002. The question is What would be the dimensions of the efficient section which requires minimum lining? So, a question on the efficient section. It also says to calculate

Also, calculate the corresponding mean velocity if Manning's n is 0.015. So, the key step in this particular question is first to draw the diagram of how it looks. It says one side is vertical, right? So, this is the width, this is the channel, and this is how the slope 1:1 looks. This is the water surface.

So, instead of regular cross-sections, we have this type, and this is the water depth y, and this is B. What are the quantities that are given? We have been given q is equal to 8 meter cube per second, s has been given as 0.0002. Manning's n is given as 0.015 and slope. So, *1:m* is given as 1:1. So, let us see the basic quantities first.

So, area A would be $0.5(B + B + my) \times y$, right. This is y, this is m means m is 1 here, all right, but just writing down the area is going to be $0.5(2B + my) \times y$. And putting m is equal to 1, area is going to be $0.5(2B + my) \times y$, that is the first thing to note down. Secondly, we can actually find the parameter as well.

Let me just clarify the area is this much. Okay, and perimeter is this one. So, parameter P is equal to $B + y + y\sqrt{1 + m^2}$ or parameter is equal to $B + y\sqrt{1 + \sqrt{2}}$. Right, or B can be written as here, alright. This is the parameter.

Using this equation, we can also write B as A/y, sorry, $A/y - y/2$, okay. Correct. So, now putting this B, P in this A/y, putting this value of A and B in the perimeter equation, which equation? This equation. So, P is $A/y - y/2 + y(1 + \sqrt{2}) \times (1 + \sqrt{2})$.

So, we know that for an efficient section, dP/dy is equal to or $d/dy\ A/y - y/2 + y(1 + \sqrt{2})$ is equal to 0. So, this A/y will come out to be $-A/y^2$, this $-y/2$ will be $1/2 + 1 + \sqrt{2}$ is equal to 0. $-A/y^2$ + this will come to be 1.914 is equal to 0 or A/y is equal to $1.914y$. This is there. If we put this value.

let us say this is star, if we put star in this one, B is equal to $A/y - y/2$, star in double star, what are we going to get? $B + y/2$ is equal to $1.914y$ or B is equal to $1.414y$. Area A will be $0.5(2B + y) \times y$ or area is equal to $0.5 \times (2 \times 1.414y + y) \times y$. or area will be $1.914\,y^2$ and parameter P is equal to $B + y\sqrt{1+\sqrt{2}}$. So, parameter will be 1.414y that is B, + $\sqrt{1+\sqrt{2}\times y}$ or P will be equal to $3.828y$. So, area is $1.914\,y^2$, parameter is this.

Hydraulic radius R will be A/P, and that means it is going to be $1.914y^2/3.828y$, and hydraulic radius will come to be $y/2$. From Manning's equation. Q is what? $1/n\ AR^{2/3}\ S_0^{1/2}$.

What was Q? 8 is equal to $1/0.015$, A is $1.914\ y^2 \times y/2$. So, because hydraulic radius was $(y/2)^{2/3} \times \sqrt{0.0002}$ or $y^{8/3}$ is equal to $8 \times 0.015 \times 2^{2/3} / \sqrt{0.0002} \times 1.914$ or $y^{8/3}$ is equal to 7.037 or y is equal to 2.079 meters, and B will come out to be 1.414 y or 2.939 meters. Channel dimensions are By here; mean flow velocity will be Q/A. So, V will be $8/(1.914 \times 2.079 \times 2.079)$ or V is 0.967 meters per second. So, this is an example where we have not only derived the efficient section, but this was also for an arbitrary shape which we have defined right, and other than just deriving, we also had some specific values where the actual cross-sections could be found out, what was the dimension for the most efficient section like this.

So, it's time to go for another question again. A concrete-lined trapezoidal channel, which shows 0.015 as the Manning's number. It is to have a side slope of one edge, *1H:1V*. The bottom slope is to be 0.0004.

Find the width of the channel necessary to carry 100 m^3/s of discharge at a normal depth of 2.5 meters. So, information like trapezoidal channel, its Manning's roughness is given, side slope 1:1 is given, bottom slope is 0.0004. Now, we have to find B, and the good thing is discharge is given, and a normal depth is also given, okay. So, we say let *B* be equal to the bottom width, okay.

Here, y_0 is equal to the normal depth, which is equal to 2.5 meters, and *m* is equal to 1. The area is equal to $(B + 2.5) \times 2.5$, as it is a trapezoidal channel, okay, $(B + 2.5) \times 2.5$. Because *m* is 1, $(B + m\,y_0) \times y_0$. The wetted perimeter *P* is equal to $B + 2\sqrt{2} \times 2.5$, which is equal to $B + 7.071$. Now, calculating the factor $AR^{2/3}$, and what is that? $Q_n/\sqrt{S_0}$, *Q* is 100, *n* is 0.015, and the bottom slope is 0.0004, and that comes out to be 75, okay. So, now this one $AR^{2/3}$, is what? $(B + 2.5) \times 2.5)\ ^{5/3} / (B + 7.071)^{2/3}$is equal to 75.

And how are we going to solve this problem? This problem will be solved by using trial and error. *B* comes out to be 16.33 meters wide. So, the question in the previous lecture, which was the last question, was where we solved for the normal depth. It was similar to this one here. Our normal depth was given, but we solved for *P*. The solution procedure is almost the same. We found the conveyance, I mean the section factor $AR^{2/3}$, using Manning's equation, and then we solved for that.

Yet another question, this was a short question. A trapezoidal channel has a side slope of 1:1. It is required to discharge 10 m^3/s of water with a bed slope of 1 in 1000. If unlined, the value of Chezy's constant *C* is 45.

If lined with concrete, *C* is 60. The cost per cubic meter of excavation is 4 times the cost of lining per square meter. The channel is to be the most efficient one. Find whether the lined canal or the unlined unlined canal will be cheaper.

What will be the dimensions of an economical canal? What are the things that are given? 10 m^3/s discharge is given. *S* is equal to 1/1000 is given. And *1:m* is 1:1.

1:1. *V:H*. We know that for an efficient trapezoidal channel, we already know that the hydraulic radius *R* is equal to $y/2$, okay. And area *A* is equal to $(B + my) \times y$ *or* $(B + y) \times y$. Perimeter *P* is equal to *B* under root 1 plus *m* square or *B* plus 2 under root 2 *y*. So, this is one of the conditions for an efficient channel for trapezoidal hydraulic radius.

So, from *R* is equal to $y/2$, which is a condition for an efficient trapezoidal channel, *R* is equal to A/P is equal to $(B + y) \times y/(B + 2\sqrt{2} \times y)$ is equal to $y/2$. So, *B* is equal to $2(\sqrt{2} - 1) \times y$ or *B* comes out to be $0.828y$, and the area will be, therefore, $0.828y^2$. $1.828y^2$ and perimeter is going to be $3.656y$. From Chezy's equation, we know that *Q* is equal to $AC\sqrt{RS}$. Here, 10 is equal to $1.828\,y^2 \times 45$ because *C* is $y/2 \times 1/1000$.

This gives us $y^{5/2}$ is equal to $10 \times \sqrt{2000}/(45 \times 1.828)$ or y as 1.968. Now, here the volume of excavation for unit meter length will be $1.828\ y^2 \times 1$, that is 7.08-meter cube. We say let the cost of 1 square meter of lining be rupees x, and then the cost of 1 meter cube of earthwork be $4x$, that is given in the question, right? That the earthwork is 4 times more expensive than the cost of lining, but however, that is in meter cube, that is meter square, ok. So, the cost of excavation is equal to $4x \times 7.08$, that is rupees $28.320x$. For lined canal, C is equal to 60, right. So, the previous one was for unlined canal, ok, and this is for lined canal. C is equal to for lined canal, C is equal to 60. So, we repeat this same procedure. Q is equal to $1.828y^2 \times 60 \times \sqrt{y/2 \times 1/1000}$ or $y^{5/2}$ is equal to $10 \times \sqrt{2000}/(1.828 \times 60)$ or y comes to be 1.755.

The volume of excavation for unit meter length is equal to $1.828\ y^2 \times 1$ is equal y is 1.755, 5.630. The cost of excavation for unit meter length is equal to $4x \times 5.630$ or rupees $22.521\ x$. Now, we also need to calculate the area of lining for unit meter length. $P \times 1$ or $3.656 \times y$ or 3.656×1.755 or 6.416. So, the cost of lining for 1 meter length, it is $6.416\ x$. So, the total cost will be rupees $22.521\ x + 6.416\ x$, that is rupees 28.937 x, right for unlined compared to the lined canal. So, in an unlined canal, there is no lining. So, only the earthwork cost is there, whereas, in the case of a lined canal, there is earthwork and the lining. As we can observe, the unlined canal is cheaper, hence it will be the most economical channel. Its dimensions are going to be, for this particular case, y as 1.968 meters and B as 1.626 meters. This is the end of one of the very lengthy questions and also marks the end of our particular lecture. So, that being said, I will see you in the next class.