Free Surface Flow
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Lecture 21

Hello students, welcome to our fifth module, the first lecture of the fifth module, and we are going to solve the problems on uniform flow. In the last classes, the last five classes, we have studied the core concepts in uniform flow. Now we will solve some of the sample and example problems so that you have a better hold on this particular concept. That being said, We will start.

The question is: a 2-meter-wide rectangular channel carries water at 20 degrees centigrade at a depth of 0.5 meters. Now, importantly, the channel is laid on a slope of 0.0004. Find the hydrodynamic nature of the surface if the channel is made of two conditions: one is very smooth concrete and the second is rough concrete. So, the difference between a) and b) is that the Manning's number is different, all right. So, we will start the solution procedure, all right. First of all, we are going to see what the hydraulic radius is, or let us say we find the area. The area is $B \times y$. So, B, how much is B?

B is 2 meters, y is 0.5 meters, and this is 2×0.5, which is 1 m^2. The perimeter is B plus $2y$, which is 2 plus 2×0.5, that is 3 meters. So, the hydraulic radius R will be A/P, which is $By/(B+2y)$ or 1/3, which is 0.333 meters. Now, another step is to calculate $\gamma R S_0$.

This is going to be 9.81 × 1000 because γ is ρg. So, $9.81 \times 1000 \times R$ is here 0.333So is 0.0004. And this comes out to be 1.308 Newton per meter square. Now, shear velocity, we have to find the $u*$, called the shear velocity first, which is $\sqrt{\tau_0/\rho}$.

$1.308/10^3$, and it comes to be 0.03617 meters per second. So, these are the general things that we need to find beforehand before proceeding to the questions or the parts that are asked. So, the first part is for a smooth concrete surface. So, you remember we showed you the tables, right, for finding the εs and the other things like diameter, but ε mainly, the ε_s for different surfaces where it was said for glass it is 3×10^{-4}, very smooth concrete surface, very rough concrete surface, or simply rough concrete. So, from that particular table, we can write the roughness value εs is equal to 0.25 millimeters or 0.00025 meters, and the υ at 20 degrees.

At 20 degrees centigrade, it is 10^{-6} meter square per second or ε_s υ, sorry $u*$, because this is the parameter on which this depends. The formula that you are going to use or with the regime, actually ε_s $u*$, you remember if it is less than 4, a different regime; if it is between 4 and 60, a transitional regime; and then greater than 4, it is a different regime itself by υ. So, 0.0025. $u*$ was 0.03617 divided by the kinematic viscosity of water, and that comes out to be 9.04 in our case, 9.04. Since this value is slightly greater than 4.0, the boundary is hydrodynamically in the early transition from a smooth to a rough surface, right. Secondly, for rough So, we did it for a smooth concrete surface where εs we took from the table. Now, we are going to find out for a rough concrete surface.

So, from here also from the table, ε_s is 3.5 millimeters or 0.0035 meters, right, and calculate this quantity $\varepsilon_s u*/\upsilon$, ε is 0.0035 into 0.03617 /10^{-6} is equal to 126.6. So, now, since this value is greater than 60, the boundary is hydrodynamically rough.

So, in the first case, it was a transition from a smooth to a rough surface, but in the second one, it is a hydrodynamically rough boundary, ok. So, now, we move to another set of problems, another problem. Which is related to the previous problem itself. So, the question is for the two cases in the previous example, which example? This example.

A 2-meter wide rectangular channel carries. Previous example, estimate the discharge in the channel using, first we have to use the Chezy formula. With Darcy-Weisbach f. And in the second case, we have to use Manning's formula. So, the question, the setup of the problem is the same.

What is the setup? A 2-meter wide rectangular channel carries water at 20 degrees centigrade at a depth of 0.5 meters. The channel is laid on a slope of 0.004, ok. So, now instead of this, find the hydrodynamic nature, the question turns out to be Estimate the discharge in the channel using the Chezy formula with Darcy-Weisbach and Manning's formula.

For the two cases, what were the two cases? Very smooth concrete and very rough concrete. So, we solve for the first smooth concrete case. Smooth concrete channel. ε_s was 0.25 millimeters, and therefore, $\varepsilon_s/4R$, you remember in pipe it was ε/D, now here $4R$ is equal to 0.25 divided by 4 radius, we will also write in terms of millimeters, okay. So, it will solve our problem because it is a non-dimensional number, 1.894×10^{-4}, right? $\varepsilon_s/4R$ is 1.894×10^{-4}. Since the boundary is in the transitional stage. How do we know the transitional stage? From the previous problem, we saw that, right? It was in the transitional stage. We are going to use this particular formula.

It is equal to 1 point, we have covered this in the slide, $-2\ log\ \varepsilon_s/4R\ +\ 21.25/Re^{0.9}$. So, the important thing to notice here Reynolds number is not known to start with, and hence, a trial and error method has to be adopted. Adopted. By trial, I mean you are going to solve it, but we see that f comes to be 0.0145 and C comes out to be $\sqrt{8g/f)}$.

So, this has been determined by trial and error, and velocity will therefore be$C\sqrt{RS_0}$, where C is 73.6 $\times \sqrt{0.333\ \times\ 0.0004}$. It is equal to 0.850 meters per second, and therefore, discharge is $A \times V$. So, this value comes to be 0.850 meter cube per second because the area was 1 in the previous case. You see, the area came out to be 1 meter square. Now, the second part was about the rough concrete channel. Here, ε_s came out to be 3.5 millimeters. And $\varepsilon_s/4R$ came out to be 2.625×10^{-3}.

We substitute the value of R here, the radius, and $\varepsilon_s/4R$ comes out to be 2.625×10^{-3}. Now, we know since the flow is in the rough turbulent state, we are going to use this equation: $1.14\ -\ 2\ log$ 2.0. If you are not sure about the equation, refer to the slides. f here needs no iteration; it comes out to be 0.025. So, C is going to be $\sqrt{8\ \times\ 9.81/0.025}$, which is 56.0.

And velocity is going to be 56 $\times \sqrt{0.333\ \times\ 0.0004}$, with one more 0. 0.647 meters per second, and Q is equal to $A \times V$, where the area is 1. So, it is still 0.647 meter cube per second. Now, the second part. So, this was all done using.

This is all done using Chezy's equation. Now, using Manning's equation. So, for Manning's a) part smooth, we need to find the value of n, right? So, if we find for the smooth case, Smooth travel finished concrete n can be taken as 0.012, and therefore, V will be $1/n\ A$ raised to the power or directly hydraulic radius.

$R^{2/3}S^{1/2}$ that is $1/0.012 \times (0.333)^{2/3}$ into $0.0004^{1/2}$ or it comes to be 0.801 meters per second, and discharge since the area is 1 meter square, discharge is 0.801 meter cube per second. And, now for the rough part using Manning's n, again using the table n is equal to 0.015 rough concrete. So, V is $1/nR^{2/3}S^{1/2}$. And therefore, V is $1/0.015 \times (0.333^{2/3}) \times (0.0004^{1/2})$ or velocity is 0.641 meters per second.

and Q is equal to AV also because since the area is 1 m^2, 0.641 m^3/s. So, this solves the complete set of problems for this case, yeah. You see how easy it is to use Manning's formula, right. In Chezy's formula, you have to use the Darcy-Weisbach equation and that is the subjectiveness of including selecting the proper values of εs right.

So, basically, we see that very good results can be obtained using Manning's formula. Now, starting with another set of problems. So, the question is: a 5-meter-wide trapezoidal channel having a side slope of $1.5H: 1V$ is laid on a slope of 0.00035, S is also given. The roughness coefficient n is equal to 0.015, find the normal depth for a discharge of 20 m^3/s through this channel. So, as you see, we have been given a lot of input parameters, that is, the trapezoidal channel, the B is 5, side slope is 1.5, and the bed slope is 0.035, and Manning's coefficient has also been given, and the important thing is the calculation of normal depth, which is what has been asked of us. So, we say let y_0 be the normal depth, which is what we have to find. This is what we have to find.

So, first, finding the area A is equal to is that $(B + 2y_0) \times (B + my_0) \times y_0$ *or* $(5 + 1.5y_0) \times y_0$ weighted parameter P is equal to $B + 2\sqrt{3.25}\, y_0$. This is or $5 + 3.606y_0$.

So, hydraulic R is equal to A/P, and that is equal to $(5 + 1.5\, y_0)\, y_0 \,/(5 + 3.606y_0)$, ok. The section factor we will calculate, $AR^{2/3}$, is equal to $Q_n/\sqrt{S_0}$. So, we can write $(5 + 1.5\, y_0)^{5/3}\ y_0\,^{5/3}/(5 + 3.606y_0)^{2/3}$ is equal to $Q_n/\sqrt{S_0}$, $(0.00035^{0.5})$, which is equal to 16.036.

We can use the trial and error method here to obtain the value of normal depth y_0, ok, and it is found to be y_0 is equal to 1.820 meters. So, this small problem is done. And I think we'll close this session now, and we'll meet in another lecture. Thank you so much.