Free Surface Flow
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Lecture 20

Welcome, students, to this last lecture of Module 4: Uniform Flow. So, in the previous lectures, we have been studying the most efficient cross sections. We have seen rectangular channels, trapezoidal channels, and triangular channels. So now it is time to move on to one of our last, you know, a regular cross section, which is a circular channel. So, the process is going to be almost the same.

We are going to, you know, minimize the perimeter, and again, I am repeating the reason would be that for the most efficient and most economical section, the perimeter has to be minimized. Why? Because the most costly thing while making a channel is the lining, the price of the lining, which means the perimeter. So, now, the most effective circular section. So, for doing that, this is a circular section. This could be a typical case in round pipes as well, a circular pipe as well, which is not completely filled, OK? So, we consider a circular section of radius R. So, this could be a pipe, could be a pipe partially filled, having a radius R, or actually a capital R, which is this one here.

All right. Water could be filled up to this level. Okay. This is the dashed line. That means the depth of this section could be D. Okay?

And this is the entire diameter. This is equal to $2R$. So, it is important to understand that in a circular section, the shape of the flow cross-section, you know, the cross-section, varies with the depth of the flow. For example, if our depth is somewhere here, right? Then our angles will vary, and the cross-section will again vary.

It will mostly depend on this angle, which is 2θ. So, due to the divergence and convergence, the depth required to achieve maximum discharge, we need an efficient channel, meaning maximum discharge and maximum velocity will be different. So, it is quite different from the other cross-sections.

So now we will start with what is the condition of maximum discharge first, okay, maximum discharge. So, the first thing is to find out the area of the flow. So, the area of the flow will be, you know, this particular part, right? And this will be, so if you want to calculate this particular area, then what we need to do is, first, we need to find out this particular area and subtract this triangular area, right.

So, based on your geometry, you know what this area will be, right? What will that area be? A_1 will be $2\theta/2\pi \times \pi R^2$, right? And what will this area be?

It is like a triangle, right? So, A_2 will be the triangle: half of the base into height, $2R\ sin\ \theta$ into $R\ cos\ \theta$. And therefore, the area of the flow A is going to be A_1 minus A_2. So, the area of the flow here is A is equal to $2\theta/2\pi \times \pi R^2 - 0.5 \times 2R\ sin\theta \times 2cos\theta$.

So, simplifying this, π π gets cancelled, 2 2 gets cancelled. So, we are left with $\theta\ R^2 - R^2\ sin\ \theta$, and the reason is $2\ sin\theta\ cos\theta$ can be written as $sin\ 2\theta$. Therefore, A will be R^2, if taken comma, it will be $\theta - 0.5\ sin\ 2\theta \times 2\ sin\theta\ cos\theta$ is equal to sin 2θ.

Now, what will be the weighted parameter? The weighted parameter will be only this much. Okay. What is it? It is very simple.

It is $2\theta \times R$. Now, if we apply Manning's equation, Manning's equations, this is the generalized formula, right? The generalized formula, the general formula for Manning's equation. So, what are we going to do? We are going to substitute P, A in Manning's equation, and R is nothing but A/P, the wetted area divided by the wetted perimeter.

So, Q is equal to $1/n$ remains the same, A is the same, R is written as $A/P^{2/3}$ and$\sqrt{S}$. And now, this will become $1/n$, A will be $A^1 \times A^{2/3}$, A becomes $A^{5/3}$, P is $P^{2/3}$, and it is still remaining. So, basically Q is $1/n \times A^{5/3} \times P^{2/3} \times \sqrt{S}$. Now, for the maximum discharge condition, see this particular equation: Q will be the radius is fixed. So, mostly it will be in terms of θ, right?

So, what are we going to do? We are going to, if we assume m and S to be constant, okay? S is the slope, okay, and yeah, assuming okay, $dQ/d\theta$ is equal to 0, and q is a function of both area and parameter, or you see, everything is constant leaving A and P. A and P depend on θ, so instead of writing $dQ/d\theta$ full equation, we can write $d/d\theta\ ((A^{5/3})/P^{2/3})$ is equal to 0. Because you see, $1/n$ is constant, S is constant.

So, differentiating Q or differentiating $((A^{5/3})/P^{2/3})$ is as good as differentiating Q directly, okay. So, this equation $dQ/d\theta$ will be $d/d\theta$ of $((A^{5/3})/P^{2/3})$, or we can write $d/d\theta$ as $(A^5)/(P^2)^{1/3}$ is equal to 0, right. Now, we actually differentiate this one. Now, so it is chain differentiation.

So, this will become 5 $A^4 \, P - 2 \, dA/d\theta$, all right, -2 So, first, we differentiate *A*, and then we differentiate *P*. So, not here. The first time, we differentiate *A*; the second time, we differentiate *P*. So, what it comes down to is 5 $A^4 \, P - 2 \, dA/d\theta, P$ is untouched, $dA/d\theta \; - \; 2$. So, A ^{5}is not touched, P^{-2} becomes $-2P^{-3} \, dP/d\theta$, or we can take away A^4. A^4 gets canceled from both sides and remains $5P \, dA/d\theta$, and so not just *A* to the power 4, we also cancel P^{-3} from both sides.

So, $5PdA/d\theta \; - \; 2A \, dP/d\theta$ is equal to 0 or $dA/d\theta$. You see, now we are trying to find out $dA/d\theta$, all right? You see what *A* is in terms of *θ*. So, $dA/d\theta$ should be $2R\theta$, sorry, $R^2 \; - \; (R^2)/2$, $sin \, 2\theta$ will be $cos \, 2\theta$ into 2. So, it becomes R^2into 2.

$1 \; - \; cos \, 2\theta$. We have the equation of *P* as well. We have the equation of *P* also in terms of *θ*. Then it becomes $dP/d\theta$ will become $2 \; \times \; R$. So $dA/d\theta$ from the equation of *A* becomes in terms of *θ* from the equation of *P* in terms of *θ* here, right?

Now, what are we going to do? We are going to substitute the value of $dA/d\theta$ and $dP/d\theta$, where? In this particular equation. So, 5, so you see it is $5P \, dA/d\theta$, $5 \; \times \; 2R\theta \, dA/d\theta \; - \; 2A$. *A* is $\theta \, R^2 \; - \; (R^2) \, sin \, 2\theta \, dP/d\theta$.

So, we put it like this. Now, 5 into 2, 10, *R* and *R* becomes *R* and R^2 becomes R^3. *θ* remains *θ*, and then $1 \; - \; cos \, 2\theta$ is equal to $2R^3$. So, this 2 and this 2 get cancelled, but this 2 comes here: $2(R^3) \, 2\theta \; - \; sin \, 2\theta$. Then we cancel R^3 as well from here, and we get $10\theta \; - \; 10\theta \; + \; cos \, 2\theta$ is equal to $4\theta \; - \; 2 \, sin \, \theta$. This becomes this side, and this comes to this side, right?

We don't need to bring, yeah, we need to bring everything to the same side. This becomes this side, and this comes to this side as well. So, $10\theta \; - \; 4\theta$ is equal to $6\theta \; - \; 10\theta \, cos \, 2\theta \; + \; 2 \, sin \, 2\theta$ is equal to 0, right? This particular equation is there. Now, if we solve by the hit-and-trial method, you see this is *θ* in sine and cosine terms as well, and *θ*. If we solve by hit-and-trial, we get *θ* is equal to 2.62636 radians or 2*θ* is equal to 302 radians.

0.22 degrees, we can write *d* as $R \; - \; R \, cos \, \theta$, or d/R is equal to $1 \; - \; cos \, \theta$, or with here taking *R* outside the bracket, all right, then d/R is equal to $1 \; - \; cos \, \theta$. And then, since we have 2*θ*, we know, we know, see, we know *θ* right here. And then, substituting the value of *θ* is equal to 636 radians, we get d/R is equal to 1.876, or instead, in terms of diameter, d/D is 0.938.

The important thing to note is that this particular equation is solved using the hit-and-trial method. Now, this is the condition for maximum discharge. Now, the condition for maximum velocity. We have seen that the area of the flow. We have come out to find out it was $1/2\, R^2\ (2\theta - sin\, 2\theta)$.

And the wetted parameter was $R \times 2\theta$. And in terms of velocity, the Manning's can be written as this particular equation, right? Then V can again be, so what R as written by A/P and after putting it comes out to this V is equal to $1/n \times A^{2/3} \times P^{2/3} \times \sqrt{S}$. So, now, the first case was we found out the condition for maximum discharge.

Now, for maximum velocity, we are going to repeat the same procedure. $dV/d\theta$ is equal to 0. Condition for maximum velocity. Here we are assuming that Manning's number n and slope are constant. Slope is constant.

So, here also V is a function of both A and P, right? So, we write V as Yes. So, V was $1/n \times A^{2/3} \times P^{2/3} \times \sqrt{S}$, right? So, instead of writing $dV/d\theta$, we can also write $d/d\theta$ of $A^{2/3}/P^{2/3}$ is 0, right?

So, So, this equation $d/d\theta$ of $A^{2/3}/P^{2/3}$. Now, it is simply $P\, dA/d\theta - A\, dP/d\theta$, right? If you differentiate it, right? In terms, so first differentiate A leaving P and then differentiate P leaving A, right?

So, $P\, dA/d\theta - A\, dP/d\theta$ is equal to 0. Now, we already know $dA/d\theta$ is $R^2(1 - cos\, 2\theta)$ from before, similar to last time, because A is $\theta R^2 - R^2\, sin\, 2\theta$. So, this $dA/d\theta$ will be $R^2 - R^2\, cos\, 2\theta$; $sin\, \theta$ becomes $cos\, 2\theta$ on differentiation, and $dP/d\theta$ becomes 2R. Now, putting these two in this one, we are going to get $2R\theta - R^2(1 - cos\, 2\theta) - R^2(2\theta - sin\, 2\theta) \times 2R$ is equal to 0. Now, this R^2 and R becomes R^3; θ remains θ, 2 is here, and this is $(1 - cos\, 2\theta)$ this 2 2 gets cancelled; R^2 R is $R^3\ (2\theta - sin\, 2\theta)$, and then this also gets cancelled.

And $2\theta - 2\theta\, cos\, \theta$ is equal to $2\theta - sin\, 2\theta$. Minus $2\theta\, cos\, 2\theta + sin\, 2\theta$ is equal to 0. And if we solve this by hit and trial, It is given here we get a value of 2θ equal to 257 degrees 22 minutes 56 seconds or in terms of radians also you can write. Now, this is another condition for maximum velocity.

Now we have to see the depth of water for maximum velocity. What is the depth? d is $R + R\, cos\ (180 - \theta)$, right? See, depth is $R + R\, cos\ (180 - \theta)$, okay? So, d is equal to $R + R\, cos\ (180 - \theta)$ degrees we had, it is in degrees, okay?

So, cos (180 - this much) it will be coming to $0.626R$ or d by is equal to $1.626R$ and d/R comes to be 1.626, or since D is equal to $2R$, d/D is equal to 0.81, right? This is also a second condition. The first condition was d/D was some other value. So, these for maximum discharge, there is one condition.

For maximum discharge, the condition was small d/D was 0.938, and for maximum velocity, the condition was d/D is 0.81, two different conditions. So in this particular section, for efficient and maximum discharge and maximum velocity in a circular section, which could be the case in pipe flow that is partially filled, we have come up with two equations: one for maximum discharge and one for maximum velocity. And for maximum discharge and maximum velocity, the conditions are not the same. Can you guess what could be the reason for it? The reason for it is that the cross-section is changing depending upon the depth, and therefore, the discharge will also change.

All right. So, now continuing with even more, you know, a little bit more complex things, there are standard lined canal sections. Okay. Now, what happens is canals are often lined, and why are they lined? To reduce seepage losses and related problems. Okay, because from canals, there could be water leakage, and to stop that, there could be other reasons as well. So now, exposed hard surface linings using materials such as cement concrete, brick tiles, asphalt concrete, and stone masonry are used.

These four are the most important categories of canal lining, especially for canals with large discharges. So, cement concrete, brick tiles, asphaltic concrete, and stone masonry. These are hard surface-lined canals. The cross-section recommended by the Indian Standard Code consists of a trapezoidal cross-section with corners rounded off with a radius equal to the full supply depth.

So, this is a standard line trapezoidal section, and how do we know about it? We know it by the Indian standards, Indian Standards Code, IS codes. You see, this is curved, right, rounded off with a radius equal to what is the radius? The radius is the full depth here.

Now, for discharges which are less than 53 meters cube per second, a triangular line section with a bottom portion. So, this is for large discharge; you understand this, okay? Secondly, for discharges which are less than 55 meters cube per second, a triangular line section. with a bottom portion rounded off with a radius equal to the full supply depth is recommended, okay? Recommended by CWC. So, this is the second type of standard line triangular channel for discharge less than 55 meters cube per second.

This is for large discharge, which is greater than 55 meters cube per second. For convenience and ease of identification, the above two channel sections are termed standard line canal sections and, in particular, as standard line trapezoidal section and standard line triangular section, respectively. Note that the standard line triangular section is the limiting case of the standard line trapezoidal section with *B* equal to 0. So, what it says is that if you put *B* equal to 0 in this, it will turn into the standard line triangular channel, and these standard line sections have interesting geometrical properties which are beneficial in the solution of uniform flow problems.

So if you are asked to design something, I think in canals, this is the standard. If you want to design something in India, these are the standard sections that you are going to use. Now, talking a little bit about the standard line trapezoidal section. So it says the full supply depth is equal to the normal depth at design discharge. Why not? And at normal depth area, you see trapezoidal, right? So, if you see this area, it will be composed of this, right, complete.

And the area will be By_0, that is the rectangular part $my_0{}^2 + y^2\,\theta$, right. Or *B* is equal to, I mean, we write in terms of this ε where ε is $m + \theta$ or $m + tan^{-1}\, 1/m$. Similarly, we can calculate the weighted parameter easily, which will be *B*. The weighted parameter will be this entire length. It will be composed of one horizontal section and these curved sections as well. So, this curve will be $P + 2my_0 + 2y\,\theta \; or \; 2y\varepsilon$.

ε is $m + \theta$. So, now the hydraulic radius is R is equal to A/P is equal to $(B + y_0\varepsilon) \times y_0/(B + 2y_0\varepsilon)$. And we apply Manning's formula similar to what we have done in other cases: *1/n*, *P/A*, *A/P*, $(A^{5/3})/(P^{2/3})$, similar to earlier cases. Same formula.

Now, if you non-dimensionalize this, it will come in terms of this particular quantity η_0, and here $\eta_0 = \varepsilon/B$. From this equation, phi 1 can be easily evaluated for various values of η_0, which is correct. Now, for the solution of this, we need a table. Or a curve of phi 1 versus η_o, okay? For these types of tables, they already exist, so you will be able to solve that. However, this will be more complex for this course to provide those tables now. The way we have done for these channels, we can also do for rectangular line channels and standard. Lined triangular channels as well, triangular sections as well. So, the area will be $2my_0{}^2$; just put in the previous one, and you will easily obtain *A* as this and *P* as this, right? Now, the hydraulic radius will come out to be *y/2*. Manning's formula will give us this. And it will again come out to be $\eta_0/\varepsilon = 0.63$.

By using this equation and tables, we can determine the uniform depth as well, okay? So, these are some of the references that you can use to study a little bit more in detail about this uniform flow. With this, we finish this particular module where we discussed the theoretical concepts of uniform flow. From the next class, we will have a problem-solving session on uniform flow. Thank you so much. See you in the next class.