Free Surface Flow
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Lecture 19

Welcome back, students, to yet another lecture on uniform flow, which is the fourth module in our free surface flows course. And in this class, we are going to start with the most efficient cross-section. We have already started with the definition of uniform flow, seen Manning's equation, Chezy's equation, and the computation of normal depth. We also saw the design charts. In the previous lecture, when we ended it, we were talking about the most efficient channel cross-sections and actually covered the first part of it, which was the rectangular section.

So, beginning this lecture, we will see what the most efficient trapezoidal section is. So, if you recall, we already derived that for the most economical and efficient channel, we must minimize the perimeter, and the reason is that the perimeter is the lining, and the most expensive part in the construction of a canal is the cost of the lining. So, in the first case, when the side slope is fixed. So, this m is fixed, which is the first case. So, if we consider a channel section of bottom width *B*, depth of flow *y*, and side slope 1:*m*, there are three variables: *y*, *B*, and *m*. But, as we have said, in our first case, *m* is fixed, so the number of variables left are two, which are *y* and *B*. So, we can vary *B* and we can vary *y*.

So, area, you see, the area is when we try to calculate the area. So, let me do it here. So, this is a rectangle. So, it will be $B \times y$. This is a triangle.

So, $0.5my \times y$, but the same triangle is here as well into 2. So, the area will become $By + my^2$. So, basically this is one way of calculating. Alternatively, you can consider $By + my + my + B/0.5 \times y$, which is also one way, but this is the area. So, basically the area is $By + my^2$. A is equal to $y + B + my$, exactly the same as $(B + my) \times y$. The top flow width will be *B*. This is $B + my + my$, which is $2my$, so $B + 2my$. The parameter is *B* plus this is the perimeter.

So, the perimeter is this B + So, this is y and this is my. So, this will be $\sqrt{y^2 + m^2y^2}$or $y\sqrt{m^2 + 1}$. And so, this multiplied by 2 plus *B* will be the parameter, you see. Now, if you substitute *B* is equal to, so we use this one. This one here.

So, instead of B, you can write, so if you use this equation, we can substitute B is equal to $A/y - my$. You see, A is equal to $(B + my) \times y$. In the first step, you bring it down here. So, it becomes A/y is equal to $B + my$. Then bring my to this side. So, it becomes $A/y - my$ is equal to B, and this is exactly what is written here. Then if we substitute B is equal to $A/y - my$ in this particular equation, you get P is equal to, instead of B, $A/y - my + 2y\sqrt{m^2 + 1}$.

So, we have A and m both are both are constant. We know that for an efficient channel, given the area, the perimeter should be minimum. So, that is our first step: dP/dy is equal to 0.

And you see, we have a complicated equation, dP. So, dP/dy becomes $-A/y^2$, the first term, my. So, it will become $-m$, that is there, and here also, y gets out. So, it becomes + $2m^2 + 1$or -. So, this is equal to 0 for P to be.

minimum dP/dy should be equal to 0. So, $-A/y^2 - m$, or you take these two towards this side, it becomes $A/y^2 + m$ is equal to $2\sqrt{m^2 + 1}$. And if you multiply it by y on both sides, it will become $A/y - my$ is equal to $2y\sqrt{m^2 + 1}$. You do multiply by y here. Now, this is the equation $A/y - my$. Now, if you substitute A/y is equal to $B + my$, in this, then you become B + you see this one.

So, instead of A/y here, you substitute $B + my$. So, it becomes $B + my + my$, which is $B + 2my$, equal to $2y\sqrt{m^2 + 1}$. Or, $B + 2my$ is equal to the top width, or the top width is equal to 2, and we also know that the top width is also equal to this. So, 1, right. So, $T/2$will become $y\sqrt{m^2 + 1}$, and this is the length of the side slope. So, $T/2$ is equal to the length of the side slope; this is the first condition for

The first condition for the most efficient rapidous section with a fixed slope is that the top width should be equal to twice the length of the side slope. This is very important: that is the first condition, or for an efficient trapezoidal channel section with a fixed side slope, half of the top width should be equal to the length of the side slope. So, for an efficient trapezoidal channel section with a fixed side slope, half of the top width should be equal to the length of the side slope, that is, the perimeter of the side slope. For the second condition, we know that the hydraulic radius R is equal to A/P. So, the area we know is $(B + my) \times y$, and $B + 2y\sqrt{m^2 + 1}$ or $(B + my) \times y$, and this is nothing but $B + T$, or $(B + my) \times y$, and what is T? It is nothing but $B + B + 2my$.

So, $(B + my) \times y$ divided by $2(B + my)$; this cancels with this. So, another condition is R is equal to $y/2$. It is very simple. Most of the cases, you will approach like this. First, minimizing the perimeter, dP/dy is equal to 0, and secondly, you approach through the hydraulic radius.

This is exactly what we did in rectangular sections, and this is what we have done in the trapezoidal section as well. So, the second condition is: For an efficient channel, the hydraulic radius R should be equal to half of the depth of flow. So, this type of understanding is very important because, for let us say, for this particular You will get weekly exercises, which will be objective in nature or fill in the blanks in exams as well.

So, these types of things, I mean, these are the types of questions that you can expect in your exams as well. What is the most efficient channel when the slopes are fixed for a trapezoidal section, right? We have given the depth of the flow as, let us say, 2 meters; what should be the hydraulic radius? So, you can use this equation, for example, and try to find out. So, if you try to look down, you see this is the trapezoid.

So, if you try to look at this area *AOP*, let us say this one. This one. Just having this θ, *sin* θ will be equal to OP/AO, that is *X* divided by the top width by 2. In *ACR*, that is *ACR*, Yeah, here, *sin* θ will be equal to, you see this one, *ACR sin* θ will be AR/AC, or *AR* is nothing but *AR* is nothing but *y* and *AC* is.

$y\sqrt{m^2 + 1}$. So, if you compare both of the above equations, you see this one *x*. If you equate *sine* θ from these two equations, you get $x/(T/2)$ is equal to $y/y\sqrt{m^2 + 1}$. *y* will get cancelled out, and you get So, you see $T/2$, the top width divided by 2, will be or if you substitute, you will get *x* is equal to *y*, and this is the third condition. So, for an efficient trapezoidal channel section, a circle of radius *y* shall be inscribed in the trapezoidal section. So, for an economical trapezoidal section with a depth of flow y and a fixed side slope, all the above three conditions should be satisfied.

This is quite an important case. The first condition is that, if you look at the first condition, $T/2$ is equal to the length of the side slope. The second condition is *R* is equal to $y/2$, and the third is that a circle of radius *y* shall be inscribed in the trapezoidal section. These are the three conditions which should be met for the most economical trapezoidal section. Now, looking at the second case when the side slope is varied.

So, we say m is no longer constant. So, if you consider a channel of width *B*, this is width *B*, flow depth *y*, and varying side slope $1{:}m$. The area will be $(B + my) \times y$, which we have seen before as well. The perimeter will be $B + 2y\sqrt{m^2 + 1}$, or similar to the previous case, the perimeter will be $A/y \; - my + 2y\sqrt{m^2 + 1}$. How?

Using this case. Put it here simply, we know that the first condition for an efficient channel is always that the first condition will be dP/dy is equal to 0. Secondly, we try to calculate the hydraulic radius, but first, dP/dy is equal to 0. So, for the most economical and cost-effective channel, we have seen that the cost of the lining should be minimum, which means the length of the perimeter should be minimum, meaning dP/dy is equal to 0. The equations are the same, but right now, when we try to do dP/dy.

This particular equation, because it differentiates with respect to A/y, becomes $-A/y2$ for the first term here, the second term becomes m, and this term, when differentiated, becomes $2\sqrt{m^2 + 1}$. Or $A/y^2 + m$ is equal to $2\sqrt{m^2 + 1}$ is equal to 0, or $A/y^2 + m$ is equal to $2\sqrt{m^2 + 1}$, or A is equal to $y^2[2\sqrt{m^2 + 1} - m]$. So, now this equation is in terms of y and m. Now, if we put this in the above equation of the perimeter, what is the perimeter equation? This one, we get P is equal to $2y\sqrt{m^2 + 1} - \; my \; - \; my + 2y\sqrt{m^2 + 1}$, or the perimeter becomes $4y - 4my$. So, this is the value of the parameter in terms of m. Now, we have already minimized the parameter with regards to minimizing the parameter with respect to the depth.

Now, for the economical channel section, we have varying side slopes. So, dP/dm should also be equal to 0. And if we use this equation, dP/dm will become $4y \times 2m/2\sqrt{m^2 \; + 1}$. So, the differential of this and this. So, this differential will be this, and $4my$ will be.

So, we are also canceling out 2 here. So, it becomes *2y* or $8my/2\sqrt{m^2 \; + 1} \; - \; 2y$ is equal to 0. Or $2my$ is equal to $y\sqrt{m^2 \; + 1}$, then you cancel out y from here, y from here, and *2m* is equal to $\sqrt{m^2 \; + 1}$ or 4. So, now squaring the whole, we get $4m^2$ is equal to $m^2 \; + 1$ or $3m^2$ is equal to 1 or m^2 is equal to 1/3 or m is equal to $1/\sqrt{3}$, indirectly meaning. tan θ is equal to $\sqrt{3}$.

So, the most effective and efficient channel is for an angle θ where tan θ is equal to $\sqrt{3}$, which means θ is equal to 60 degrees, and this is the fourth condition. So, what would be the most effective channel? The first three conditions plus the fourth condition that this θ is equal to 60 degrees. So, for an economical trapezoidal section with varying side slope, the value of the side slope should be 1 is to $\sqrt{3}$, or the angle should be 60 degrees. So, to sum up, the most economical trapezoidal section should satisfy the following conditions the most.

So, for example, we had only one condition for rectangular; we have here four conditions. The first condition is that the length of the side slope should be equal to the top width divided by 2, or the length of the side slope should be half of the top width. The second hydraulic radius should be depth by 2. Third, a circle of radius y should be inscribed in the trapezoidal section, and the last condition is β is equal to 60 degrees.

All these four conditions are very, very important, and it becomes One of the easy ways to ask you questions also. So, in your exercises, when you do expect many questions based on these. So, just a little bit of recap. We, among the most economical sections, started with the rectangular section and trapezoidal section. Now, we are going to see another one, the most effective, most efficient triangular section, efficient or economical or effective, whatever you want to say. So, the important thing to note is that the cross-section here is a triangle. So, here the depth of flow is y, this is the top width, this is the slope, and this is the angle θ and this is 1 vertical and m horizontal.

So, the top width is very simple. this is y, right. So, the top width will this will be my, and this also will be my. So, the top width will be $2my$ area is nothing but twice, you know, the I mean it means you can consider 2 triangles or 1 triangle. So, $2my$ base height half. So, the area is my^2

Perimeter again. So, this plus this. So, parameter this one will be y, my this particular. So, it will be $y\sqrt{m^2 + 1}$. And since there are.

For the most efficient economical section, the first thing to do is minimize the, you see it is depending on m and also y. So, we do dP/dy is equal to 0. So, dP/dm is equal to 0. So, P is equal to $2y\sqrt{m^2 + 1}$. Here when we do that, so it will become A is equal to my^2, sorry. So, we will keep this, and you see this area, area was my^2.

So, we are going to substitute y in terms of area is constant A/m, and this we substitute it here, then our perimeter $2\sqrt{A/m}$ and $\sqrt{m^2 + 1}$ or P is equal to this. We further simplify $2\sqrt{A} \times \sqrt{(1 + m^2)/m}$. So, the idea is to have, so what are we trying to do? Here we see we have two parameters, two variables, one is y and one is m. Both can vary.

So, we want to make it in only one area where 'see' is constant. So, we are trying to transform and put y in terms of area so that when we differentiate, we do not have to worry about area. So, this is the equation that we get. So, then we do dP/dm is equal to 0, $2\sqrt{A}$, you see, it will remain the same. Now, the difficult part is this one.

So, the difficult part is to differentiate this one. So, it becomes $1/(2\sqrt{m + 1/m}) \times (1 - 1/m^2)$. This is in chain differentiation. Now, we substitute this, now this is equal to 0 or this is equal to 0. The solution of this will be $m + 1/m$ is equal to 0, which is not possible.

This is not possible. Second is $m - 1/m$ is equal to 0 or m is equal to plus or minus 1 or tan θ is equal to 1. That means θ is equal to So, let me just go back again here, for this to be 0, hold on, for this part to be 0, a is non-zero, correct? This actually is anyways in the denominator, so cannot be 0.

The only part is this to be 0, $m + 1/m^2$ square, which gives *m* equal to plus or minus 1. This is what is given here, and then it gives $tan\theta$ equal to 1. So, theta is tan inverse 1, which is θ equal to 45 degrees. This is one condition. The hydraulic radius thing is *R* equal to A/P, you know, hydraulic radius $(my^2)/y\sqrt{1 + 1/m^2})$.

So, instead of *m*, you write take *m* equal to plus or minus 1. So, it becomes *R* equal to $y/2\sqrt{2}$, that is the second condition. So, for the most efficient triangular section of depth of flow *y* and varying side slope $1\!:m$, side slope and height. So, *m* should be equal to 1, and the hydraulic radius should be $y/2\sqrt{2}$.

This is for the triangular section. So, I think I will end my lecture here, and in the next class, we will start with the most effective circular section and hope to try to complete the theoretical aspect of our uniform flow in the next lecture itself. So, thank you so much, and see you in the next class.