Free Surface Flow
Dr. Mohammad Saud Afzal
Department of Civil Engineering
Indian Institute of Technology Kharagpur

Lecture 18

Welcome back, students, to yet another lecture on uniform flow, the theoretical part where we are covering the core concepts and trying to see some solved examples as well in this slide. In the last class, we were covering the design curves to compute our normal depth. Similar to what we did for critical flow, a similar design curve is available as well. So, that being said, we will now start looking at one of the solved examples. So, the problem is to compute the normal depth in a trapezoidal channel having a bottom width of 10 meters.

Side slope of 2:1. So, the side slope is given. First of all, the important thing to note is we have to see what information is given in the question. It is given as a trapezoidal channel, and the bottom width B is given. So, I will just write it down.

Trapezoidal channel, first thing, B is given as 10 meters, m or s, whatever you want to call it, is given as 2:1, and the discharge is given as 30 m^3/s, and the slope is also given, S_b is given as 0.001. And Manning's n is also given as 0.013. This is the information that is already given to us. Now, let us see. So, if we are given Q as 30 meters cubed per second, B_o as 10, S_b as 0.01, n as 0.3.

So, first we are going to calculate the section factor $AR^{(2/3)}$, which can be found using $nQ/S_b^{0.5}$. n is known, Q is known, and S_b is also known. These three things we know; using that, we are going to calculate the value of $A\ R^{2/3}$. And if we do that, n = 0.013, here Q = 30, and $0.001^{0.5}$, we get a value of 12.33. The second step is to divide this value. This is a trapezoidal channel, so we will divide by B_o, which is this B or B_o, whatever you want to call it, and then we will get the value of 0.0266. Now, with this particular value, you are going to look it up in the chart and try to find out for a 2:1 slope what is the corresponding y_n. y_n/B_o Let us go to that chart. It is easier.

So, let me just again see what the value was. So, our value was 0.0266, 0.010203 somewhere, I think. 0.5. So, somewhere here, is it right? So, it is almost in the range of 0.1. I think y_n/B_o, if you look and try to draw at the curve, is 0.09.

Anyways, we need to have a better look. So, yes, so it was 0.11. So, from the design curve for the trapezoidal channel 2:1, the value of $\frac{y_n}{B_o}$ is 0.11, clearly observed from a better design curve. And therefore, $\frac{y_n}{B_o}$ is equal to 0.11, where B_o is 10, so the normal depth comes to 1.1 meters. Now, the concept of equivalent Manning's roughness coefficient: natural channels or rivers usually have sand beds in the main deeper portion of the channel, and floodplains on both sides may have vegetation.

As such, *n* values in different portions along the parameters may vary because it is highly unlikely to have one common roughness or surface in the entire canal lining, river lining, or whatever it may be; it will change. Even, let us say, if there is growth of vegetation or any plant, the tree surface will change, right? So, for such types of channels, an equivalent roughness parameter is obtained, which is valid for the entire perimeter of the channel section. This is the concept behind the equivalent Manning's roughness coefficient. So, there are several ways.

One of the ways is Horton's method of equivalent roughness estimation. So, this is, let us say, this is free surface. This is the cross section. You see different lengths at different parameters.

Different roughness. So, Horton devised a method using which an equivalent Manning's roughness could be found. So, here consider a channel having its parameter composed of *n* types of roughness, the unlimited *n* number of sections are there. P_1, P_2, P_3 until P_n are lengths of these *n* parts and n_1, n_2 until n_n are the respective roughness coefficients. This is what I explained just now above.

Let each part P_i be associated with partial area A_i such that the sum of all the areas equals the total area. Make sense? And the sum of all the parameters equals the total parameter. It is assumed that the mean velocity in each partial area is the mean velocity *V* for the entire flow area. This is one assumption that the mean velocity in each partial area is the mean velocity *V* for the entire flow area; the velocity is the same.

So, V_1 is equal to V_2 is equal to V_n; all the velocities in all the areas are the same. So, if we apply Manning's formula, $S_b^{0.5}$ is $\frac{n_1 V_1}{R_1^{2/3}}$; all of these will remain constant for all, right? The bed slope, because that is what we have considered common: $\frac{n_1 V_1}{R_1^{2/3}}$ equal to $\frac{n_2 V_2}{R_2^{2/3}}$ is equal to all equals equal to $\frac{nV}{R^{2/3}}$, and this is what we are after. This is equivalent Manning's roughness, and we have also said V_1, V_2, V_n will be common. So, where n is the equivalent roughness coefficient.

Following equation 2 for the i^{th} part, we can write $\frac{n_i V_i}{R_i^{2/3}}$ is equal to $\frac{nV}{R^{2/3}}$. But V_i is equal to V; hence, the above equation becomes $\frac{n_i}{R_i^{2/3}}$ is equal to $\frac{n}{R^{2/3}}$ or $\frac{A_i}{A}$. is equal to $\frac{n_i}{n}\frac{P_i}{P}$. So, we write the hydraulic radius R_i as $\frac{A_i}{P_i}$ and substitute it here. And therefore, we get A_i by $A^{(2/3)}$ is equal to $n_i P_i^{(2/3)}$ divided by $nP^{(2/3)}$ or $\frac{A_i}{A}$. So, we do this 3/2 and also do this 3/2. Okay?

And we get $\frac{A_i}{A}$ is equal to $\frac{P_i n_i^{3/2}}{Pn^{3/2}}$ or A_i is equal to $\frac{A \times P_i n_i^{3/2}}{Pn^{3/2}}$. I hope until this point is very clear. So, simply we equated the Manning's equation, I mean the part of the Manning's equation where S_b raised to the power half was constant. Until now. But we also know that sigma A_i is equal to A; sigma of A_i is equal to A. So, we can do sigma; hence, A is equal to, so sigma A_i, A is equal to, so A_i $P_n^{(3/2)}$ becomes, it is very simple, you see.

So, we put sigma here, and we put sigma here. So, this sigma A_i will become A; A is equal to $\frac{A}{P_n^{3/2}}$ will come out $P_i n_i^{3/2}$. This taking away right now because So, this is what we get: A is equal to $AP_n^{(3/2)}$ or simply n; this is what we find out, right. So, ((sigma $\frac{\left(P_i n_i^{3/2}\right)^{2/3}}{P^{2/3}}$, and this is important.

Important expression of equivalent roughness, all right. Now, a question: an earthen trapezoidal channel with n equal to 0.025 has a bottom width of 5 meters, a side slope of 1.5:1, and a uniform flow depth of 1.1 meters. In an economic study to remedy excessive seepage from the canal, two proposals are considered: one is to line the sides only, and the second is to line the bed only. If the lining is of smooth concrete, determine the equivalent roughness in the above two cases. Very simple; they have already given the existing values of Manning's roughness for the earthen trapezoidal channel.

Bottom width they have given; it's trapezoidal, right? So, it will be something like this. This they have given as *B*, right? Side slope also they have given here, and actually, this depth is also given as normal depth, 1.1 meters. So, this they have already given. This they have given: 5 meters, 1.1 meters, and dip they 1.5 horizontal to 1 vertical, and yeah, existing n is also given as 0.025, right. So, in the first proposal, to line the sides only.

So, in the first proposal, they want to line it with the sides only with smooth concrete having a definite roughness. In the second proposal, to line the bed only. In that quest, we have to calculate the equivalent roughness. So, we have just seen the formula. So, let *e* denote the earthen material, and *c* represents concrete. Make sense.

ne is given as earth 0.025 and concrete is 0.012. *B* is 5 written before *y*, normal depth 1.1, and *Z* is equal to that, which is the slope 1.5. So, what is the wetted parameter $B+2y\sqrt{1+z^2}$? That right now comes to be the wetted parameter 8.96 meters. And in the first case, when we have to line only the sides. Line the sides only.

So, the earthen parameter earthen. So, if they line these sides, the parameter of the earthen will be only *B*, equal to 5 meters. Therefore, what will be the parameter of concrete, this plus this. So, $2\times y$, that is the depth, into $\sqrt{1+z^2}$. So, the perimeter of the earthen is 5 meters, and the parameter of concrete is 3.966 meters.

Now, it is a very simple formula: $n=\frac{\left(P_e n_e^{3/2}+\ P_c n_c^{3/2}\right)^{2/3}}{P^{2/3}}$. This is the formula, but let me go and just show you for a minute this particular formula: $\frac{\left(\text{sigma}\ P_i n_i^{3/2}\right)^{2/3}}{P^{2/3}}$. So, it is just a matter of putting in the values because we know P_e, we know ne, P_e is known, n_e is known, P_c is known, n_c is known, and the whole parameter is also known, this 8.96 meters. So, $\frac{\left(5 * 0.025^{3/2} + 3.966 * 0.012^{3/2}\right)^{2/3}}{8.96^{2/3}}$, and the value comes out to be 0.0198 meters. Now, in the second, smoothing, see the concrete lining is only on the bed.

So, the earthen material will be on the sides only. Earth and this is concrete. P_e becomes 3.966, P_c is 5. The equivalent roughness coefficient formula will remain the same. So, just substitute the values. Only P_e and P_c values have changed, and therefore, we get 0.0183. In the other case, Manning's coefficient was 0.0198, and it is 0.0183.

Very slightly, 0.019 and 0.018. So, which is smoother, in which the concrete is only on the bed? So, this will give you an idea. Calculate the equivalent Manning's roughness. Now, starting with economical sections, what is an economical channel section?

So, any section is said to be economical when its construction cost is minimum for a given discharge. So, a section is said to be most efficient if, for a given cross-sectional area, the discharge carrying capacity is maximum. So, one is an economical section, and one is the most efficient. So, as we know, the highest component of the total construction cost in a channel section is the cost of lining, and if the parameter is kept minimum. The cost of lining will be minimum; hence, it will be the most economical section.

So, first we define what an economical section is. I mean, we understand the cost should be minimum, right. And for effective channel section, there is a cross section that is used. The discharge carrying capacity is maximum. Thirdly, since in the construction the highest total construction cost is for the cost of lining, and lining means the perimeter. So, that means if we keep the perimeter to a minimum, the cost of lining will also be minimum; hence, it will be the most economical section.

So, if we see Manning's equation, Q is equal to $\frac{1}{n} A R^{2/3} S^{1/2}$. So, let us put R, I mean, let us put hydraulic radius in terms of area and perimeter. So, it becomes $\frac{1}{n} A \times \left(A/P\right)^{2/3} \times S^{1/2}$.

Or $\frac{1}{n}\frac{A^{5/2}}{P^{2/3}}S^{1/2}$, which implies that Q or discharge is inversely proportional to the wetted perimeter.

So, for maximum discharge using the equation for a given area, the perimeter should be minimum, right? For maximum discharge, it is inversely related. So, we can observe that Q is inversely proportional to $P^{2/3}$. Also, if the perimeter is minimum, the resistance offered by the perimeter will be minimum; hence, discharge will be maximum. Not only equation-wise, but if we see, if the perimeter is minimum, the resistance offered by the perimeter will also be less; hence, discharge will be more. Therefore, for the most economical and most efficient channel section, the perimeter should be minimum. Important conclusion here is if we are asked for the most economical and most efficient channel, most efficient channel, we should minimize the perimeter of the cross section. So, that being said, we will start with most efficient channels of different cross sections. So, if we consider a rectangular channel section that has a width B and flow depth as y, what is going to be the area? Area is $B \times y$ given here.

The perimeter we have seen many times it will be $y + B + y$. *So,* $y + B + y$, that is $B + 2y$ is also given here. Instead of B, we can write if we use this then B can be written as $\frac{A}{y}$. So, in this equation, if we put B as $\frac{A}{y}$, this will come to like this and so P becomes $\frac{A}{y}+2y$. Area is constant because we have to minimize P. We know that the channel will be most efficient if for a given area P is minimum, that is correct. So, what we do for minimizing $\frac{dP}{dy}$ is equal to 0.

For a rectangular channel, what was P? P was $\frac{A}{y}$ plus So, $\frac{dP}{dy}$ will be A's constant $\frac{-1}{y^2}$, right? $\frac{1}{y}$ is $\frac{-1}{y^2}+2$, and this is exactly here 2 for minima. For P to be minimum, $\frac{dP}{dy}$ must be equal to 0, if you remember from your math class. So, $\frac{-A}{y^2}+2=0$, this one.

So, it will $\frac{-A}{y^2}+2=0$ Or A is equal to, so if you take this on this side, it will be $2=\frac{A}{y^2}$ or A is equal to $2y^2$, as similarly given here. And the area is nothing but $B \times y$. The area is, this is $B \times y$. So, B_y is equal to $2y^2$, and cancel y on one side, y is equal to $B/2$. This means, for the most efficient and most economical channel, the depth should be half of the width of the channel. So, also the hydraulic radius R is equal to $\frac{A}{P}$. So, $\frac{B \times y}{B+2y}$, and since we have already found out that y is equal to $B/2$.

So, $2y$ is equal to B; the hydraulic radius is also equal to $y/2$. So, for the most efficient rectangular channel section, the depth of flow y is equal to $B/2$, and the hydraulic radius R is equal to $y/2$. So, for this, this was for the rectangular section. We will also see this type of analysis for other sections as well. So, I think, at this particular point in time, we will end our lecture here and start with the most efficient trapezoidal section in the next class.

Thank you so much.