

Free Surface Flow
Dr. Mohammad Saud Afzal
Department of Civil Engineering
Indian Institute of Technology Kharagpur

Lecture 17

Welcome students. So, until now we have covered three main topics, which were the introduction to the concepts of free surface flows. Secondly, we covered critical flows, and the third one was the problems on critical flow. The fourth one is uniform flow. So, among that, I mean within this module, we have already covered one lecture.

Now, we are heading to another round where we are going to see mostly the friction factor. Today, we are going to start with a solved example because it is much easier, you know, in this slide itself to see the concept and how it is being implemented. So, the rectangular channel, which is 2 meters wide, carries water at a depth of 0.5 meters. The bed is laid at a slope of 0.0004. If Manning's n is given, find the bed shear stress, shear velocity, and C , and f .

So, basically, what I can say here is. This is a direct implementation of the concept covered in the previous lecture. So, let us begin. So, the hydraulic radius will be $\frac{A}{P}$, or area is $B \times y$, and perimeter is, it is a rectangular channel, right? This is B , this is y , this is y . So, the area will be $B \times y$, and the perimeter will be $B + 2y$, and B here is given as 2 meters, and y is 0.5.

So, $\frac{2 \times 0.5}{2 + 2 \times 0.5}$ that gives us $1/3$ meter. So, bed shear stress τ_o is τ_o is $\gamma R S_b$, γ is $1000R$. So, γ is 1009 point ρg . And R is $1/3$, which we calculated here, and S_b is 0.0004, which is also given here. It comes out to be 1.308 N/m^2 .

Frictional velocity, shear velocity that is u^* is $\sqrt{\frac{\tau_o}{\rho}}$ is 1.308, which we got from here, put here. And divided by ρ is 1000. So, we get 0.0361 meters per second. So, C , if you

remember from this particular formula, this formula C was related as $\frac{1}{n} R^{1/6}$ is equal to

$\sqrt{\frac{8g}{f}}$. C is $\frac{R^{1/6}}{n}$ because Manning's n is given.

So, we have found out R is $\frac{(1/3)^{1/6}}{n}$, n is 0.012. So, it comes out to be 69.39 $m^{(0.5)}s$. And,

C from the same equation $\sqrt{\frac{8g}{f}}$ that is f , therefore, so C is nothing but $\sqrt{\frac{8g}{f}}$. So, if you

do square on both sides, you get f is equal to $\frac{8g}{C^2}$ and the value comes out to be 0.0163.

Another direct implementation problem, a rectangular channel 6 meters wide carries a discharge of 5 meters cube per second at a slope of 0.006. So, discharge Q is given, slope is given, compute the normal depth and maximum shear on the bed taking Manning's n is equal to 0.014.

So, Manning's n is given. So, we need to calculate the normal depth. What is the solution? We will use Manning's equation. So, if we have Manning's equation, it says Q is equal to $A \times V$. A is, let us say A , and V is V by Manning's $\frac{1}{n} R^{2/3} S_b^{0.5}$.

And A here is By_n ; area is By_n . So, putting this value Q as 5, B is given as 6. So, this was 6, this was B , y_n , and so hydraulic radius is nothing but A/P , and A is $\frac{By_n}{B + 2y_n}$. This is area;

this is wetted parameter. And is (Note: check the gap) also given as 0.006 after substituting in the values. So, we see this is a non-linear equation in terms of normal depth, and the only way to solve it is.

So, the solution is done using trial and error method, and after that trial and error, we get the normal depth as 0.336. How to check if it is correct? Substitute the value here and here and see if it is satisfied. the above equation, all right.

So, the maximum another part if it says The depth we have calculated right now, calculate the maximum shear stress on the bed. So, the maximum bed shear is given by τ_o is equal to $\gamma R S_b$, correct. So, τ_o is γ is 1000×9 . So, this is ρ , this is g .

We have now determined everything. So, our hydraulic radius is $b \times B \times y_n$. This is the normal depth. This is B , this is B , and this is y_n , and this is S_b . So, τ_o comes out to be 17.78, 17.78 N/m^2 . So, now coming to the Darcy-Weisbach friction factor F again here. It is important to note that incompressible turbulent flow over plates, in pipes, and ducts has been extensively studied in the fluid mechanics discipline. From the time of Prandtl in 1875 and Von Karman, there have been various people who have enabled, you know, our considerable understanding of turbulent flow and associated practical applications, right. So, a lot of studies have been done, and especially this Darcy-Weisbach friction factor f is mainly used in pipe flow.

What is in pipe flow? There are certain terms like pipe. How is that defined? Is it a smooth surface, or is it a rough surface? So, a surface in pipe flow can be termed hydraulically smooth. Or rough, or in transition, depending on the relative thickness of the roughness magnitude to the thickness of the laminar sublayer. On what does it depend? It depends on the relative thickness of the roughness magnitude to the thickness of the laminar sublayer.

So, the classifications for these are given as follows. So, this is just for your information. So, $\epsilon_s \nu^*$, this is viscosity, kinematic viscosity, kinematic eddy viscosity. So, if this particular quantity, right? Is less than 4, it is called hydraulically smooth. And if this quantity is between 4 and 60, it is in transition. If it is greater than 60, it is called the fully rough flow turbulent regime.

Here, what is ϵ_s ? It is equivalent sand roughness, sand grain roughness. This is nothing new. u^* is, you know, u^* is $\sqrt{\frac{\tau_o}{\rho}}$, shear velocity, and ν is the kinematic viscosity. For pipe

flow, Darcy's Weisbach equation can be used: h_f is equal to $\frac{fLV^2}{2gD}$. For a smooth pipe, f is

found to be a function of the Reynolds number $\frac{\rho VD}{\nu}$.

So, this formula is used for rough turbulent flows. f is a function of relative roughness $\frac{\varepsilon_s}{D}$. In pipe flow in hydraulics, you can go and revise my NPTEL course on hydraulics and especially take up that pipe flow. You will understand what the roughness element is, what the type of roughness is, and that it is independent of, you know, other factors. These are the parameters. So, importantly, for rough turbulent flows, when the flow is fully turbulent, f is a function of roughness alone and will not depend on the Reynolds number. In the transition regime, both the Reynolds number and relative roughness play an important role. For the smooth case only, so the smooth case, only the Reynolds number; for rough turbulent, only $\frac{\varepsilon_s}{D}$; for transition, both. So, the roughness magnitude for commercial pipes is expressed as equivalent sand grain roughness ε_s . The extensive experimental investigation of pipe flow has yielded the following generally accepted relations for the variation of f in various regimes of flow. So, we are covering this because, you know, many times, what is going to happen is that pipes are not fully filled, and it will also comprise a part of open channel flow. So, the formulas, the Blasius formulas, the Karman-Prandtl equation, these are all based on extensive experimental investigations.

And these are the most widely accepted. For a smooth wall where the Reynolds number is less than 10^5 , Darcy's friction factor f , this formula can be used. For a smooth wall, So, for a smooth wall and f , when the Reynolds number is greater than this, we can use the Karman-Prandtl equation: $\frac{1}{f^{0.5}} = 2 \log \text{Re} (f^{0.5}) - 0.8$. Now, for rough boundaries, rough boundaries fully turbulent, with Reynolds number greater than 10^5 , we use the Karman-Prandtl equation again, which is $\frac{1}{f^{0.5}} = -2 \log \frac{\varepsilon_s}{D} - 1.14$.

And for the transition zone, we can use the Colebrook-White equation, which depends on both depth $\frac{D}{\varepsilon_s}$ and f . You see, for rough boundaries, it depends only on $\frac{\varepsilon_s}{D}$. Here, it depends on both, and in this one, it depends only on the Reynolds number for smooth walls. This is an important thing to note. It is usual to show the variation of f with Re and $\frac{\varepsilon_s}{D}$ by

a 3-parameter graph known as the Moody chart. So, for this, refer to the hydraulic engineering course in NPTEL, alright, yeah. So, studies on non-circular conduits, such as rectangular, oval, and triangular shapes, have shown that by introducing the hydraulic radius R , the formula developed for pipes is applicable for non-circular ducts also. And the reason for studying this formula in open channel flow is also that it has been shown that if we adopt the value of diameter correspondingly, which we have seen before as well, we are also able to use these formulas for other cases as well. Since, for a circular shape, $D/4$, if we replace D by $4R$, equations can be used for any duct shape provided the conduit areas are close enough to the area of a subcircular circle or semicircle.

So, if we replace D by $4R$, we should be able to use the formula of a pipe for open channel flow. Now, as I said, we are going to apply the formula developed for pipe flow to open channels as well. So, for the purpose of flow resistance, which essentially takes place in the thin layer adjacent to the wall, an open channel can be considered to be a conduit cut into two. The hydraulic radius would then be the appropriate length parameter.

So, what would be the length parameter, hydraulic radius, instead of the diameter of the pipe? The appropriate length parameter and prediction of the friction factor f can be done by using the above equations, the equations that we showed. It should be remembered that the Reynolds number is $\frac{4RV}{\nu}$ and the relative roughness is $\frac{\epsilon_s}{D}$; we use $\frac{\epsilon_s}{4R}$.

Therefore, the Darcy-Weisbach equation, it was $\frac{fLV^2}{2gD}$. So, instead of D , we write $4R$,

which will result in this particular equation. Now, if we rearrange and try to find out V , we can get V is equal to $\sqrt{\frac{8g}{f}} \sqrt{R} \sqrt{\frac{h_f}{L}}$. Now, noting that for uniform flow in an open channel,

$\frac{h_f}{L}$ is equal to the slope of the energy line, that is, S_f is equal to S_o . It may be seen that the

previous equation is the same as Chezy, you see $C\sqrt{R}\sqrt{\frac{h_f}{L}}$. It was related to the non-dimensional parameter; this was a slope. So, if we see, we can write C is equal to $\sqrt{\frac{8g}{f}}$ for convenience of use. The above equation, along with the equations, can be used to prefer a modified Moody chart showing the variation of C with.

So, Moody's chart has the Reynolds number and $\frac{\epsilon_s}{D}$ or $\frac{D}{\epsilon_s}$. So, we can try to prepare a new Moody's chart for an open channel by using D is equal to $4R$ and do different experiments. If f is to be calculated by using one of the previous equations, it is inconvenient to use, as f is involved on both sides of the equations. So, simplified empirical forms, which are accurate enough for all practical purposes, are given by Jain as follows. So, we, I mean, we said, I mean, it is, most of these equations are, you know, we, they have, they have f on both sides of the equation. So, it is not so easy to solve.

Therefore, we are even going to solve equations like this. For a smooth wall, we can use this particular equation: $\frac{1}{\sqrt{f}}$ is equal to $1.8 \log Re - 1.5146$. For the transition zone, we can simply use, you know, f is only, you see, f is. So, in both of these equations, f is not on both sides of the equation. And these two equations are very useful for obtaining explicit solutions of many flows.

The earlier equations were implicit in nature where f was present on both the left-hand side and the right-hand side. Generally, the open channels that are encountered in the field are very large in size and also in the magnitude of roughness elements. Consequently, high Reynolds numbers and large relative roughness are operative, with the result that most open channels have a rough turbulent flow regime. So, in principle, most of the flows in nature, especially the open channel flows that occur in open regions like rivers, streams, and canals, will be technically rough turbulent. Flow regime.

So, mostly depending, you can use this equation as well for transition, meaning it could be used for either of those, whether it is a smooth wall or, I mean, whether it is a lower

Reynolds number or a higher Reynolds number. So, due to the paucity of reliable experimental or field data on channels covering a wide range of parameter values of ϵ_s . Are not available to the same degree of confidence as for pipe material. So, we are talking about open channel flow. So, ϵ_s for open channel flow is not that readily available.

However, the table can be used to estimate the value of ϵ_s . For some common open channel surfaces. So, people have done many experiments and estimated that these could be the values: for glass, it is $3 * 10^{-4}$, very, very little, very, very smooth; concrete surface, gas sewer pipe. You know, as it becomes more rough, ϵ_s increases in size. See, rubble masonry is 6, untreated granite is 10, 3 to 10, earth channels, rough concrete, you know, smooth concrete is points, and rough is.

So, the rougher the element, the higher the ϵ_s value in millimeters. Now, after this particular concept, we are going to start with the computation of normal depth. So, normal depth is mostly calculated using Chezy's equation or Manning's equation. So, Manning's equation for uniform flow in terms of discharge can be written as very simple: $Q = \frac{1}{n} AR^{2/3} S_b^{1/2}$ in the above equation. A and R are functions of the flow depth y and channel cross-section.

This is wetted area, hydraulic radius, bed slope. For a given channel section and specified bottom slope, only one discharge is possible for a given normal depth. So, if we have a fixed channel cross-section and bottom slope, it implies only one discharge. Per depth or conversely, for one normal depth, there will be only one possible discharge, not unlike cubic equations.

If the values of the normal depth are known, discharge can be computed directly from Manning's equation. Simply, if the normal depth is known, then there is no case of trial and error. We may rewrite Manning's equation in terms of discharge. So, if we want to rewrite Manning's equation in terms of discharge, we can write $K Q$ is equal to $K S_b^{0.5}$, where K is the conveyance factor for the channel section and is given by a very simple K , which is nothing but $\frac{1}{n} AR^{2/3}$. So, what is the conveyance factor?

You should understand. Also, from Manning's equation, $AR^{(2/3)}$ can also be written as $\frac{nQ}{S_b^{1/2}}$

. And this K is a function of the normal depth, properties of the channel section, and Manning's coefficient. And this is nothing but a section factor. So, we have studied two terms: conveyance factor and section factor.

The section factor is $AR^{(2/3)}$, and when this section factor is multiplied by n , it becomes the conveyance factor, or $K \times n$ is nothing but the section factor. So, the design curves for the computation of normal depth are based on expressions of the section factor, this section factor, which is quite important: $\frac{nQ}{S_b}$. So, whatever the design curves are for the

computation of normal depth, they are based on expressions of the section factor like this. You see, if you recall the critical, we had a similar thing where we had something for and then, triangle, not triangle, circle, rectangular, and also trapezoid, you know, so here also.

So, based on the section factor, you see $\frac{AR^{2/3}}{B_o^{8/3}}$ against values of $\frac{y}{B_o}$. So, similar to critical depth design curves, we have design curves for normal depth as well. Something like this. So, for a given Q , if we have no discharge n and S_b , then we can calculate $AR^{(2/3)}$.

You see $AR^{(2/3)}$ depends on n , Q , and $S_b^{0.5}$. So, we can calculate $AR^{(2/3)}$. This value is divided by either $B_o^{8/3}$ for trapezoidal sections or D_o for circular sections. The resulting value is $\frac{AR^{2/3}}{B_o^{8/3}}$, or in the case of circular, then we find out this value, which will lie somewhere here. Let us say it lies somewhere here, then we draw a straight line, and let us say circular

And if it is trapezoidal or something, we just draw a line here. And therefore, we find out these values, which will be equal to y_o by $\frac{y}{B_o}$ or $\frac{y}{D_o}$, depending on whether it is a trapezoidal section or a circular section. Then the value of, as I said, y by $\frac{y_n}{B_o}$ or $\frac{y_n}{D_o}$ is obtained directly from the design curves corresponding to the already computed values of

these values. So, I think I will end today's lecture here, and in the next lecture, we will start with the solution of the design curve problem about the computation of the normal depth. Thank you so much.

See you in the next lecture.