Free Surface Flow
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Lecture 16

Welcome students to the fourth module of our free surface flow course, and today we are going to start with the introduction of uniform flow. So, the important thing to understand is what is uniform flow. So, a flow is said to be uniform when its properties do not change with distance. You remember in our first module, when we were having the introduction section, we talked about steady flow, unsteady flow, uniform and non-uniform. So, if you remember from that particular class, uniform means properties do not change in space.

So, where in reality practically it is to be found out, it essentially occurs in prismatic channels. And what is a prismatic channel, if you try to recall from your lectures, it has fixed cross-section areas. So, a flow in an open channel is said to be uniform if, practically speaking, any properties do not change, but in an open channel, we say that the depth is also a flow property, the depth of the flow also does not change and it remains the same at every section of the channel. These are important things that I am underlining. And what is this constant depth called?

This constant depth is called normal depth. So, this entire chapter is dedicated to finding this particular value of normal depth given by y_n. So, the flow velocity V, the discharge, and the wetted area are also constant in this channel, as we said all the properties. So, properties like velocity, discharge Q, and wetted area are also constant in the channel reach. So, this is a sample figure, this is nothing, this is just the bed.

This is the riverbed, and we are seeing the water surface flowing like this, you know, this is the water surface. So, this is the water surface. And this is one section that we study, and this is another section that we study. And we assume that the velocity is V_1 and discharge Q_1 at section 1, z_1 and z_2 are the datum of the bed at these two sections. y_1 is the water depth; this is the velocity head, and this is the corresponding velocity head here. This is the corresponding water depth here, and this is the datum, and this is the energy loss due to the friction that has happened. This is a general figure, nothing particularly related to the uniform flow. But if we assume uniform flow characteristics in the channel reach, what are the things that we can write? We can write y_1 is equal to, so this y_1 and this y_2 should be equal, and this should be equal to y_1 is equal to y_2 is equal to y_n, this normal depth. We can write this V_1

So, this V_1 and this V_2 here will also be equal to the same, I mean, they will both exactly be the same, and we can call it V. This discharge Q_1 and this discharge Q_2 will also be the same for uniform flow, and similar will be the area as well for both cross sections. So, again, since y, V, and Q are constant at any section along the flow direction, we can always write $\frac{dy}{dx}$ is equal to 0, $\frac{dV}{dx}$ is equal to 0, and $\frac{dQ}{dx}$ is also equal to 0, since these are constant and do not change with distance. And the slope of the channel bed, S_b, is equal to the water surface slope, S_w. So, these are the features of uniform flow. So, you see this last line that we have seen: the slope of the channel bed, S_b, is also equal to the water surface slope, S_w.

So, how are we the next problem is dedicated to. Now, show that the slope of the energy line S_f is equal to the bed slope in uniform channel flow. This is what we need to prove. So, you see first we draw this particular figure, I will just repeat again, this is the section 1 and 2 which are separated by a distance dx, datum at section 1, z_1, water depth y_1 and this is velocity head. Similarly, at section 2, z_2 depth y_2 and this is the so this should be actually $\frac{V_2^2}{2g}$, at anyways, since it is uniform flow we can write $\frac{V_1^2}{2g}$ and this is this loss this is and this is the normal depth right.

So, if we apply Bernoulli section at 1-1 and 2-2, we can write $z_1+\frac{V_1^2}{2g}$, $z+\frac{V_1^2}{2g}+\ y_1$ is equal to $z_2+\frac{V_2^2}{2g}+y_2+h_f$. So, these are standard terms and this is the loss energy. or head loss due to friction. But for uniform flow, we know y_1 is equal to y_2 is equal to y_n and V_1 is equal to V_2 is equal to V. How is this our equation number 1 coming out to be this particular equation? z_1 - z_2 is equal to h_f because here this so this is not same but $\frac{V_1^2}{2g}$ and this $\frac{V_2^2}{2g}$ is same y_1 and y_2 are the same because of uniform flow condition. So, this goes away, this goes away, this goes away, this goes away.

So, z_1 minus z_2 is equal to h_f. or instead of z we can write dz, h_f is equal to h_f, z_1 minus z_1 is nothing but dz like x_1 minus x_2 is dx. Now, if we divide both sides by, so dz is equal to h_f and if we divide both sides by dx. we can write dz dx is equal to h_f/dx and what is $\frac{dz}{dx}$? It is nothing but the s_b bed slope and this is $\frac{h_f}{dx}$ is the energy slope.

And this is S_f, and this is what we needed to prove. Now, after a very simple thing, we are going to see the shear stress distribution in uniform flow. So, if we recall the momentum equation for flows in an open channel, we see it is given by this diagram, which we need to understand. This is the bed. Channel bed, this is the bed slope angle. These are F_1 and F_2, the hydrostatic pressure forces.

This is F_4, which is nothing, and W is the weight. F_4 is the component of weight. In the x-direction, not in the x-direction actually, in the flow direction. This entire section is called the control volume, denoted by a dash sign. So, now let me erase this one because

So, the equation is F_1 minus F_2. So, one important term that I missed was F_3. F_3 is nothing but In layman's terms, frictional force or more scientifically, shear force. This is basically water friction due to the bed.

So, $F_1 – F_2$: F_1 is in the positive direction in this direction, F_2 is in the opposite direction, F_3 is the shear force + F_4, that weight component is supporting the flow. As equal to the rate of change in momentum, the rate of change of momentum. This is nothing but Newton's second law. So, the assumption of uniform flow ensures the equality of pressure forces F_1 and F_2. So, in a uniform flow, F_1 and F_2 will be the same because the depth and everything is the same, and the momentum fluxes M_1 and M_2 at sections 1 and 2.

So, both forces and momentum fluxes are going to be the same. Thus, the momentum equations become F_4 is equal to F_3. What is F_4 and what is F_3? The weight component and F_3 is the shear stress. So, what is F_4?

That is *W* sin θ, sorry, F_4 is $W \sin \theta$, and F_3 is F_f, that is the shear force at the boundary. So, if we consider the channel reach of length *L* here, this one shown above, and say if to is the average shear stress. So, what we are doing now is we are trying to further simplify this particular equation, F_4 is equal to F_3, or in simple words, $W \sin \theta$ is equal to *F*. So, consider the channel reach of length *L* as shown above this one here. And if τo is the average shear stress on the wetted perimeter, then shear stress into the wetted perimeter *P* into the length of the reach.

So, force is equal to shear stress multiplied by area. We also know that weight can be written as γ, which is the unit weight of water multiplied by area multiplied by length. Weight multiplied by volume and sin θ can be written as bed slope for small angles of θ. Gamma is nothing but the unit weight of water, and *A* is the wetted area. Thus, the momentum equation can be written as γALS_b equals $\tau_o\ P \times L$.

If we rearrange, γALS_b equals τo $P \times L$ or τ_o. So, if you keep this one here and bring everything else down, it will be $\gamma ALS_b/PL$. This is *L*, *L* cancels, and area by perimeter is nothing but So, τ_o will become γRS_b, and this is the expression of average shear stress on the channel boundary. This is one of the very important equations. And another thing, as I told you before, the hydraulic radius *R* is equal to *A/P*. We are mostly going to work with these terms: hydraulic radius, area, and perimeter. Wetted area and wetted perimeter.

So, this is important. So, there is a parameter called shear velocity u_*, a new term which is critical for turbulent flow analysis and is obtained from average bed shear stress. So, this u_* is found from this average bed shear stress, and the formula is u_* equals $\sqrt{\frac{\tau o}{\rho}}$. Quite an important value, shear velocity. Now, we will see the equations of average velocity.

So, in uniform flow, the average flow velocity *V* is expressed approximately as This is the normal form; this is the general form: *V* is equal to *CR* raised to the power *x*, S_f raised to the power *y*, where *C* is a constant, *R* is the hydraulic radius, and S_f is the slope of the energy slope line. Now, C is the sum coefficient dependent on channel roughness and flow parameters like viscosity and surface. I say it is a constant; it is constant. But, on what parameters does it depend?

It depends on channel roughness and flow parameters like viscosity and surface tension, mainly these three things. R is the hydraulic radius; hydraulic radius is what? It is the ratio of the wetted area to the wetted perimeter. And S_f is the energy slope, but in the case of uniform flow, S_f is equal to S_p is equal to S_w. This we have already shown in one of the first problems of this lecture. We derived this condition.

Hence, the uniform flow formula can be conveniently written as V is equal to $CR^x S_b^{\ y}$. And the most important practical formula for uniform flow in open channels was named after a French engineer, Antoine Chezy. And also Robert Manning's. And these two important formulas are V is equal to $C\sqrt{RS_p}$. Here x for Chezy's equation, R for Chezy's equation, x is 0.5 and y is also 0.5. And for Manning's equation, x is 2/3 and y is 1/2. This is Chezy's formula, and this is Manning's formula. C is Chezy's coefficient, and n is Manning's coefficient. This is Chezy's formula, and therefore, C is Chezy's coefficient. This is Manning's formula, and therefore, n is Manning's coefficient. The values of C and n for different surfaces with different roughness are determined by experiments.

So, scientists have conducted several experiments, numerous ones, and they came up with the values of C for different surfaces and the same for n. Still, there are experiments being revised, and these values are also being changed. An important thing to note is that C and n are not dimensionless, and their dimensions are not $(L^{0.5})(T^{-1})$ and n is $(L^{-1/3})(T^{1})$. How do we find this out? It is very simple; I am just telling you.

Let us see; I am just letting you know. See, we write down velocity as $L(T^{-1})$. Let us see for Chezy, and we need to find out the dimension of this. $R^{0.5}$, so $L^{0.5}$, and the bed has no unit; it is a slope, so no unit. So, C will be $(L^{0.5})(T^{-1})$, you see.

This is what we find out, and similarly, you can find $L(T^{-1})$ is equal to $1/n$. This is $(L^{2/3})$, right. So, n will be $(L^{2/3})/L$. This is T. So, it becomes $(L^{-1/3})(T)$. So, this is how easily we can derive the units, I mean the dimensions, of C and n. If we see, this was for open channel flow, but for pipe flow, there is a very famous equation called Darcy-Weisbach equation. And that equation, what it shows, is it gives us a relationship between head loss.

and the velocity head. $\frac{V^2}{2g}$, it gives us a relationship between head loss and velocity head. So, according to some hydraulicians, Darcy's friction coefficient f can be, so this coefficient *f* can be satisfactorily used for open channel flow. When the pipe diameter *D* is replaced by 4 times the hydraulic radius in the above equation. So, what it says is this Darcy's equation can also be used for open channel.

How? It says replace *D* by 4*R*, where *R* is hydraulic radius. This is what it says. So, for a fully developed pipe flow, the hydraulic radius is one fourth of the diameter. Very simple, it can be said alternatively as well. So, *R* is equal to, I mean, just to show for a pipe flow, *R* is $\frac{A}{P}$, that is $\frac{\pi}{4}D^2$, right, for a circle, for a pipe.

A is $\frac{\pi D^2}{4}$, right, and the perimeter is πD. So, this is one of the reasons to get it down and why we know why. Hence, we can write the following Darcy-Weisbach equation: h_f is equal to this original equation $\frac{fLV^2}{2gD}$. We can write h_f is equal to $\frac{fLV^2}{2gD}$ and *D* has been replaced by *4R*. Or, so we have written h_f is equal to $\frac{fLV^2}{2g(4R)}$, and we send this one and *fL* to this side. Then, we can write V^2 is equal to $\frac{8gRh_f}{fL}$, same like this. Or, $V^2 = \frac{8gRS_f}{f}$ because h_f, you know, $\frac{h_f}{L}$ is nothing but the energy slope line. Or, we can also write *V* is equal to $\sqrt{\frac{8g}{f}}\sqrt{RS_f}$.

So, as I told you, remember S_f is nothing but $\frac{h_f}{L}$. Now, if we compare the above equation with Chezy's and Manning's equation, which equation is this equation? If we compare this with Chezy's and Manning's equation, we find, let us say C is equal to $\frac{1}{n} R^{1/6} = \sqrt{\frac{8g}{f}}$. And this is the relationship between C, n, and f. And these are some typical values of Manning's n for steel, for cast iron, you know, non-metals. You do not need to remember, but you see the order of magnitude 0.03 is one of the standard things. You see, this is what we use.

This is one of the most common ones. So, I think I will end this lecture here, and from the next lecture, we will start with one of the solved examples and see how the problem, I mean, the concept that we have studied about uniform flow, is implemented. Thank you so much, and I will see you in the next class.